Chapter 32

1. We use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -\left(-1\,\text{Wb} + 2\,\text{Wb} - 3\,\text{Wb} + 4\,\text{Wb} - 5\,\text{Wb}\right) = +3\,\text{Wb}\ .$$

2. (a) The flux through the top is +(0.30 T)πr^2 where r = 0.020 m. The flux through the bottom is +0.70 mWb as given in the problem statement. Since the *net* flux must be zero then the flux through the sides must be negative and exactly cancel the total of the previously mentioned fluxes. Thus (in magnitude) the flux though the sides is 1.1 mWb.

(b) The fact that it is negative means it is inward.

3. (a) We use Gauss' law for magnetism: $\oint \vec{B}\cdot d\vec{A} = 0$. Now,

$$\oint \vec{B}\cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C\ ,$$

where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is Φ_1 = –25.0 μWb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. Its value is

$$\Phi_2 = \pi(0.120\,\text{m})^2\left(1.60\times10^{-3}\,\text{T}\right) = +7.24\times10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}\ .$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}\ .$$

Thus, the magnitude is $|\Phi_C| = 47.4\,\mu\text{Wb}$.

(b) The minus sign in Φ_C indicates that the flux is inward through the curved surface.

4. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the xz plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\Phi_B(S_1)=\Phi_B(S_2)=\int_{-r}^{r} B(x)L\,dx = 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx = 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx = \frac{\mu_0 iL}{\pi}\ln 3\,.$$

5. We use the result of part (b) in Sample Problem — "Magnetic field induced by changing electric field,"

$$B=\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}, \qquad (r\ge R)$$

to solve for dE/dt:

$$\frac{dE}{dt}=\frac{2Br}{\mu_0\varepsilon_0 R^2}=\frac{2\left(2.0\times10^{-7}\ \text{T}\right)\left(6.0\times10^{-3}\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.0\times10^{-3}\text{m}\right)^2}=2.4\times10^{13}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

6. The integral of the field along the indicated path is, by Eq. 32-18 and Eq. 32-19, equal to

$$\mu_0 i_d\left(\frac{\text{enclosed area}}{\text{total area}}\right)=\mu_0(0.75\ \text{A})\frac{(4.0\ \text{cm})(2.0\ \text{cm})}{12\ \text{cm}^2}=52\ \text{nT}\cdot\text{m}\,.$$

7. (a) Inside we have (by Eq. 32-16) $B=\mu_0 i_d r_1/2\pi R^2$, where $r_1=0.0200$ m, $R=0.0300$ m, and the displacement current is given by Eq. 32-38 (in SI units):

$$i_d=\varepsilon_0\frac{d\Phi_E}{dt}=(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{-3}\ \text{V/m}\cdot\text{s})=2.66\times10^{-14}\ \text{A}\,.$$

Thus we find

$$B=\frac{\mu_0 i_d r_1}{2\pi R^2}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})(0.0200\ \text{m})}{2\pi(0.0300\ \text{m})^2}=1.18\times10^{-19}\ \text{T}\,.$$

(b) Outside we have (by Eq. 32-17) $B=\mu_0 i_d/2\pi r_2$ where r_2 = 0.0500 cm. Here we obtain

$$B=\frac{\mu_0 i_d}{2\pi r_2}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})}{2\pi(0.0500\ \text{m})}=1.06\times10^{-19}\ \text{T}$$

8. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

$$B(2\pi r)=\varepsilon_0\mu_0\left(0.60\ \text{V}\cdot\text{m/s}\right)\frac{r}{R}\,.$$

Using r = 0.0200 m (which, in any case, cancels out) and R = 0.0300 m, we obtain

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi R} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0300\ \text{m})}$$
$$= 3.54\times10^{-17}\ \text{T}.$$

(b) For a value of r larger than R, we must note that the flux enclosed has already reached its full amount (when $r = R$ in the given flux expression). Referring to the equation we wrote in our solution of part (a), this means that the final fraction (r/R) should be replaced with unity. On the left hand side of that equation, we set r = 0.0500 m and solve. We now find

