# Chapter 31

1. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If $Q$ is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{\left(2.90\times10^{-6}\,\text{C}\right)^2}{2\left(3.60\times10^{-6}\,\text{F}\right)} = 1.17\times10^{-6}\,\text{J}.$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If $I$ is the maximum current, then $U = LI^2/2$ leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.168\times10^{-6}\,\text{J}\right)}{75\times10^{-3}\,\text{H}}} = 5.58\times10^{-3}\,\text{A}.$$

2. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 31-1. The values of $t$ when plate $A$ will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00\times10^3\,\text{Hz}} = n(5.00\,\mu\text{s}),$$

where $n$ = 1, 2, 3, 4, $\ldots$. The earliest time is ($n$ = 1) $t_A = 5.00\,\mu$s.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps $a$ and $e$ in Fig. 31-1). This is when plate $A$ acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + (n-1)T = \frac{1}{2}(2n-1)T = \frac{(2n-1)}{2f} = \frac{(2n-1)}{2\left(2\times10^3\,\text{Hz}\right)} = (2n-1)(2.50\,\mu\text{s}),$$

where $n$ = 1, 2, 3, 4, $\ldots$. The earliest time is ($n$ = 1) $t = 2.50\,\mu$s.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps $a$ and $c$ in Fig. 31-1). Later this will repeat every half-period (compare steps $c$ and $g$ in Fig. 31-1). Therefore,

$$t_L = \frac{T}{4} + \frac{(n-1)T}{2} = (2n-1)\frac{T}{4} = (2n-1)(1.25\,\mu\text{s})\,,$$

where $n$ = 1, 2, 3, 4, $\ldots$. The earliest time is ($n$ = 1) $t = 1.25\,\mu$s.

3. (a) The period is $T$ = 4(1.50 $\mu$s) = 6.00 $\mu$s.

(b) The frequency is the reciprocal of the period: $f = \frac{1}{T} = \frac{1}{6.00\,\mu\text{s}} = 1.67 \times 10^5\ \text{Hz}$.

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or 3.00 $\mu$s.

4. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{\left(1.60 \times 10^{-6}\,\text{C}\right)^2}{2\left(140 \times 10^{-6}\,\text{J}\right)} = 9.14 \times 10^{-9}\ \text{F}.$$

5. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00 \times 10^{-6}\,\text{C}}{\sqrt{\left(1.10 \times 10^{-3}\,\text{H}\right)\left(4.00 \times 10^{-6}\,\text{F}\right)}} = 4.52 \times 10^{-2}\ \text{A}\,.$$

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\,\text{N}}{\left(2.0 \times 10^{-13}\,\text{m}\right)\left(0.50\,\text{kg}\right)}} = 89\,\text{rad/s}.$$

(b) The period is 1/$f$ and $f = \omega/2\pi$. Therefore,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{89\,\text{rad/s}} = 7.0 \times 10^{-2}\,\text{s}.$$

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{(89\,\text{rad/s})^2(5.0\,\text{H})} = 2.5 \times 10^{-5}\ \text{F}.$$

7. Table 31-1 provides a comparison of energies in the two systems. From the table, we see the following correspondences:

$$x \leftrightarrow q, \quad k \leftrightarrow \frac{1}{C}, \ m \leftrightarrow L, \quad v = \frac{dx}{dt} \leftrightarrow \frac{dq}{dt} = i,$$
$$\frac{1}{2}kx^2 \leftrightarrow \frac{q^2}{2C}, \quad \frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}Li^2.$$

(a) The mass $m$ corresponds to the inductance, so $m$ = 1.25 kg.

(b) The spring constant $k$ corresponds to the reciprocal of the capacitance. Since the total energy is given by $U = Q^2/2C$, where $Q$ is the maximum charge on the capacitor and $C$ is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{\left(175 \times 10^{-6}\ \text{C}\right)^2}{2\left(5.70 \times 10^{-6}\ \text{J}\right)} = 2.69 \times 10^{-3}\ \text{F}$$

and

$$k = \frac{1}{2.69 \times 10^{-3}\ \text{m/N}} = 372\ \text{N/m}.$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\max} = 1.75 \times 10^{-4}$ m.

