CHAPTER 17: Temperature, Thermal Expansion, and the Ideal Gas Law

Responses to Questions

2.   Properties of materials that could be exploited in making a thermometer include:

    a.   thermal expansion, both linear and volume

    b.   the proportionality between temperature and pressure (for an ideal gas when volume is

           held constant)

    c.   temperature dependence of resistivity

    d.   frequency of emitted radiation from a heated object (blackbody radiation curve).

    (Note: resistivity and blackbody radiation are defined in Volume II.)

5.   No. We can only infer that the temperature of C is different from that of A and B. We cannot infer
     anything about the relationship of the temperatures of A and B.

9.   Lower. Mercury has a larger coefficient of volume expansion than lead. When the temperature
     rises, mercury will expand more than lead. The density of mercury will decrease more than the
     density of lead will decrease, and so the lead will need to displace more mercury in order to
     balance its weight.

11. If water is added quickly to an overheated engine, it comes into contact with the very hot metal
     parts of the engine. Some areas of the metal parts will cool off very rapidly; others will not. Some of
     the water will quickly turn to steam and will expand rapidly. The net result can be a cracked engine
     block or radiator, due to the thermal stress, and/or the emission of high temperature steam from
     the radiator. Water should always be added slowly, with the engine running. The water will mix
     with the hotter water already in the system, and will circulate through the engine, gradually cooling
     all parts at about the same rate.

15. Slow. On a hot day, the brass rod holding the pendulum will expand and lengthen, increasing the
     length and therefore the period of the pendulum, causing the clock to run slow.

16. Soda is essentially water, and water (unlike most other substances) expands when it freezes. If the can is full while the soda is liquid, then as the soda freezes and expands, it will push on the inside surfaces of the can and the ends will bulge out.

17. The coefficient of volume expansion is much greater for alcohol than for mercury. A given temperature change will therefore result in a greater change in volume for alcohol than for mercury. This means that smaller temperature changes can be measured with an alcohol thermometer.

Solutions to Problems

6. Assume that the temperature and the length are linearly related. The change in temperature per unit length change is as follows.

$$\frac{\Delta T}{\Delta l} = \frac{100.0°C - 0.0°C}{21.85 \text{ cm} - 11.82 \text{ cm}} = 9.970 \text{C°}/\text{cm}$$

Then the temperature corresponding to length L is $T(l) = 0.0°C + (l - 11.82 \text{ cm})(9.970 \text{C°}/\text{cm})$.

(a) $T(18.70 \text{ cm}) = 0.0°C + (18.70 \text{ cm} - 11.82 \text{ cm})(9.970 \text{C°}/\text{cm}) = \boxed{68.6°C}$

(b) $T(14.60 \text{ cm}) = 0.0°C + (14.60 \text{ cm} - 11.82 \text{ cm})(9.970 \text{C°}/\text{cm}) = \boxed{27.7°C}$

7. Take the 300 m height to be the height in January. Then the increase in the height of the tower is given by Eq. 17-1a.

$$\Delta l = \alpha l_0 \Delta T = (12 \times 10^{-6}/\text{C°})(300 \text{ m})(25°C - 2°C) = \boxed{0.08 \text{ m}}$$

10. The increase in length of the rod is given by Eq. 17-1a.

$$\Delta l = \alpha l_0 \Delta T \quad \rightarrow \quad \Delta T = \frac{\Delta l}{\alpha l_0} \quad \rightarrow \quad T_f = T_i + \frac{\Delta l}{\alpha l_0} = 25°C + \frac{0.010}{19 \times 10^{-6}/\text{C°}} = 551.3°C \approx \boxed{550°C}$$

11. The density at $4°C$ is $\rho = \frac{M}{V} = \frac{1.00 \times 10^3 \text{ kg}}{1.00 \text{ m}^3}$. When the water is warmed, the mass will stay the same, but the volume will increase according to Eq. 17-2.

$$\Delta V = \beta V_0 \Delta T = \left(210 \times 10^{-6}/C^\circ \right) \left(1.00\,\mathrm{m}^3 \right) \left(94^\circ C - 4^\circ C \right) = 1.89 \times 10^{-2}\,\mathrm{m}^3$$

