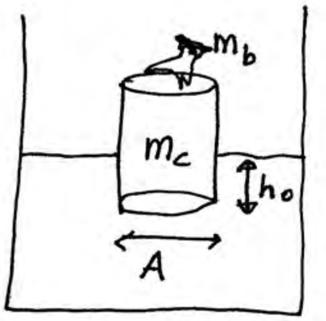
Physics 4B | Final Exam Solutions | Spring 2013

Problem 1 :

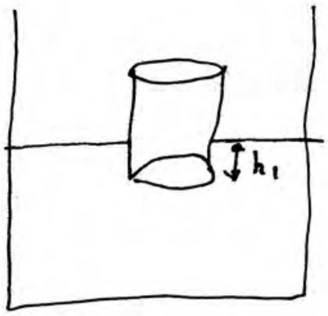


Initially, $F_B = \rho g A h_o = (m_c + m_b) g$

$\rho = 1 g/cm^3$, $A = 200\ cm^2$, $h_o = 5 cm$, $g = 9.8\ m/s^2$, $m_b = 300g$

$\Rightarrow m_c = \rho A h_o - m_b = \frac{1g}{cm^3} \times 200\ cm^2 \times 5cm - 300g = 700g$

$\boxed{m_c = \text{mass of cylinder} = 700g}$ (a)



(b) $F_B = \rho g A h_1 = m_c g \Rightarrow$ using $\rho \cdot A = \frac{(m_c + m_b)}{h_o}$

$h_1 = \frac{m_c}{m_c + m_b} h_o = \frac{700}{1000} \times 5cm = 3.5cm$

$\boxed{h_1 = \text{length submerged w/out bird} = 3.5cm}$ (b)

(c) The force due to the bird's weight is $F_b = m_b g$ is (−) force pushing cylinder up when bird is gone

It causes a displacement $x = h_o - h_1 = 1.5cm$

and for any h, the buoyant force is proportional to $h - h_1$

So, $F_b = +kx = +k(h_o - h_1) \Rightarrow k = \frac{m_b g}{h_o - h_1} \Rightarrow$ the frequency of oscillation

is $\omega = \sqrt{\frac{k}{m_c}} = \sqrt{\frac{m_b}{m_c} \frac{g}{h_o - h_1}} = \sqrt{\frac{300g}{700g} \frac{980\ cm}{s^2 \cdot 1.5cm}} = 16.73 s^{-1}$

So, $\boxed{\omega = 16.73\ rad/s}$ (c)

(d) In harmonic oscillation, $x = a \sin\omega t$, $v = \omega a \cos\omega t \Rightarrow$ maximum

speed is $v_{max} = \omega a = \omega(h_o - h_1) = 1.5cm \times 16.73 s^{-1}$

$\Rightarrow \boxed{v_{max} = 25.1 cm/s}$

(e) The kinetic energy gets dissipated as heat, increases the entropy.

$K = \frac{1}{2} m_c v_{max}^2 = \frac{1}{2} \cdot 0.7kg \cdot (0.251 \frac{m}{s})^2 = 0.022 J$

$\Delta S = \frac{Q}{T} = \frac{K}{T}$

$\boxed{\Delta S = \frac{K}{T} = \frac{0.022J}{293K} = 7.5 \times 10^{-5} \frac{J}{K}}$

Problem 2

Sound velocity is $v = \sqrt{\frac{B}{\rho}}$

$B = -V \left.\frac{dP}{dV}\right)_S$ adiabatic bulk modulus. ρ = mass density

Assume we have n moles of ideal gas,

$PV = nRT$ for both monoatomic and diatomic. $N = N_A n$ is # of particles

Mass density $\rho = \frac{m \cdot N}{V} = \frac{m \cdot N_A \cdot n}{V}$. m = mass of atom or molecule
$\Rightarrow m_{diat} = 2\, m_{monoat}$