(d) The maximum speed $v_{\max}$ corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175 \times 10^{-6}\ \text{C}}{\sqrt{(1.25\ \text{H})\left(2.69 \times 10^{-3}\ \text{F}\right)}} = 3.02 \times 10^{-3}\ \text{A}.$$

Consequently, $v_{\max} = 3.02 \times 10^{-3}$ m/s.

8. We apply the loop rule to the entire circuit:

$$\varepsilon_{\text{total}} = \varepsilon_{L_1} + \varepsilon_{C_1} + \varepsilon_{R_1} + \cdots = \sum_j \left(\varepsilon_{L_j} + \varepsilon_{C_j} + \varepsilon_{R_j}\right) = \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j\right) = L\frac{di}{dt} + \frac{q}{C} + iR$$

with

$$L = \sum_j L_j, \quad \frac{1}{C} = \sum_j \frac{1}{C_j}, \quad R = \sum_j R_j$$

and we require $\varepsilon_{\text{total}} = 0$. This is equivalent to the simple *LRC* circuit shown in Fig. 31-26(b).

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}.$ Consequently,

$$t=\frac{T}{4}=\frac{2\pi\sqrt{LC}}{4}=\frac{2\pi\sqrt{(0.050\,\text{H})(4.0\times10^{-6}\,\text{F})}}{4}=7.0\times10^{-4}\ \text{s}.$$

10. We find the inductance from $f=\omega/2\pi=\left(2\pi\sqrt{LC}\right)^{-1}$.

$$L=\frac{1}{4\pi^2 f^2C}=\frac{1}{4\pi^2\left(10\times10^3\ \text{Hz}\right)^2\left(6.7\times10^{-6}\ \text{F}\right)}=3.8\times10^{-5}\ \text{H}.$$

11. (a) Since the frequency of oscillation *f* is related to the inductance *L* and capacitance *C* by $f=1/2\pi\sqrt{LC}$, the smaller value of *C* gives the larger value of *f*. Consequently, $f_{\max}=1/2\pi\sqrt{LC_{\min}}$, $f_{\min}=1/2\pi\sqrt{LC_{\max}}$, and

$$\frac{f_{\max}}{f_{\min}}=\frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}}=\frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}}=6.0.$$

(b) An additional capacitance *C* is chosen so the ratio of the frequencies is

$$r=\frac{1.60\,\text{MHz}}{0.54\,\text{MHz}}=2.96.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If *C* is in picofarads (pF), then

$$\frac{\sqrt{C+365\,\text{pF}}}{\sqrt{C+10\,\text{pF}}}=2.96.$$

The solution for *C* is

$$C=\frac{(365\,\text{pF})-(2.96)^2(10\,\text{pF})}{(2.96)^2-1}=36\,\text{pF}.$$

(c) We solve $f=1/2\pi\sqrt{LC}$ for *L*. For the minimum frequency, *C* = 365 pF + 36 pF = 401 pF and *f* = 0.54 MHz. Thus

$$L=\frac{1}{(2\pi)^2 Cf^2}=\frac{1}{(2\pi)^2\left(401\times10^{-12}\ \text{F}\right)\left(0.54\times10^{6}\ \text{Hz}\right)^2}=2.2\times10^{-4}\ \text{H}.$$

12. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1-75.0\%)=25.0\%$, then

$$\frac{U_E}{U}=\frac{q^2/2C}{Q^2/2C}=25.0\%$$

which leads to $q/Q=\sqrt{0.250}=0.500.$

(b) From

$$\frac{U_B}{U}=\frac{Li^2/2}{LI^2/2}=75.0\%,$$

we find $i/I=\sqrt{0.750}=0.866.$

13. (a) The charge (as a function of time) is given by $q=Q\sin\omega t$, where $Q$ is the maximum charge on the capacitor and $\omega$ is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i=\frac{dq}{dt}=\omega Q\cos\omega t,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega=1/\sqrt{LC},$

$$Q=I\sqrt{LC}=(2.00\,\text{A})\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})}=1.80\times10^{-4}\,\text{C}.$$

