CHAPTER 14: Oscillations

Responses to Questions

2. The acceleration of a simple harmonic oscillator is momentarily zero as the mass passes through the equilibrium point. At this point, there is no force on the mass and therefore no acceleration.

5. The maximum speed of a simple harmonic oscillator is given by $v = A\sqrt{\frac{k}{m}}$. The maximum speed can be doubled by doubling the amplitude, *A*.

8. The tire swing is a good approximation of a simple pendulum. Pull the tire back a short distance and release it, so that it oscillates as a pendulum in simple harmonic motion with a small amplitude. Measure the period of the oscillations and calculate the length of the pendulum from the expression $T = 2\pi\sqrt{\frac{l}{g}}$. The length, l, is the distance from the center of the tire to the branch. The height of the branch is l plus the height of the center of the tire above the ground.

11. The two masses reach the equilibrium point simultaneously. The angular frequency is independent of amplitude and will be the same for both systems.

14. When you rise to a standing position, you raise your center of mass and effectively shorten the length of the swing. The period of the swing will decrease.

Solutions to Problems

3. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(68\text{ kg})(9.80\text{ m/s}^2)}{5.0\times10^{-3}\text{m}} = 1.333\times10^5\text{ N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.333\times10^5\text{ N/m}}{1568\text{ kg}}} = 1.467\text{ Hz} \approx \boxed{1.5\text{ Hz}}$$

4. (*a*) The motion starts at the maximum extension, and so is a cosine. The amplitude is the displacement at the start of the motion.

$$x = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T}t\right) = (8.8\,\text{cm})\cos\left(\frac{2\pi}{0.66}t\right) = (8.8\,\text{cm})\cos(9.520t)$$

$$\approx \boxed{(8.8\,\text{cm})\cos(9.5t)}$$

(*b*) Evaluate the position function at *t* = 1.8 s.

$$x = (8.8\,\text{cm})\cos\left(9.520\text{s}^{-1}(1.8\,\text{s})\right) = -1.252\,\text{cm} \approx \boxed{-1.3\,\text{cm}}$$

5. The period is 2.0 seconds, and the mass is 35 kg. The spring constant can be calculated from Eq. 14-7b.

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow T^2 = 4\pi^2\frac{m}{k} \rightarrow k = 4\pi^2\frac{m}{T^2} = 4\pi^2\frac{35\,\text{kg}}{(2.0\,\text{s})^2} = \boxed{350\,\text{N/m}}$$

10. The spring constant is the same regardless of what mass is attached to the spring.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{k}{4\pi^2} = mf^2 = \text{constant} \rightarrow m_1 f_1^2 = m_2 f_1^2 \rightarrow$$

$$(m\ \text{kg})(0.83\,\text{Hz})^2 = (m\ \text{kg} + 0.68\ \text{kg})(0.60\ \text{Hz})^2 \rightarrow m = \frac{(0.68\,\text{kg})(0.60\,\text{Hz})^2}{(0.83\,\text{Hz})^2 - (0.60\,\text{Hz})^2} = \boxed{0.74\,\text{kg}}$$

$\boxed{13.}$ (*a*) For A, the amplitude is $A_A = \boxed{2.5\,\text{m}}$. For B, the amplitude is $A_B = \boxed{3.5\,\text{m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_A = \boxed{0.25\,\text{Hz}}$.

For B, the frequency is 1 cycle every 2.0 seconds, so $f_B = \boxed{0.50\,\text{Hz}}$.

(*c*) For C, the period is $T_A = \boxed{4.0\,\text{s}}$. For B, the period is $T_B = \boxed{2.0\,\text{s}}$

(*d*) Object A has a displacement of 0 when $t = 0$, so it is a sine function.

$$x_A = A_A\sin(2\pi f_A t) \rightarrow \boxed{x_A = (2.5\,\text{m})\sin\left(\tfrac{1}{2}\pi t\right)}$$

Object B has a maximum displacement when $t = 0$, so it is a cosine function.

$$x_B = A_B\cos(2\pi f_B t) \rightarrow \boxed{x_B = (3.5\,\text{m})\cos(\pi t)}$$

14. Eq. 14-4 is $x = A\cos(\omega t + \phi)$.

(*a*) If $x(0) = -A$, then $-A = A\cos\phi \rightarrow \phi = \cos^{-1}(-1) \rightarrow \boxed{\phi = \pi}$.

(*b*) If $x(0)=0$, then $0=A\cos\phi \rightarrow \phi=\cos^{-1}(0) \rightarrow \boxed{\phi=\pm\frac{1}{2}\pi}$.