$\Rightarrow \boxed{\rho_{diat} = 2\, \rho_{monoat}}$

For bulk moduli, use that for adiabatic process

$PV^{\gamma} = \text{const} \Rightarrow P = \frac{\text{const}}{V^{\gamma}} \Rightarrow \left.\frac{dP}{dV}\right)_S = -\gamma \frac{\text{const}}{V^{\gamma+1}} = -\frac{\gamma P}{V}$

$\Rightarrow \boxed{B = \gamma P}$ $\quad \gamma = C_P / C_V = (C_V + R)/C_V$

For monoatomic gas, $C_V = \frac{3}{2} R \Rightarrow \gamma_{monoat} = \frac{5}{3} = \frac{5}{2} / \frac{3}{2}$

For diatomic gas, $C_V = \frac{5}{2} R \Rightarrow \gamma_{diat} = \frac{7}{5} = 7/2 / 5/2$

$\Rightarrow \gamma_{diat} = \frac{7}{5} = \frac{5}{3} \cdot \frac{3}{5} \cdot \frac{7}{5} = \frac{5}{3} \cdot \frac{21}{25} = \frac{21}{25} \gamma_{monoat}$

$\Rightarrow \boxed{B_{diat} = \frac{21}{25} B_{monoat}}$ for the same pressure

So, $v_{diat} = \sqrt{\frac{B_{diat}}{\rho_{diat}}} = \sqrt{\frac{21}{25}} \frac{1}{\sqrt{2}} \sqrt{\frac{B_{monoat}}{\rho_{monoat}}} = \sqrt{\frac{21}{50}}\, v_{monoat}$

$\Rightarrow \boxed{\frac{v_{diat}}{v_{monoat}} = \sqrt{\frac{21}{50}} = 0.648}$

Problem 3

The frequency, wavelength and velocity of the wave are related by $\boxed{v = f\lambda}$

The fundamental frequency is the smallest frequency, corresponds to largest λ.

The allowed wavelengths satisfy $n\lambda_n = 2l$, with $l = 20\text{cm}$ = string length

Largest λ is for $n = 1$, $\boxed{\lambda_1 = 2l = 40\text{ cm}}$

Speed of transverse wave in string: $v = \sqrt{\frac{F_T}{\mu}}$, F_T = tension force
μ = mass / unit length

So: $v = f\lambda \Rightarrow \sqrt{\frac{F_T}{\mu}} = f_1 \cdot 2l \Rightarrow \boxed{F_T = 4 f_1^2 l^2 \mu}$

$A = 0.1\text{ mm}^2$ = crossection of string

mass density: $\mu = \frac{m}{l}$. $m = \rho_{steel} \cdot A \cdot l = 8000 \frac{kg}{m^3} \cdot 0.1 \times 10^{-6} m^2 \times 0.2\, m$

$\Rightarrow m = 1.6 \times 10^{-4}\, kg \Rightarrow \boxed{\mu = 8 \times 10^{-4} \frac{kg}{m}}$

$\Rightarrow F_T = 4 \cdot \frac{1000^2}{s^2} \times (0.2\, m)^2 \times 8 \times 10^{-4} \frac{kg}{m} = 128\text{ N}$

So tension is $\boxed{F_T = 128\text{ N}}$ in upper string

(b) Cooling from 25°C to 0°C changes length according to $\Delta l = \alpha l_0 \Delta T$

$\alpha = 12 \times 10^{-6} / °C$ $\Rightarrow$ $\Delta l = 12 \times 10^{-6} \times 20\text{cm} \times 2\ °C = 0.06\text{ mm}$

So length changes by $\boxed{\Delta l = 0.06\text{ mm}} \Rightarrow \boxed{l' = l - \Delta l = 19.994\text{ cm}}$

(c) To expand it back, we need to apply extra tension:

$F_T' = \frac{\Delta l}{l_0} \cdot A \cdot E$, with E = elastic modulus $= 200 \times 10^9\, N/m^2$