(b) The energy stored in the capacitor is given by

$$U_E=\frac{q^2}{2C}=\frac{Q^2\sin^2\omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt}=\frac{Q^2\omega\ \sin\omega t\cos\omega t}{C}$$

We use the trigonometric identity $\cos\omega t\sin\omega t=\frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt}=\frac{\omega Q^2}{2C}\sin(2\omega t).$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2$ rad. This means

$$t=\frac{\pi}{4\omega}=\frac{\pi}{4}\sqrt{LC}=\frac{\pi}{4}\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})}=7.07\times10^{-5}\,\text{s}.$$

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\ \text{H})(2.70\times10^{-6}\ \text{F})} = 5.655\times10^{-4}\,\text{s},$ so

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{\pi\left(1.80\times10^{-4}\ \text{C}\right)^2}{\left(5.655\times10^{-4}\ \text{s}\right)\left(2.70\times10^{-6}\ \text{F}\right)} = 66.7\,\text{W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

14. The capacitors $C_1$ and $C_2$ can be used in four different ways: (1) $C_1$ only; (2) $C_2$ only; (3) $C_1$ and $C_2$ in parallel; and (4) $C_1$ and $C_2$ in series.

(a) The smallest oscillation frequency is

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1+C_2)}} = \frac{1}{2\pi\sqrt{\left(1.0\times10^{-2}\ \text{H}\right)\left(2.0\times10^{-6}\ \text{F}+5.0\times10^{-6}\ \text{F}\right)}} = 6.0\times10^2\ \text{Hz}.$$

(b) The second smallest oscillation frequency is

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{\left(1.0\times10^{-2}\,\text{H}\right)\left(5.0\times10^{-6}\,\text{F}\right)}} = 7.1\times10^2\,\text{Hz}.$$

(c) The second largest oscillation frequency is

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{\left(1.0\times10^{-2}\,\text{H}\right)\left(2.0\times10^{-6}\,\text{F}\right)}} = 1.1\times10^3\,\text{Hz}.$$

(d) The largest oscillation frequency is

$$f_4 = \frac{1}{2\pi\sqrt{LC_1C_2/(C_1+C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0\times10^{-6}\,\text{F}+5.0\times10^{-6}\,\text{F}}{\left(1.0\times10^{-2}\,\text{H}\right)\left(2.0\times10^{-6}\,\text{F}\right)\left(5.0\times10^{-6}\,\text{F}\right)}} = 1.3\times10^3\,\text{Hz}.$$

15. (a) The maximum charge is $Q = CV_{\max} = (1.0\times10^{-9}\ \text{F})(3.0\ \text{V}) = 3.0\times10^{-9}\ \text{C}$.

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0\times10^{-9}\ \text{C}}{\sqrt{(3.0\times10^{-3}\ \text{H})(1.0\times10^{-9}\ \text{F})}} = 1.7\times10^{-3}\ \text{A}.$$

(c) When the current is at a maximum, the magnetic energy is at a maximum also:

$$U_{B,\max} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0\times10^{-3}\ \text{H})(1.7\times10^{-3}\ \text{A})^2 = 4.5\times10^{-9}\ \text{J}.$$

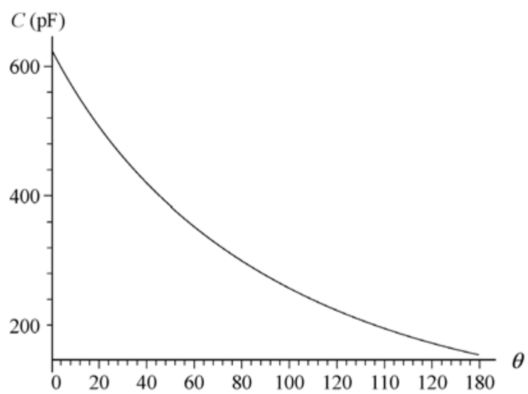
16. The linear relationship between $\theta$ (the knob angle in degrees) and frequency $f$ is

$$f = f_0\left(1+\frac{\theta}{180°}\right) \Rightarrow \theta = 180°\left(\frac{f}{f_0}-1\right)$$

where $f_0 = 2\times10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for $C$ in terms of $\theta$:

$$C = \frac{1}{4\pi^2 L f_0^2\,(1+\theta/180°)^2} = \frac{81}{400000\pi^2\,(180°+\theta)^2}$$

with SI units understood. After multiplying by $10^{12}$ (to convert to picofarads), this is plotted below:




17. (a) After the switch is thrown to position $b$ the circuit is an $LC$ circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(54.0\times10^{-3}\ \text{H})(6.20\times10^{-6}\ \text{F})}} = 275\ \text{Hz}.$$

(b) When the switch is thrown, the capacitor is charged to $V$ = 34.0 V and the current is zero. Thus, the maximum charge on the capacitor is $Q = VC$ = (34.0 V)(6.20 × $10^{-6}$ F) = 2.11 × $10^{-4}$ C. The current amplitude is

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(3.60\times10^{-3}\ \mathrm{H}\right)\left(4.00\times10^{-6}\ \mathrm{F}\right)}} = 1.33\times10^{3}\,\mathrm{Hz}.$$

(c) Referring to Fig. 31-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4\left(1.33\times10^{3}\ \mathrm{Hz}\right)} = 1.88\times10^{-4}\ \mathrm{s}.$$

21. (a) We compare this expression for the current with $i = I\sin(\omega t+\phi_0)$. Setting $(\omega t+\phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56 \times 10^{-4}$ s.

(b) Since $\omega = 2500$ rad/s $= (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\mathrm{rad/s})^2\left(64.0\times10^{-6}\ \mathrm{F}\right)} = 2.50\times10^{-3}\ \mathrm{H}.$$

(c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(2.50\times10^{-3}\ \mathrm{H}\right)\left(1.60\,\mathrm{A}\right)^2 = 3.20\times10^{-3}\ \mathrm{J}.$$

22. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\begin{aligned}\omega' &= \frac{1}{\sqrt{L_{\mathrm{eq}}C_{\mathrm{eq}}}} = \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}}\\ &= \frac{1}{\sqrt{L_1C_1}}\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega,\end{aligned}$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

23. (a) The total energy $U$ is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2L}{2} = \frac{\left(3.80\times10^{-6}\,\mathrm{C}\right)^2}{2\left(7.80\times10^{-6}\,\mathrm{F}\right)} + \frac{\left(9.20\times10^{-3}\,\mathrm{A}\right)^2\left(25.0\times10^{-3}\,\mathrm{H}\right)}{2} = 1.98\times10^{-6}\,\mathrm{J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2\left(7.80\times10^{-6}\text{ F}\right)\left(1.98\times10^{-6}\text{ J}\right)} = 5.56\times10^{-6}\text{ C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.98\times10^{-6}\text{ J}\right)}{25.0\times10^{-3}\text{ H}}} = 1.26\times10^{-2}\text{ A}.$$

(d) If $q_0$ is the charge on the capacitor at time $t = 0$, then $q_0 = Q \cos \phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80\times10^{-6}\text{ C}}{5.56\times10^{-6}\text{ C}}\right) = \pm 46.9°.$$

For $\phi = +46.9°$ the charge on the capacitor is decreasing, for $\phi = -46.9°$ it is increasing. To check this, we calculate the derivative of $q$ with respect to time, evaluated for $t = 0$. We obtain $-\omega Q \sin \phi$, which we wish to be positive. Since sin(+46.9°) is positive and sin(–46.9°) is negative, the correct value for increasing charge is $\phi = -46.9°$.

(e) Now we want the derivative to be negative and $\sin \phi$ to be positive. Thus, we take $\phi = +46.9°$.