The density at the higher temperature is $\rho = \dfrac{M}{V} = \dfrac{1.00 \times 10^3\,\mathrm{kg}}{1.00\,\mathrm{m}^3 + 1.89 \times 10^{-2}\,\mathrm{m}^3} = \boxed{981\,\mathrm{kg/m}^3}$

14. Assume that each dimension of the plate changes according to Eq. 17-1a.

$$\Delta A = A - A_0 = \left(l + \Delta l \right)\left(w + \Delta w \right) - l\,w = l\,w + l\,\Delta w + w\Delta l + \Delta l\,\Delta w - l\,w$$
$$= l\,\Delta w + w\Delta l + \Delta l\,\Delta w$$

Neglect the very small quantity $\Delta l\,\Delta w$.

$$\Delta A = l\,\Delta w + w\Delta l = l\left(\alpha w\Delta T \right) + w\left(\alpha l\,\Delta T \right) = \boxed{2\alpha l\,w\Delta T}$$

15. The change in volume of the aluminum is given by the volume expansion formula, Eq. 17-2.

$$\Delta V = \beta V_0 \Delta T = \left(75 \times 10^{-6}/C^\circ \right) \left(\tfrac{4}{3}\pi \left(\frac{8.75\ \mathrm{cm}}{2} \right)^3 \right) \left(180^\circ C - 30^\circ C \right) = \boxed{3.9\,\mathrm{cm}^3}$$

20. (*a*) When a substance changes temperature, its volume will change by an amount given by

Eq. 17-2. This causes the density to change.

$$\Delta \rho = \rho_f - \rho = \frac{M}{V} - \frac{M}{V_0} = \frac{M}{V_0 + \Delta V} - \frac{M}{V_0} = \frac{M}{V_0 + \beta V_0 \Delta T} - \frac{M}{V_0} = \frac{M}{V_0}\left(\frac{1}{1 + \beta \Delta T} - 1 \right)$$
$$= \rho \left(\frac{1}{1 + \beta \Delta T} - \frac{1 + \beta \Delta T}{1 + \beta \Delta T} \right) = \rho \left(\frac{-\beta \Delta T}{1 + \beta \Delta T} \right)$$

If we assume that $\beta \Delta T = 1$, then the denominator is approximately 1, so
$\boxed{\Delta \rho = -\rho \beta \Delta T}$.

(*b*) The fractional change in density is

$$\frac{\Delta \rho}{\rho} = \frac{-\rho \beta \Delta T}{\rho} = -\beta \Delta T = -\left(87 \times 10^{-6}/^\circ C \right)\left(-55^\circ C - 25^\circ C \right) = 6.96 \times 10^{-3}$$

This is a $\boxed{0.70\%\ \text{increase}}$.

22. (*a*) The original surface area of the sphere is given by $A = 4\pi r^2$. The radius will expand with

temperature according to Eq. 17-1b, $r_{new} = r(1 + \alpha\Delta T)$. The final surface area is $A_{new} = 4\pi r_{new}^2$

, and so the change in area is $\Delta A = A_{new} - A$.

$$\Delta A = A_{new} - A = 4\pi r^2 (1 + \alpha\Delta T)^2 - 4\pi r^2 = 4\pi r^2 \left[(1 + \alpha\Delta T)^2 - 1 \right]$$
$$= 4\pi r^2 \left[1 + 2\alpha\Delta T + \alpha^2 (\Delta T)^2 - 1 \right] = 4\pi r^2 (2\alpha\Delta T) \left[1 + \tfrac{1}{2}\alpha\Delta T \right]$$

If the temperature change is not large, $\tfrac{1}{2}\alpha\Delta T = 1$, and so $\boxed{\Delta A = 8\pi r^2 \alpha\Delta T}$

(*b*) Evaluate the above expression for the solid iron sphere.

