54. For four heads: $W = 1 \;\rightarrow\; \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 1 = \boxed{0}$

For 3 heads, 1 tail:

$W = 4 \;\rightarrow\; \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 4 = \boxed{1.91\times10^{-23}\text{ J/K}}$

For 2 heads, 2 tails: $W = 6 \;\rightarrow\; \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 6 = \boxed{2.47\times10^{-23}\text{ J/K}}$

For 1 head, 3 tails:

$W = 4 \;\rightarrow\; \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 4 = \boxed{1.91\times10^{-23}\text{ J/K}}$

For four tails: $W = 1 \;\rightarrow\; \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 1 = \boxed{0}$

55. From the table below, we see that there are a total of $2^6 = 64$ microstates.

Macrostate	Possible Microstates (H = heads, T = tails)						Number of Microstates
6 heads, 0 tails	H H H H H H						1
5 heads, 1 tails	H H H H H T	H H H H T H	H H H T H H	H H T H H H	H T H H H H	T H H H H H	6
4 heads, 2 tails	H H H H T T	H H H T H T	H H T H H T	H T H H H T	T H H H H T		15
	H H H T T H	H H T H T H	H T H H T H	T H H H T H	H H T T H H		
	H T H T H H	T H H T H H	H T T H H H	T H T H H H	T T H H H H		
3 heads, 3 tails	H H H T T T	H H T H T T	H T H H T T	T H H H T T	H H T T H T		20
	H T H T H T	T H H T H T	H T T H H T	T H T H H T	T T H H H T		
	T T T H H H	T T H T H H	T H T T H H	H T T T H H	T T H H T H		
	T H T H T H	H T T H T H	T H H T T H	H T H T T H	H H T T T H		
2 heads, 4 tails	T T T T H H	T T T H T H	T T H T T H	T H T T T H	H T T T T H		15
	T T T H H T	T T H T H T	T H T T H T	H T T T H T	T T H H T T		
	T H T H T T	H T T H T T	T H H T T T	H T H T T T	H H T T T T		
1 heads, 5 tails	T T T T T H	T T T T H T	T T T H T T	T T H T T T	T H T T T T	H T T T T T	6
0 heads, 6 tails	T T T T T T						1

(*a*) The probability of obtaining three heads and three tails is $\boxed{20/64}$ or $\boxed{5/16}$.

(*b*) The probability of obtaining six heads is $\boxed{1/64}$.

56. When throwing two dice, there are 36 possible microstates.

(*a*) The possible microstates that give a total of 7 are: (1)(6) , (2)(5) , (3)(4), (4)(3), (5)(2), and

(6)(1). Thus the probability of getting a 7 is $6/36 = \boxed{1/6}$.

(*b*) The possible microstates that give a total of 11 are: (5)(6) and (6)(5). Thus the probability of

getting an 11 is $2/36 = \boxed{1/18}$.

(*c*) The possible microstates that give a total of 4 are: (1)(3) , (2)(2) , and (3)(1). Thus the probability of getting a 5 is $3/36 = \boxed{1/12}$.

57. (*a*) There is only one microstate for 4 tails: TTTT. There are 6 microstates with 2 heads and 2

tails: HHTT, HTHT, HTTH, THHT, THTH, and TTHH. Use Eq. 20-14 to calculate the entropy change.

$$\Delta S = k\ln W_2 - k\ln W_1 = k\ln\frac{W_2}{W_1} = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 6 = \boxed{2.47\times10^{-23}\text{ J/K}}$$

(*b*) Apply Eq. 20-14 again. There is only 1 final microstate, and about 1.0×10^{29} initial microstates.

$$\Delta S = k\ln W_2 - k\ln W_1 = k\ln\frac{W_2}{W_1} = \left(1.38\times10^{-23}\text{ J/K}\right)\ln\left(\frac{1}{1.0\times10^{29}}\right) = \boxed{-9.2\times10^{-22}\text{ J/K}}$$

(*c*) These changes are much smaller than those for ordinary thermodynamic entropy changes.

For ordinary processes, there are many orders of magnitude more particles than we have considered in this problem. That leads to many more microstates and larger entropy values.

58. The number of microstates for macrostate A is $W_{\text{A}} = \dfrac{10!}{10!0!} = 1$. The number of microstates for macrostate B is $W_{\text{B}} = \dfrac{10!}{5!5!} = 252$.

(*a*) $\Delta S = k\ln W_B - k\ln W_A = k\ln\dfrac{W_B}{W_A} = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 252 = \boxed{7.63\times10^{-23}\text{ J/K}}$

Since $\Delta S > 0$, this can occur naturally.