24. The charge $q$ after $N$ cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 31-25:

$$\begin{aligned} q &= Qe^{-Rt/2L}\cos\left(\omega' t + \phi\right) = Qe^{-RNT/2L}\cos\left[\omega'\left(2\pi N/\omega'\right) + \phi\right] \\ &= Qe^{-RN\left(2\pi\sqrt{L/C}\right)/2L}\cos\left(2\pi N + \phi\right) \\ &= Qe^{-N\pi R\sqrt{C/L}}\cos\phi. \end{aligned}$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q \cos \phi$, where $q_0 = 6.2\ \mu$C is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0 \exp\left(-N\pi R\sqrt{C/L}\right)$.

(a) For $N = 5$, $q_5 = \left(6.2\,\mu\text{C}\right)\exp\left(-5\pi\left(7.2\Omega\right)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.85\,\mu\text{C}$.

(b) For $N = 10$, $q_{10} = \left(6.2\,\mu\text{C}\right)\exp\left(-10\pi\left(7.2\Omega\right)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.52\,\mu\text{C}$.

(c) For $N = 100$, $q_{100} = \left(6.2\,\mu\text{C}\right)\exp\left(-100\pi\left(7.2\Omega\right)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 1.93\,\mu\text{C}$.

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$t = 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) = 50\left(2\pi\sqrt{\left(220\times10^{-3}\,\text{H}\right)\left(12.0\times10^{-6}\,\text{F}\right)}\right)$$
$$= 0.5104\,\text{s}.$$

The maximum charge on the capacitor decays according to $q_{\max} = Qe^{-Rt/2L}$ (this is called the *exponentially decaying amplitude* in Section 31-5), where $Q$ is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 31-25). Dividing by $Q$ and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2\left(220\times10^{-3}\,\text{H}\right)}{0.5104\,\text{s}}\ln\left(0.99\right) = 8.66\times10^{-3}\ \Omega\,.$$

26. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 31-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{\max}^2/2C$, where $q_{\max}$ is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{\max}^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \;\Rightarrow\; q_{\max} = \frac{Q}{\sqrt{2}}.$$

Now $q_{\max}$ (referred to as the *exponentially decaying amplitude* in Section 31-5) is related to $Q$ (and the other parameters of the circuit) by

$$q_{\max} = Qe^{-Rt/2L} \Rightarrow \ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}.$$

Setting $q_{\max} = Q/\sqrt{2}$, we solve for $t$:

$$t = -\frac{2L}{R}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{L}{R}\ln 2\ .$$

The identities $\ln(1/\sqrt{2}) = -\ln\sqrt{2} = -\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

27. Let $t$ be a time at which the capacitor is fully charged in some cycle and let $q_{\max 1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t)=\frac{q_{\max 1}^2}{2C}=\frac{Q^2}{2C}e^{-Rt/L}$$

where

$$q_{\max 1}=Qe^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in Section 31-5). One period later the charge on the fully charged capacitor is

$$q_{\max 2}=Qe^{-R(t+T)2/L} \qquad \text{where } T=\frac{2\pi}{\omega'},$$

and the energy is

$$U(t+T)=\frac{q_{\max 2}^2}{2C}=\frac{Q^2}{2C}e^{-R(t+T)/L}\ .$$

The fractional loss in energy is

$$\frac{|\Delta U|}{U}=\frac{U(t)-U(t+T)}{U(t)}=\frac{e^{-Rt/L}-e^{-R(t+T)/L}}{e^{-Rt/L}}=1-e^{-RT/L}\ .$$

Assuming that $RT/L$ is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L}\approx 1-\frac{RT}{L}+\frac{R^2T^2}{2L^2}+\cdots.$$

If we approximate $\omega \approx \omega'$, then we can write $T$ as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U}\approx 1-\left(1-\frac{RT}{L}+\cdots\right)\approx\frac{RT}{L}=\frac{2\pi R}{\omega L}.$$

28. (a) We use $I = \varepsilon/X_c = \omega_d C\varepsilon$:

$$I=\omega_d C\varepsilon_m=2\pi f_d C\varepsilon_m=2\pi(1.00\times10^3\,\text{Hz})(1.50\times10^{-6}\,\text{F})(30.0\ \text{V})=0.283\ \text{A}\ .$$

(b) $I = 2\pi(8.00 \times 10^3\ \text{Hz})(1.50 \times 10^{-6}\ \text{F})(30.0\ \text{V}) = 2.26\ \text{A}$.