$$\Delta A = 8\pi r^2 \alpha\Delta T = 8\pi \left(60.0 \times 10^{-2}\,\mathrm{m} \right)^2 \left(12 \times 10^{-6}/{}^\circ\mathrm{C} \right) \left(275^\circ\mathrm{C} - 15^\circ\mathrm{C} \right) = \boxed{2.8 \times 10^{-2}\,\mathrm{m}^2}$$

25. The thermal stress must compensate for the thermal expansion. E is Young's modulus for the

aluminum.

$$\mathrm{Stress} = F/A = \alpha E \Delta T = \left(25 \times 10^{-6}/\mathrm{C}^\circ \right) \left(70 \times 10^9\,\mathrm{N/m}^2 \right) \left(35^\circ\mathrm{C} - 18^\circ\mathrm{C} \right) = \boxed{3.0 \times 10^7\,\mathrm{N/m}^2}$$

31. Assume the gas is ideal. Since the amount of gas is constant, the value of $\dfrac{PV}{T}$ is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow \quad V_2 = V_1 \frac{P_1}{P_2} \frac{T_2}{T_1} = \left(3.80\,\mathrm{m}^3 \right) \left(\frac{1.00\ \mathrm{atm}}{3.20\ \mathrm{atm}} \right) \frac{(273 + 38.0)\,\mathrm{K}}{273\,\mathrm{K}} = \boxed{1.35\,\mathrm{m}^3}$$

33. Assume the nitrogen is an ideal gas. From Example 17-10, the volume of one mole of nitrogen gas
at STP is $22.4 \times 10^{-3}\,\mathrm{m}^3$. The mass of one mole of nitrogen, with a molecular mass of 28.0 u, is 28.0
grams. Use these values to calculate the density of the oxygen gas.

$$\rho = \frac{M}{V} = \frac{28.0 \times 10^{-3}\,\mathrm{kg}}{22.4 \times 10^{-3}\,\mathrm{m}^3} = \boxed{1.25\,\mathrm{kg/m}^3}$$

37. (a) Assume the nitrogen is an ideal gas. The number of moles of nitrogen is found from the atomic

weight, and then the ideal gas law is used to calculate the volume of the gas.

$$n = (28.5 \text{ kg}) \frac{1 \text{ mole N}_2}{28.01 \times 10^{-3} \text{kg}} = 1017 \text{ mol}$$

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{(1017 \text{ mol})(8.314 \text{ J/molgK})(273 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 22.79 \text{ m}^3 \approx \boxed{22.8 \text{ m}^3}$$

(b) Hold the volume and temperature constant, and again use the ideal gas law.

$$n = (28.5 \text{ kg} + 25.0 \text{ kg}) \frac{1 \text{ mole N}_2}{28.01 \times 10^{-3} \text{kg}} = 1910 \text{ mol}$$

$$PV = nRT \quad \rightarrow$$

$$P = \frac{nRT}{V} = \frac{(1910 \text{ mol})(8.314 \text{ J/molgK})(273 \text{ K})}{22.79 \text{ m}^3} = \boxed{1.90 \times 10^5 \text{ Pa} = 1.88 \text{ atm}}$$

41. We assume that the gas is ideal, that the amount of gas is constant, and that the volume of the gas is constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \rightarrow \quad T_2 = T_1 \frac{P_2}{P_1} = [(273.15 + 20.0) \text{ K}] \left(\frac{2.00 \text{ atm}}{1.00 \text{ atm}} \right) = 586.3 \text{ K} = 313.15^\circ\text{C} \approx \boxed{313^\circ\text{C}}$$

42. Assume that the air is an ideal gas. The pressure and volume are held constant. From the ideal gas law, the value of $\frac{PV}{R} = nT$ is held constant.

$$n_1 T_1 = n_2 T_2 \quad \rightarrow \quad \frac{n_2}{n_1} = \frac{T_1}{T_2} = \frac{(273 + 38) \text{ K}}{(273 + 15) \text{ K}} = \frac{311}{288} = 1.080$$

Thus $\boxed{8.0\%}$ must be removed.