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi r} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0500\ \text{m})}$$
$$= 2.13\times10^{-17}\ \text{T}.$$

9. (a) Application of Eq. 32-7 with $A = \pi r^2$ (and taking the derivative of the field expression given in the problem) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi r^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

For r = 0.0200 m, this gives

$$B = \frac{1}{2}\varepsilon_0\mu_0 r(0.00450\ \text{V/m}\cdot\text{s})$$
$$= \frac{1}{2}(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.0200\ \text{m})(0.00450\ \text{V/m}\cdot\text{s})$$
$$= 5.01\times10^{-22}\ \text{T}.$$

(b) With $r > R$, the expression above must replaced by

$$B(2\pi r) = \varepsilon_0\mu_0\pi R^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

Substituting r = 0.050 m and R = 0.030 m, we obtain $B = 4.51\times10^{-22}$ T.

10. (a) Here, the enclosed electric flux is found by integrating

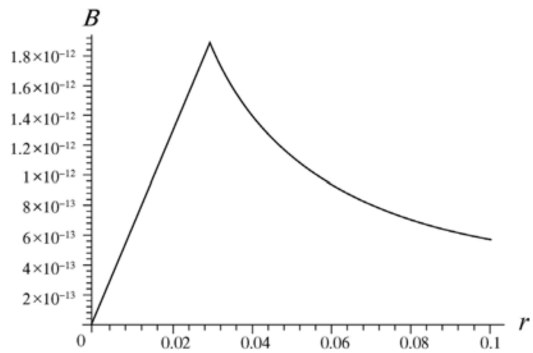
$$\Phi_E = \int_0^r E\ 2\pi r dr = t(0.500\ \text{V/m}\cdot\text{s})(2\pi)\int_0^r\left(1-\frac{r}{R}\right)r dr = t\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Then (after taking the derivative with respect to time) Eq. 32-3 leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right).$$

$$B_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}V_{\max}\omega\cos(\omega t)\right)_{\max}$$
$$= \frac{\mu_0\varepsilon_0 R^2 V_{\max}\omega}{2rd} \quad (\text{for } r \geq R)$$

(note the $B \propto r^{-1}$ dependence — see also Eqs. 32-16 and 32-17). The plot (with SI units understood) is shown below.



12. From Sample Problem — "Magnetic field induced by changing electric field," we know that $B \propto r$ for $r \leq R$ and $B \propto r^{-1}$ for $r \geq R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of $B_{\max}$. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$.

(a) Inside the capacitor, $0.75\, B_{\max}/B_{\max} = r_1/R$, or $r_1 = 0.75\, R = 0.75$ (40 mm) =30 mm.

(b) Outside the capacitor, $0.75\, B_{\max}/B_{\max} = (r_2/R)^{-1}$, or

$$r_2 = R/0.75 = 4R/3 = (4/3)(40 \text{ mm}) = 53 \text{ mm}.$$

(c) From Eqs. 32-15 and 32-17,

$$B_{\max} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(6.0\ \text{A})}{2\pi(0.040\ \text{m})} = 3.0\times10^{-5}\ \text{T}.$$

13. Let the area plate be A and the plate separation be d. We use Eq. 32-10:

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0\frac{d}{dt}(AE) = \varepsilon_0 A\frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right),$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\ \text{A}}{2.0\times10^{-6}\ \text{F}} = 7.5\times10^{5}\ \text{V/s}.$$

Therefore, we need to change the voltage difference across the capacitor at the rate of 7.5×10^5 V/s.