$\Rightarrow F_T' = \frac{0.06 \times \cancel{10^{-3}}\, m}{0.2\, m} \times 0.1 \times \cancel{10^{-6}}\, m^2 \times 200 \times \cancel{10^9} \frac{N}{m^2} = 6\text{ N}$

so extra tension is $\boxed{F_T' = 6\text{ N}}$

(d) Using (a), (b), (c), we conclude that tension in lower string is

$$\boxed{F_T = 128\,N + 6\,N = 134\,N}$$

Since frequency is proportional to $F_T^{1/2}$ we have for lower string

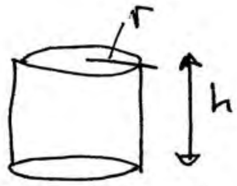
$$\boxed{f_1' = \sqrt{\frac{134}{128}}\, f_1 = 1.023\, f_1 = 1023\,Hz}$$ fundamental frequency

(e) Beat frequency is $f_1' - f_1 = 23\,Hz$

Time difference between amplitude maxima

$$\boxed{T = \frac{1}{f_1' - f_1} = 0.043\,s}$$

Problem 4



$P = \frac{\Delta Q}{\Delta t} = \varepsilon \sigma A T^4$; $\varepsilon = 1$ for black body

A = surface area of cylinder

P = power emitted by radiation

$A = 2\pi r h + 2\pi r^2$; $r = 0.2\,m$, $h = 1.5\,m$

$\Rightarrow \boxed{A = 2.14\,m^2}$ $\quad \sigma = 5.67\times10^{-8}\,\frac{W}{m^2K^4}$. $T = (273+36)\,K = 309\,K$

For 1 day, $\Delta t = 24\times3600\,s$ $\Rightarrow$

$\Delta Q = \sigma A T^4 \Delta t = 5.67\times10^{-8}\times2.14\times309^4\times24\times3600\,J$

$\Rightarrow \boxed{\Delta Q = 9.56\times10^7\,J}$ heat radiated in 1 day. Compensate with hamburgers.

Each hamburger: $q_H = 800\times10^3\times4.168\,J = 3.33\times10^6\,J$

$\Rightarrow$ # of hamburgers per day $\boxed{n_H = \frac{\Delta Q}{q_H} = 28.7}$ (a)

(b) As the person radiates without eating, temperature decreases. $\boxed{\Delta Q = C\cdot\Delta T}$

Heat capacity: $C = 4.168\,\frac{J}{g\,°C}\times70{,}000\,g = \boxed{2.92\times10^5\,\frac{J}{°C}}$

The heat radiated per second is $\boxed{\frac{\Delta Q}{\Delta t} = \sigma A T^4 = 1106.2\,\frac{J}{s}}$

So: $\frac{\Delta Q}{\Delta t} = \frac{C\cdot\Delta T}{\Delta t} = 1106.2\,\frac{J}{s} \Rightarrow \quad \Delta t = \frac{C\cdot\Delta T}{1106.2\,J}\cdot s$; $\Delta T = 2°C \Rightarrow$

$(\Delta T = 36°C - 34°C)$

$$\boxed{\Delta t = \frac{2.92\times10^5\,\frac{J}{°C}}{1106.2\,\frac{J}{s}}\cdot 2°C = 528\,s}$$

This is an approximation because it assumes the heat radiated per second is constant throughout this process, while in fact as T starts to decrease the heat radiated per second also decreases.

(c) If we use the same procedure as in (b) we find, for $\Delta t = 24 \times 3600 s$

$$\Delta T = \frac{\Delta t}{C}\left(\frac{\Delta Q}{\Delta t}\right) = \frac{24 \times 3600 s}{2.92 \times 10^5 J/°C} \cdot 1106.2 \frac{J}{s} = 327.3 K$$

That can't be right, since it would give negative absolute temperature (309 K − 327 K)