(*c*) If $x(0)=A$, then $A=A\cos\phi \rightarrow \phi=\cos^{-1}(1) \rightarrow \boxed{\phi=0}$.

(*d*) If $x(0)=\frac{1}{2}A$, then $\frac{1}{2}A=A\cos\phi \rightarrow \phi=\cos^{-1}\left(\frac{1}{2}\right) \rightarrow \boxed{\phi=\pm\frac{1}{3}\pi}$.

(*e*) If $x(0)=-\frac{1}{2}A$, then $-\frac{1}{2}A=A\cos\phi \rightarrow \phi=\cos^{-1}\left(-\frac{1}{2}\right) \rightarrow \boxed{\phi=\pm\frac{2}{3}\pi}$.

(*f*) If $x(0)=A/\sqrt{2}$, then $A/\sqrt{2}=A\cos\phi \rightarrow \phi=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \rightarrow \boxed{\phi=\pm\frac{1}{4}\pi}$.

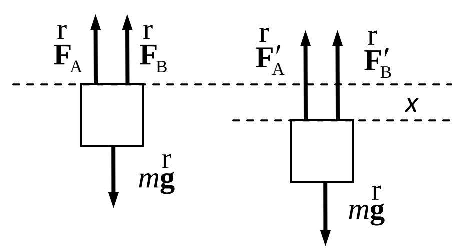
The ambiguity in the answers is due to not knowing the direction of motion at $t = 0$.

19. When the object is at rest, the magnitude of the spring force is equal to the force of gravity. This determines the spring constant. The period can then be found.

$$\sum F_{\text{vertical}} = kx_0 - mg \rightarrow k = \frac{mg}{x_0}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\frac{mg}{x_0}}} = 2\pi\sqrt{\frac{x_0}{g}} = 2\pi\sqrt{\frac{0.14\text{m}}{9.80\,\text{m/s}^2}} = \boxed{0.75\text{s}}$$

24. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's second law for vertical forces, with up as positive, gives this.



$$\sum F_y = F_A + F_B - mg = 0 \rightarrow F_A + F_B = mg$$

Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x<0$. Again write Newton's second law for vertical forces.

$$\sum F_y = F_{net} = F'_A + F'_B - mg = F_A - kx + F_B - kx - mg = -2kx + (F_A + F_B - mg) = -2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows.

$$f = \frac{1}{2\pi}\sqrt{k_{\text{effective}}/m} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

28. (*a*) The total energy is the maximum potential energy.

$$U = \tfrac{1}{2}E \quad \rightarrow \quad \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(\tfrac{1}{2}kA^2\right) \quad \rightarrow \quad \boxed{x = A/\sqrt{2} \approx 0.707A}$$

(b) Now we are given that $x = \tfrac{1}{3}A$.

$$\frac{U}{E} = \frac{\tfrac{1}{2}kx^2}{\tfrac{1}{2}kA^2} = \frac{x^2}{A^2} = \frac{1}{9}$$

Thus the energy is divided up into $\boxed{\tfrac{1}{9}\text{ potential and }\tfrac{8}{9}\text{ kinetic}}$.

31. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{95.0\text{ N}}{0.175\text{ m}} = 542.9\,\text{N/m}$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$E_i = E_f \quad \rightarrow \quad \tfrac{1}{2}kx_{\text{max}}^2 = \tfrac{1}{2}mv_{\text{max}}^2 \quad \rightarrow \quad v_{\text{max}} = x_{\text{max}}\sqrt{\frac{k}{m}} = (0.175\,\text{m})\sqrt{\frac{542.9\,\text{N/m}}{0.160\text{ kg}}} = \boxed{10.2\,\text{m/s}}$$

35. (*a*) The work done in compressing the spring is stored as potential energy. The compressed location corresponds to the maximum potential energy and the amplitude of the ensuing motion.

$$W = \tfrac{1}{2}kA^2 \quad \rightarrow \quad k = \frac{2W}{A^2} = \frac{2(3.6\,\text{J})}{(0.13\,\text{m})^2} = 426\,\text{N/m} \approx \boxed{430\,\text{N/m}}$$

(*b*) The maximum acceleration occurs at the compressed location, where the spring is exerting the maximum force. If the compression distance is positive, then the acceleration is negative.

$$F = -kx = ma \quad \rightarrow \quad m = -\frac{kx}{a} = -\frac{(426\,\text{N/m})(0.13\,\text{m})}{15\,\text{m/s}^2} = \boxed{3.7\,\text{kg}}$$

41. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Mars}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \quad \rightarrow$$