(*b*) $\Delta S = k\ln W_A - k\ln W_B = -k\ln\dfrac{W_B}{W_A} = -\left(1.38\times10^{-23}\text{ J/K}\right)\ln 252 = \boxed{-7.63\times10^{-23}\text{ J/K}}$

Since $\Delta S < 0$, this cannot occur naturally.

60. The required area is $\left(22\dfrac{10^3\text{ W h}}{\text{day}}\right)\left(\dfrac{1\text{ day}}{9\text{ h Sun}}\right)\left(\dfrac{1\text{ m}^2}{40\text{ W}}\right) = 61\text{ m}^2 \approx \boxed{60\text{ m}^2}$. A small house with 1000 ft^2 of floor space, and a roof tilted at 30°, would have a roof area of $\left(1000\text{ ft}^2\right)\left(\dfrac{1}{\cos 30^\circ}\right)\left(\dfrac{1\text{ m}}{3.28\text{ ft}}\right)^2 = 110\text{ m}^2$, which is about twice the area needed, and so the cells would fit on the house. But not all parts of the roof would have 9 hours of sunlight, so more than the minimum number of cells would be needed.

77. We need to find the efficiency in terms of the given parameters, T_{H}, T_{L}, V_{a}, and V_{b}. So we must find the net work done and the heat input to the system. The work done during an isothermal process is given by Eq. 19-8. The work done during an isovolumentric process is 0. We also use the first law of thermodynamics.

ab (isothermal): $\Delta E_{\substack{\text{int}\\\text{ab}}} = 0 = Q_{\text{ab}} - W_{\text{ab}} \rightarrow Q_{\text{ab}} = W_{\text{ab}} = nRT_{\text{H}}\ln\dfrac{V_{\text{b}}}{V_{\text{a}}} > 0$

bc (isovolumetric):

$$\Delta E_{\substack{\text{int}\\\text{bc}}} = Q_{\text{bc}} - 0 \rightarrow Q_{\text{bc}} = \Delta E_{\substack{\text{int}\\\text{bc}}} = nC_V\left(T_{\text{L}} - T_{\text{H}}\right) = \tfrac{3}{2}nR\left(T_{\text{L}} - T_{\text{H}}\right) < 0$$

cd (isothermal):

$$\Delta E_{\substack{\text{int}\\\text{cd}}} = 0 = Q_{\text{cd}} - W_{\text{cd}} \quad \rightarrow \quad Q_{\text{cd}} = W_{\text{cd}} = nRT_{\text{L}} \ln\frac{V_{\text{a}}}{V_{\text{b}}} = -nRT_{\text{L}} \ln\frac{V_{\text{b}}}{V_{\text{a}}} < 0$$

da (isovolumetric):

$$\Delta E_{\substack{\text{int}\\\text{da}}} = Q_{\text{da}} - 0 \quad \rightarrow \quad Q_{\text{da}} = \Delta E_{\substack{\text{int}\\\text{da}}} = nC_V\left(T_{\text{H}} - T_{\text{L}}\right) = \tfrac{3}{2} nR\left(T_{\text{H}} - T_{\text{L}}\right) > 0$$

$$W = W_{\text{ab}} + W_{\text{cd}} = nRT_{\text{H}} \ln\frac{V_{\text{b}}}{V_{\text{a}}} - nRT_{\text{L}} \ln\frac{V_{\text{b}}}{V_{\text{a}}} = nR\left(T_{\text{H}} - T_{\text{L}}\right)\ln\frac{V_{\text{b}}}{V_{\text{a}}}$$

$$Q_{\text{in}} = Q_{\text{ab}} + Q_{\text{da}} = nRT_{\text{H}} \ln\frac{V_{\text{b}}}{V_{\text{a}}} + \tfrac{3}{2} nR\left(T_{\text{H}} - T_{\text{L}}\right)$$

$$e_{\text{Sterling}} = \frac{W}{Q_{\text{in}}} = \frac{\left(T_{\text{H}} - T_{\text{L}}\right)\ln\frac{V_{\text{b}}}{V_{\text{a}}}}{T_{\text{H}} \ln\frac{V_{\text{b}}}{V_{\text{a}}} + \frac{3}{2}\left(T_{\text{H}} - T_{\text{L}}\right)} = \boxed{\left(\frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}}\right)\left[\frac{\ln\frac{V_{\text{b}}}{V_{\text{a}}}}{\ln\frac{V_{\text{b}}}{V_{\text{a}}} + \frac{3}{2}\left(\frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}}\right)}\right]}$$

$$= e_{\text{Carnot}}\left[\frac{\ln\frac{V_{\text{b}}}{V_{\text{a}}}}{\ln\frac{V_{\text{b}}}{V_{\text{a}}} + \frac{3}{2}\left(\frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}}\right)}\right]$$

Since the factor in [ ] above is less than 1, we see that $\boxed{e_{\text{Sterling}} < e_{\text{Carnot}}}$.