Need to integrate differential equation, since heat radiated decreases as T decreases

$$\frac{dQ}{dt} = -\sigma A T^4 \quad , \quad dQ = C dT \Rightarrow$$

$$\Rightarrow C\frac{dT}{dt} = -\sigma A T^4 \Rightarrow \frac{dT}{T^4} = -\frac{\sigma A}{C} dt \Rightarrow \int_{T_0}^{T_1} \frac{dT}{T^4} = -\frac{\sigma A}{C} \Delta t \Rightarrow$$

$$\frac{1}{3}\left(\frac{1}{T_1^3} - \frac{1}{T_0^3}\right) = \frac{\sigma A}{C} \Delta t \Rightarrow \boxed{\frac{1}{T_1^3} = \frac{3\sigma A}{C}\Delta t + \frac{1}{T_0^3}}$$

where $T_0 = 309 K$ is initial temperature, T_1 is final temperature

$$\frac{3\sigma A}{C} = \frac{3 \times 5.67 \times 10^{-8} \times 2.14}{2.92 \times 10^5} = 1.247 \times 10^{-12} s/K^3$$

Using this formula with $\Delta t = 528 s$, gives $T_1 = 307.03 K$ in agreement with (b)

Using this formula with $\Delta t = 24 \times 3600 s$ gives

$$\boxed{T_1 = 191.9 K}$$

Problem 5

N atoms. Total kinetic energy is conserved. Initially,

$$E = \frac{1}{3} N m_{He} \frac{v_x^2}{2} + \frac{2}{3} N m_{He} \frac{v_z^2}{2}$$

with $m_{He} = 6.64 \times 10^{-27}$ kg . $k_B = 1.38 \times 10^{-23}$ J/K
$R = 8.314$ J/mol K

By equipartition, in equilibrium

$$E = \frac{3}{2} N k_B T = \frac{N m_{He}}{3 \cdot 2} (v_x^2 + 2 v_z^2) \Rightarrow$$

$$\boxed{T = \frac{m_{He}}{9 k_B} (v_x^2 + 2 v_z^2) = 43.30\ K} \quad (a)$$

(b) $PV = RT$, $P = \frac{F}{A}$; $V = A \cdot l$, with $l = 2$ m

$$\Rightarrow F = \frac{A \cdot RT}{V} = \frac{RT}{l} = \frac{8.314 \times 43.30}{2} \frac{J}{m} \Rightarrow \boxed{F = 180.02\ N}$$

(c) $$\sqrt{\overline{v^2}} = v_{rms} = \sqrt{\frac{1}{3} v_x^2 + \frac{2}{3} v_z^2} = 519.6 \frac{m}{s}$$

It is $\boxed{\text{the same}}$ initially and after equilibrium is reached (see (a))

(d) Initially, the most probable speed is $\boxed{600\ m/s}$

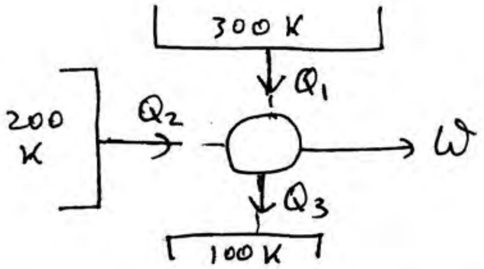
In equilibrium, speed distribution is

$$F(v) = C v^2 e^{-mv^2/2k_BT} \text{, maximum at}$$

$$0 = F'(v_m) = 2 v_m - \frac{2m}{2k_BT} v_m^3 \Rightarrow \boxed{v_m = \sqrt{\frac{2k_BT}{m_{He}}}}$$

$$\Rightarrow \boxed{v_m = 424.2\ m/s}$$ most probable speed in equilibrium

Problem 6



(a) $Q_1 = 90\,J$, $W = 40\,J$, $Q_2 = 30\,J$

$W = Q_1 + Q_2 - Q_3 \Rightarrow \boxed{Q_3 = Q_1 + Q_2 - W = 80\,J}$

Change in entropy of engine in 1 cycle : $\boxed{0}$ (entropy is function of state)

Change in entropy of heat reservoirs:

$$\Delta S = -\frac{Q_1}{T_1} - \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = -\frac{90}{300}\frac{J}{K} - \frac{30}{200}\frac{J}{K} + \frac{80}{100}\frac{J}{K}$$

$\Rightarrow \boxed{\Delta S = 0.35\,J/K}$ this is also the change in entropy of the universe in one cycle.