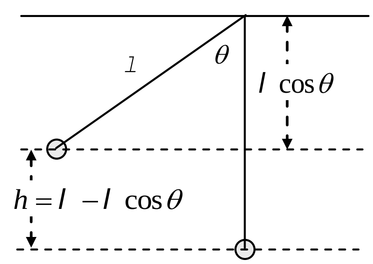
$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (1.35\,\text{s})\sqrt{\frac{1}{0.37}} = \boxed{2.2\,\text{s}}$$

45. If we consider the pendulum as starting from its maximum displacement, then the equation of motion can be written as $\theta = \theta_0 \cos\omega t = \theta_0 \cos\frac{2\pi t}{T}$. Solve for the time for the position to decrease to half the amplitude.

$$\theta_{1/2} = \tfrac{1}{2}\theta_0 = \theta_0 \cos\frac{2\pi t_{1/2}}{T} \quad \rightarrow \quad \frac{2\pi t_{1/2}}{T} = \cos^{-1}\tfrac{1}{2} = \frac{\pi}{3} \quad \rightarrow \quad t_{1/2} = \tfrac{1}{6}T$$

It takes $\frac{1}{6}T$ for the position to change from $+10^\circ$ to $+5^\circ$. It takes $\frac{1}{4}T$ for the position to change from $+10^\circ$ to 0. Thus it takes $\frac{1}{4}T - \frac{1}{6}T = \frac{1}{12}T$ for the position to change from $+5^\circ$ to 0. Due to the symmetric nature of the cosine function, it will also take $\frac{1}{12}T$ for the position to change from 0 to -5°, and so from $+5^\circ$ to -5° takes $\frac{1}{6}T$. The second half of the cycle will be identical to the first, and so the total time spent between $+5^\circ$ and -5° is $\frac{1}{3}T$. So the pendulum spends one-third of its time between $+5^\circ$ and -5°.

47. Use energy conservation to relate the potential energy at the maximum height of the pendulum to the kinetic energy at the lowest point of the swing. Take the lowest point to be the zero location for gravitational potential energy. See the diagram.



$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad K_{\text{top}} + U_{\text{top}} = K_{\text{bottom}} + U_{\text{bottom}} \quad \rightarrow$$

$$0 + mgh = \tfrac{1}{2}mv_{\text{max}}^2 \quad \rightarrow \quad v_{\text{max}} = \sqrt{2gh} = \boxed{\sqrt{2gl\,(1-\cos\theta)}}$$

52. The meter stick used as a pendulum is a physical pendulum. The period is given by Eq. 14-14, $T = 2\pi\sqrt{\frac{I}{mgh}}$. Use the parallel axis theorem to find the moment of inertia about the pin. Express the distances from the center of mass.

$$I = I_{CM} + mh^2 = \tfrac{1}{12}ml^2 + mh^2 \quad \rightarrow \quad T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{12}ml^2 + mh^2}{mgh}} = \frac{2\pi}{\sqrt{g}}\left(\tfrac{1}{12}\frac{l^2}{h} + h\right)^{1/2}$$

$$\frac{dT}{dh} = 2\pi\left(\tfrac{1}{2}\right)\left(\tfrac{1}{12}\frac{l^2}{h} + h\right)^{-1/2}\left(-\tfrac{1}{12}\frac{l^2}{h^2} + 1\right) = 0 \quad \rightarrow \quad h = \sqrt{\tfrac{1}{12}}\,l = 0.2887\,\text{m}$$

$$x = \tfrac{1}{2}l - h = 0.500 - 0.2887 \approx \boxed{0.211\,\text{m}} \text{ from the end}$$

Use the distance for *h* to calculate the period.

$$T = \frac{2\pi}{\sqrt{g}}\left(\tfrac{1}{12}\frac{l^2}{h} + h\right)^{1/2} = \frac{2\pi}{\sqrt{9.80\,\text{m/s}^2}}\left(\tfrac{1}{12}\frac{(1.00\,\text{m})^2}{0.2887\,\text{m}} + 0.2887\,\text{m}\right)^{1/2} = \boxed{1.53\,\text{s}}$$

59. (*a*) The energy of the oscillator is all potential energy when the cosine (or sine) factor is 1, and so

$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}kA_0^2 e^{-\frac{bt}{m}}$. The oscillator is losing 6.0% of its energy per cycle. Use this to find the actual frequency, and then compare to the natural frequency.