48. From the ideal gas equation, we have $P_1 V_1 = nRT_1$ and $P_2 V_2 = nRT_2$, since the amount of gas is constant. Use these relationships along with the given conditions to find the original pressure and temperature.

$$P_1V_1 = nRT_1 \; ; \; P_2V_2 = nRT_2 \;\; \rightarrow \;\; P_1V_1 - P_2V_2 = nRT_1 - nRT_2 = nR(T_1 - T_2) \;\; \rightarrow$$

$$P_1V_1 - (P_1 + 450\,\text{Pa})V_2 = nR(T_1 - T_2) \;\; \rightarrow \;\; P_1(V_1 - V_2) - V_2(450\,\text{Pa}) = nR(9.0\,\text{K}) \;\; \rightarrow$$

$$P_1 = \frac{nR(9.0\,\text{K}) + V_2(450\,\text{Pa})}{V_1 - V_2} = \frac{(4.0\,\text{mol})(8.314\,\text{J/mol}\cancel{\text{K}})(9.0\,\text{K}) + (0.018\,\text{m}^3)(450\,\text{Pa})}{0.002\,\text{m}^3}$$

$$= 1.537 \times 10^5\,\text{Pa} \approx \boxed{1.5 \times 10^5\,\text{Pa}}$$

$$T_1 = \frac{P_1V_1}{nR} = T_1 = \frac{(1.537 \times 10^5\,\text{Pa})(0.020\,\text{m}^3)}{(4.0\,\text{mol})(8.314\,\text{J/mol}\cancel{\text{K}})} = \boxed{92.4\,\text{K}} \approx -181°\text{C}$$

52.   We assume that the water is at $4°\text{C}$ so that its density is $1000\,\text{kg/m}^3$.

$$1.000\,\text{L}\left(\frac{10^{-3}\,\text{m}^3}{1\,\text{L}}\right)\left(\frac{1000\,\text{kg}}{1\,\text{m}^3}\right)\left(\frac{1\,\text{mol}}{(15.9994 + 2 \times 1.00794) \times 10^{-3}\,\text{kg}}\right) = \boxed{55.51\,\text{mol}}$$

$$55.51\,\text{mol}\left(\frac{6.022 \times 10^{23}\,\text{molecules}}{1\,\text{mol}}\right) = \boxed{3.343 \times 10^{25}\,\text{molecules}}$$

53. We use Eq. 17-4.

$$PV = NkT \;\; \rightarrow \;\; P = \frac{N}{V}kT = \left(\frac{1\,\text{molecule}}{1\,\text{cm}^3}\right)\left(\frac{100^3\,\text{cm}^3}{\text{m}^3}\right)\left(1.38 \times 10^{-23}\,\frac{\text{J}}{\text{K}}\right)(3\,\text{K}) = \boxed{4 \times 10^{-17}\,\text{Pa}}$$

55.   Assume the gas is ideal at those low pressures, and use the ideal gas law.

$$PV = NkT \;\; \rightarrow \;\; \frac{N}{V} = \frac{P}{kT} = \frac{1 \times 10^{-12}\,\text{N/m}^2}{(1.38 \times 10^{-23}\,\text{J/K})(273\,\text{K})} = \left(3 \times 10^8\,\frac{\text{molecules}}{\text{m}^3}\right)\left(\frac{10^{-6}\,\text{m}^3}{1\,\text{cm}^3}\right)$$

$$= \boxed{300\,\text{molecules/cm}^3}$$

56. We assume an ideal gas at STP. Example 17-10 shows that the molar volume of this gas is 22.4 L.
We calculate the actual volume of one mole of gas particles, assuming a volume of l_0^3, and then
find the ratio of the actual volume of the particles to the volume of the gas.

$$\frac{V_{\text{molecules}}}{V_{\text{gas}}} = \frac{(6.02 \times 10^{23}\,\text{molecules})\left((3.0 \times 10^{-10}\,\text{m})^3/\text{molecule}\right)}{(22.4\,\text{L})(1 \times 10^{-3}\,\text{m}^3/1\,\text{L})} = \boxed{7.3 \times 10^{-4}}$$

The molecules take up less than 0.1% of the volume of the gas.