Since $\Delta S > 0$ and energy is conserved, it is consistent with the laws of thermodynamics.

(b) If it is operating in a reversible way, $\Delta S = 0$

$$-\frac{Q_1}{T_1} - \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = 0$$, and $W = Q_1 + Q_2 - Q_3$; find Q_2, Q_3 :

$$Q_2 = W + Q_3 - Q_1 \Rightarrow -\frac{Q_1}{T_1} - \frac{W}{T_2} - \frac{Q_3}{T_2} + \frac{Q_1}{T_2} + \frac{Q_3}{T_3} = 0 \Rightarrow$$

$$Q_3\left(\frac{1}{T_3} - \frac{1}{T_2}\right) = Q_1\left(\frac{1}{T_1} - \frac{1}{T_2}\right) + \frac{W}{T_2} \Rightarrow$$

$$\boxed{Q_3 = 10\,J, \quad Q_2 = -40\,J}$$

Check : $Q_1 + Q_2 - Q_3 = 90\,J - 40\,J - 10\,J = 40\,J = W$ ✓

$$\Delta S = -\frac{Q_1}{T_1} - \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = -\frac{90}{300} + \frac{40}{200} + \frac{10}{100} = 0$$ ✓

Problem 7

(a) $PV = nRT$ in 1. $T = T_1$

In 3, $\frac{P}{2} \cdot 2V = nRT \Rightarrow \boxed{T_3 = 300K}$. In 2, $P \cdot 2V = 2nRT = nRT_2 \Rightarrow \boxed{T_2 = 600K}$

(b) Change in entropy of the gas between 1 and 3: follow isotherm rather than path in the figure. $\delta Q = PdV$

$$\Delta S = \int_1^3 \frac{PdV}{T} = \int_1^3 \frac{nR}{V} dV = nR \ln 2$$

$$\Rightarrow \boxed{\Delta S = 3 \times 8.314 \frac{J}{K} \cdot \ln 2 = 17.29 \frac{J}{K}}$$

(c) Since it is a reversible process $\Delta S_{univ} = 0$

$$\Rightarrow \boxed{\Delta S_{env} = -\Delta S_{gas} = -17.29 \text{ J/K}}$$ decreased

(d) Heat is not a function of state, so we need to follow given path.

For gas, $\Delta E_{int} = Q - W$. Since $T_1 = T_3 \Rightarrow \Delta E_{int} = 0 \Rightarrow$

$\Rightarrow Q = W$ where Q = heat absorbed by gas
W = work performed by gas. (Only 1→2 part does work)

$$W = W_{12} = P\Delta V = \frac{nRT_1}{V}(2V - V) = nRT_1 = 7483 \text{ J}$$

$\Rightarrow$ $\boxed{\text{heat absorbed by gas} = 7483 \text{ J in going from 1 to 3}}$

Problem 8

A : 3 coins

B : 4 coins

E_A	microstates : #
0	ttt : 1
1	htt, ... : 3
2	hht, ... : 3
3	hhh : 1
total :	8

E_B	microstates
0	tttt : 1
1	httt : 4
2	hhtt : 6
3	hhht : 4
4	hhhh : 1
	total : 16

(b)

The total # of microstates for the entire system if the energy can be anything is $8 \times 16 = \boxed{128 \text{ states}}$

(c) Total number of microstates if total energy is 3 :

$E_A = 0, \quad E_B = 3 \qquad 1 \times 4 = 4$ states

$E_A = 1, \quad E_B = 2 \qquad 3 \times 6 = 18$ states

$E_A = 2, \quad E_B = 1 \qquad 3 \times 4 = 12$ states

$E_A = 3, \quad E_B = 0 \qquad 1 \times 1 = 1$ state

$\boxed{\text{total: } 35 \text{ microstates}}$

(d) The most probable macrostate with $E_{tot} = 3\,J$ is

$\boxed{E_A = 1\,J,\ E_B = 2\,J, \text{ with } 18 \text{ microstates}}$