$$E(t+T) = 0.94E(t) \quad \rightarrow \quad \tfrac{1}{2}kA_0^2 e^{-\frac{b(t+T)}{m}} = 0.94\left(\tfrac{1}{2}kA_0^2 e^{-\frac{bt}{m}}\right) \quad \rightarrow \quad e^{-\frac{bT}{m}} = 0.94 \quad \rightarrow$$

$$\frac{b}{2m} = -\frac{1}{2T}\ln(0.94) = -\frac{\omega_0}{4\pi}\ln(0.94)$$

$$\frac{f' - f_0}{f_0} = \frac{\frac{1}{2\pi}\sqrt{\omega_0^2 - \frac{b^2}{4m^2}} - \frac{\omega_0}{2\pi}}{\frac{\omega_0}{2\pi}} = \sqrt{1 - \frac{b^2}{4m^2\omega_0^2}} - 1 = \sqrt{1 - \frac{[\ln(0.94)]^2}{16\pi^2}} - 1 \approx 1 - \tfrac{1}{2}\frac{[\ln(0.94)]^2}{16\pi^2} - 1$$

$$= -\tfrac{1}{2}\frac{[\ln(0.94)]^2}{16\pi^2} = -1.2\times10^{-5} \quad \rightarrow \quad \%\text{ diff} = \left(\frac{f' - f_0}{f_0}\right)100 = \boxed{\left(-1.2\times10^{-3}\right)\%}$$

(*b*) The amplitude's decrease in time is given by $A = A_0 e^{-\frac{bt}{2m}}$. Find the decrease at a time of *nT*, and solve for *n*. The value of $\frac{b}{2m}$ was found in part (*a*).

$$A = A_0 e^{-\frac{bt}{2m}} \rightarrow A_0 e^{-1} = A_0 e^{-\frac{bnT}{2m}} \rightarrow 1 = \frac{b}{2m} nT = -\frac{1}{2T}\ln(0.94) nT \rightarrow$$

$$n = -\frac{2}{\ln(0.94)} = 32.32 \approx \boxed{32 \text{ periods}}$$

60. The amplitude of a damped oscillator decreases according to $A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$. The data can be used to find the damping constant.

$$A = A_0 e^{-\frac{bt}{2m}} \rightarrow b = \frac{2m}{t}\ln\left(\frac{A_0}{A}\right) = \frac{2(0.075\,\text{kg})}{(3.5\,\text{s})}\ln\left(\frac{5.0}{2.0}\right) = \boxed{0.039\,\text{kg/s}}$$

$\boxed{67.}$ Apply the resonance condition, $\omega = \omega_0$, to Eq. 14-23, along with the given condition of $A_0 = 23.7\frac{F_0}{m}$. Note that for this condition to be true, the value of 23.7 must have units of s^2.

$$A_0 = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + b^2\omega^2/m^2}} \rightarrow$$

$$A_0(\omega = \omega_0) = \frac{F_0}{m\sqrt{b^2\omega_0^2/m^2}} = \frac{F_0}{m\frac{b\omega_0}{m}} = \frac{F_0}{m\frac{b\omega_0^2}{m\omega_0}} = \frac{F_0}{\frac{m\omega_0^2}{Q}} = Q\frac{F_0}{k} = 23.7\frac{F_0}{k} \rightarrow Q = \boxed{23.7}$$

81. Assume the block has a cross-sectional area of *A*. In the equilibrium position, the net force on the block is zero, and so $F_{\text{buoy}} = mg$. When the block is pushed into the water (downward) an additional distance Δx, there is an increase in the buoyancy force (F_{extra}) equal to the weight of the additional water displaced. The weight of the extra water displaced is the density of water times the volume displaced.

$$F_{\text{extra}} = m_{\substack{\text{add.}\\\text{water}}}\, g = \rho_{\text{water}} V_{\substack{\text{add.}\\\text{water}}}\, g = \rho_{\text{water}} gA\Delta x = \left(\rho_{\text{water}} gA\right)\Delta x$$

This is the net force on the displaced block. Note that if the block is pushed down, the additional force is upwards. And if the block were to be displaced upwards by a distance Δx, the buoyancy force would actually be less than the weight of the block by the amount F_{extra}, and so there would be a net force downwards of magnitude F_{extra}. So in both upward and downward displacement, there is a net force of magnitude $\left(\rho_{\text{water}} gA\right)\Delta x$ but opposite to the direction of displacement. As a vector, we can write the following.

$$\vec{\mathbf{F}}_{\text{net}} = -\left(\rho_{\text{water}} g A\right) \Delta\vec{\mathbf{x}}$$

This is the equation of simple harmonic motion, with a "spring constant" of $\boxed{k = \rho_{\text{water}} g A}$