29. (a) The current amplitude $I$ is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \varepsilon_m$. Therefore,

$$I=\frac{V_L}{X_L}=\frac{\varepsilon_m}{2\pi f_d L}=\frac{30.0\,\text{V}}{2\pi(1.00\times10^3\,\text{Hz})(50.0\times10^{-3}\text{H})}=0.0955\,\text{A}=95.5\ \text{mA}.$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance $X_L$ is eight times larger and the current is one-eighth as much. The current is now

$$I = (0.0955 \text{ A})/8 = 0.0119 \text{ A} = 11.9 \text{ mA}.$$

30. (a) The current through the resistor is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0 \text{ V}}{50.0\,\Omega} = 0.600 \text{ A}.$$

(b) Regardless of the frequency of the generator, the current is the same, $I = 0.600 \text{ A}$.

31. (a) The inductive reactance for angular frequency $\omega_d$ is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0\times10^{-3}\text{ H})(10\times10^{-6}\text{F})}} = 6.5\times10^{2} \text{ Hz}.$$

(b) The inductive reactance is

$$X_L = \omega_d L = 2\pi f_d L = 2\pi(650 \text{ Hz})(6.0 \times 10^{-3} \text{ H}) = 24\ \Omega.$$

The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free $LC$ oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

32. (a) The circuit consists of one generator across one inductor; therefore, $\varepsilon_m = V_L$. The current amplitude is

$$I = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25.0 \text{ V}}{(377 \text{ rad/s})(12.7 \text{ H})} = 5.22\times10^{-3} \text{ A}.$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 30-35 gives $\varepsilon_L = 0$ at that instant. Stated another way, since $\varepsilon(t)$ and $i(t)$ have a 90° phase difference, then $\varepsilon(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 31-28 with $\varepsilon = -\varepsilon_m/2$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that $\varepsilon$ is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 31-28 with respect to time (and demanding the result be negative) we

(b) We describe three methods here (each using information from different points on the graph):

<u>method 1</u>: At $\omega_d$ = 50 rad/s, we have $Z \approx 700\ \Omega$, which gives $C = (\omega_d\sqrt{Z^2 - R^2}\,)^{-1} = 41\ \mu$F.

<u>method 2</u>: At $\omega_d$ = 50 rad/s, we have $X_C \approx 500\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

<u>method 3</u>: At $\omega_d$ = 250 rad/s, we have $X_C \approx 100\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

37. The rms current in the motor is

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + X_L^2}} = \frac{420\,\text{V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61\,\text{A}.$$

38. (a) The graph shows that the resonance angular frequency is 25000 rad/s, which means (using Eq. 31-4)

$$C = (\omega^2 L)^{-1} = [(25000)^2 \times 200 \times 10^{-6}]^{-1} = 8.0\ \mu\text{F}.$$

(b) The graph also shows that the current amplitude at resonance is 4.0 A, but at resonance the impedance $Z$ becomes purely resistive ($Z = R$) so that we can divide the emf amplitude by the current amplitude at resonance to find $R$: 8.0/4.0 = 2.0 Ω.

39. (a) Now $X_L = 0$, while $R = 200\ \Omega$ and $X_C = 1/2\pi f_d C = 177\ \Omega$. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200\,\Omega)^2 + (177\,\Omega)^2} = 267\,\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{200\,\Omega}\right) = -41.5°$$

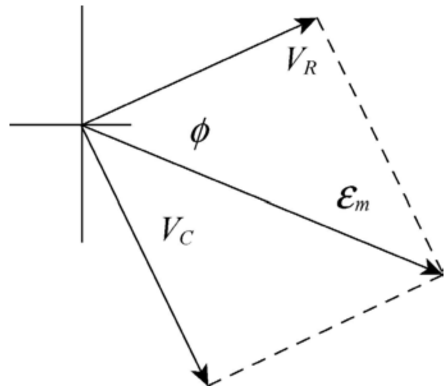
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{267\,\Omega} = 0.135\ \text{A}\ .$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.135\,\text{A})(200\Omega) \approx 27.0\,\text{V}$$
$$V_C = IX_C = (0.135\,\text{A})(177\Omega) \approx 23.9\,\text{V}$$

The circuit is capacitive, so $I$ leads $\varepsilon_m$. The phasor diagram is drawn to scale next.