14. Consider an area A, normal to a uniform electric field $\vec{E}$. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation, $i_d = \varepsilon_0 A(dE/dt)$, so

$$J_d = \frac{1}{A}\varepsilon_0 A\frac{dE}{dt} = \varepsilon_0\frac{dE}{dt}.$$

15. The displacement current is given by $i_d = \varepsilon_0 A(dE/dt)$, where A is the area of a plate and E is the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. Thus,

$$i_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}.$$

Now, $\varepsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor (not filled with a dielectric), so

$$i_d = C\frac{dV}{dt}.$$

16. We use Eq. 32-14: $i_d = \varepsilon_0 A(dE/dt)$. Note that, in this situation, A is the area over which a changing electric field is present. In this case $r > R$, so $A = \pi R^2$. Thus,

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{i_d}{\varepsilon_0 \pi R^2} = \frac{2.0\,\text{A}}{\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.10\,\text{m}\right)^2} = 7.2\times10^{12}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

17. (a) Using Eq. 27-10, we find $E = \rho J = \dfrac{\rho i}{A} = \dfrac{\left(1.62\times10^{-8}\,\Omega\cdot\text{m}\right)\left(100\,\text{A}\right)}{5.00\times10^{-6}\,\text{m}^2} = 0.324\,\text{V/m}.$

(b) The displacement current is

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0 A\frac{d}{dt}\left(\frac{\rho i}{A}\right) = \varepsilon_0\rho\frac{di}{dt} = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.62\times10^{-8}\,\Omega\right)\left(2000\,\text{A/s}\right)$$
$$= 2.87\times10^{-16}\,\text{A}.$$

(c) The ratio of fields is $\dfrac{B\left(\text{due to } i_d\right)}{B\left(\text{due to } i\right)} = \dfrac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r} = \dfrac{i_d}{i} = \dfrac{2.87\times10^{-16}\ \text{A}}{100\,\text{A}} = 2.87\times10^{-18}.$

18. From Eq. 28-11, we have $i = (\varepsilon / R)\, e^{-t/\tau}$ since we are ignoring the self-inductance of the capacitor. Equation 32-16 gives

$$B = \frac{\mu_0 i_d r}{2\pi R^2}\,.$$

Furthermore, Eq. 25-9 yields the capacitance

$$C = \frac{\varepsilon_0 \pi (0.05\text{ m})^2}{0.003\text{ m}} = 2.318\times10^{-11}\,\text{F}\,,$$

so that the capacitive time constant is

$$\tau = (20.0 \times 10^6\,\Omega)(2.318 \times 10^{-11}\,\text{F}) = 4.636 \times 10^{-4}\,\text{s}.$$

At $t = 250 \times 10^{-6}$ s, the current is

$$i = \frac{12.0\text{ V}}{20.0 \text{ x } 10^6\,\Omega}\; e^{-t/\tau} = 3.50 \times 10^{-7}\,\text{A}\,.$$

Since $i = i_d$ (see Eq. 32-15) and $r = 0.0300$ m, then (with plate radius $R = 0.0500$ m) we find

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(3.50\times10^{-7}\text{ A})(0.030\text{ m})}{2\pi(0.050\text{ m})^2} = 8.40\times10^{-13}\text{ T}\,.$$

19. (a) Equation 32-16 (with Eq. 26-5) gives, with $A = \pi R^2$,

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{\mu_0 J_d A r}{2\pi R^2} = \frac{\mu_0 J_d (\pi R^2) r}{2\pi R^2} = \frac{1}{2}\mu_0 J_d r$$
$$= \frac{1}{2}(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(6.00\text{ A/m}^2)(0.0200\text{ m}) = 75.4\text{ nT}\,.$$

(b) Similarly, Eq. 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = \dfrac{\mu_0 J_d \pi R^2}{2\pi r} = 67.9\text{ nT}\,.$

20. (a) Equation 32-16 gives $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 2.22\ \mu\text{T}\,.$

(b) Equation 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = 2.00\ \mu\text{T}\,.$

21. (a) Equation 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\text{enc}}$. It is the enclosed portion of the displacement current, and if we related this to the displacement current density J_d, then

$$i_{d\text{ enc}} = \int_0^r J_d\, 2\pi r\, dr = (4.00\text{ A/m}^2)(2\pi)\int_0^r \left(1 - r/R\right) r\, dr = 8\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$