40. A phasor diagram very much like Fig. 31-11(d) leads to the condition:

$$V_L - V_C = (6.00\text{ V})\sin(30^\circ) = 3.00\text{ V}.$$

With the magnitude of the capacitor voltage at 5.00 V, this gives a inductor voltage magnitude equal to 8.00 V. Since the capacitor and inductor voltage phasors are 180° out of phase, the potential difference across the inductor is $-8.00\text{ V}$.

41. (a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\text{ Hz})(70.0\times10^{-6}\text{ F})} = 37.9\,\Omega\ .$$

The inductive reactance 86.7 Ω is unchanged. The new impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\,\Omega)^2 + (37.9\,\Omega - 86.7\,\Omega)^2} = 206\,\Omega.$$

(b) The phase angle is

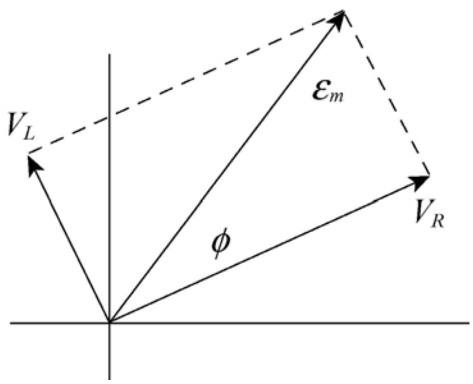
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 37.9\,\Omega}{200\,\Omega}\right) = 13.7^\circ.$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\text{ V}}{206\,\Omega} = 0.175\text{ A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.175\text{ A})(200\,\Omega) = 35.0\text{ V}$$
$$V_L = IX_L = (0.175\text{ A})(86.7\,\Omega) = 15.2\text{ V}$$
$$V_C = IX_C = (0.175\text{ A})(37.9\,\Omega) = 6.62\text{ V}$$




44. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\text{ Hz})(24.0\times10^{-6}\text{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - X_C)^2} \\ &= \sqrt{(220\,\Omega)^2 + [2\pi(400\text{ Hz})(150\times10^{-3}\text{ H}) - 16.6\ \Omega]^2} = 422\ \Omega\ . \end{aligned}$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\text{ A}\ .$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, $X_C$ increases as $C_{\text{eq}}$ decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\ \Omega)^2 + [2\pi(400\text{ Hz})(150\times10^{-3}\text{ H}) - 2(16.6\ \Omega)]^2} = 408\ \Omega < 422\ \Omega\ .$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

45. (a) Yes, the voltage amplitude across the inductor can be much larger than the amplitude of the generator emf.

(b) The amplitude of the voltage across the inductor in an *RLC* series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\text{ H}}{\sqrt{(1.0\text{ H})(1.0\times10^{-6}\text{F})}} = 1000\ \Omega\ .$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

$$I = \left.\frac{\varepsilon_m}{Z}\right|_{\text{resonance}} = \frac{\varepsilon_m}{R} = \frac{10\text{ V}}{10\ \Omega} = 1.0\text{ A}\ .$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\,\text{A})(1000\ \Omega) = 1.0\times10^3\ \text{V}$$

which is much larger than the amplitude of the generator emf.

46. (a) A sketch of the phasors would be very much like Fig. 31-9(c) but with the label "$I_C$" on the green arrow replaced with "$V_R$."

(b) We have $I\,R = I\,X_C$, or

$$I\,R = I\,X_C \;\rightarrow\; R = \frac{1}{\omega_d\,C}$$

which yields $f = \dfrac{\omega_d}{2\pi} = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(50.0\,\Omega)(2.00\times10^{-5}\text{ F})} = 159\text{ Hz}\,.$

(c) $\phi = \tan^{-1}(-V_C/V_R) = -\,45°$.

(d) $\omega_d = 1/RC = 1.00\times10^3$ rad/s.

(e) $I = (12\text{ V})/\sqrt{R^2 + X_C{}^2} = 6/(25\sqrt{2}) \approx 170$ mA.

47. (a) For a given amplitude $\varepsilon_m$ of the generator emf, the current amplitude is given by

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}\,.$$

We find the maximum by setting the derivative with respect to $\omega_d$ equal to zero:

$$\frac{dI}{d\omega_d} = -(E)_m[R^2 + (\omega_d L - 1/\omega_d C)^2]^{-3/2}\left[\omega_d L - \frac{1}{\omega_d C}\right]\left[L + \frac{1}{\omega_d^2 C}\right].$$

49. (a) Since $L_{eq} = L_1 + L_2$ and $C_{eq} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\begin{aligned}\omega &= \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_1+C_2+C_3)}} \\ &= \frac{1}{2\pi\sqrt{(1.70\times10^{-3}\ \text{H}+2.30\times10^{-3}\ \text{H})(4.00\times10^{-6}\ \text{F}+2.50\times10^{-6}\ \text{F}+3.50\times10^{-6}\ \text{F})}} \\ &= 796\,\text{Hz}.\end{aligned}$$

(b) The resonant frequency does not depend on $R$ so it will not change as $R$ increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as $L_1$ increases.

(d) Since $\omega \propto C_{eq}^{-1/2}$ and $C_{eq}$ decreases as $C_3$ is removed, $\omega$ will increase.

50. (a) A sketch of the phasors would be very much like Fig. 31-10(c) but with the label "$I_L$" on the green arrow replaced with "$V_R$."

(b) We have $V_R = V_L$, which implies

$$I\,R = I\,X_L \;\rightarrow\; R = \omega_d\,L$$

which yields $f = \omega_d/2\pi = R/2\pi L = 318$ Hz.

(c) $\phi = \tan^{-1}(V_L/V_R) = +45°$.

(d) $\omega_d = R/L = 2.00\times10^3$ rad/s.

(e) $I = (6\text{ V})/\sqrt{R^2 + X_L^2} = 3/(40\sqrt{2}) \approx 53.0$ mA.

51. We use the expressions found in Problem 31-47:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}, \quad \omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} .$$

We also use Eq. 31-4. Thus,

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}.$$

For the data of Problem 31-47,

$$\frac{\Delta\omega_d}{\omega} = (5.00\,\Omega)\sqrt{\frac{3(20.0\times10^{-6}\text{ F})}{1.00\text{ H}}} = 3.87\times10^{-2}.$$

This is in agreement with the result of Problem 31-47. The method of Problem 31-47, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

52. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

53. (a) Using Eq. 31-61, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12.0\,\Omega)^2 + (1.30\,\Omega - 0)^2} = 12.1\,\Omega.$$

(b) The average rate at which energy has been supplied is

$$P_{\text{avg}} = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2} = \frac{(120\text{ V})^2(12.0\,\Omega)}{(12.07\,\Omega)^2} = 1.186\times10^3\text{ W} \approx 1.19\times10^3\text{ W}.$$

54. The amplitude (peak) value is

$$V_{\max} = \sqrt{2}V_{\text{rms}} = \sqrt{2}(100\text{ V}) = 141\text{ V}.$$

55. The average power dissipated in resistance $R$ when the current is alternating is given by $P_{\text{avg}} = I_{\text{rms}}^2 R$, where $I_{\text{rms}}$ is the root-mean-square current. Since $I_{\text{rms}} = I/\sqrt{2}$, where $I$ is the current amplitude, this can be written $P_{\text{avg}} = I^2R/2$. The power dissipated in the same resistor when the current $i_d$ is direct is given by $P = i_d^2 R$. Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\text{ A}}{\sqrt{2}} = 1.84\text{ A}.$$

56. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{\max}/P_{\min} = (I/I_{\min})^2 = 5$, or

$$\left(\frac{I}{I_{\min}}\right)^2 = \left(\frac{\varepsilon_m/Z_{\min}}{\varepsilon_m/Z_{\max}}\right)^2 = \left(\frac{Z_{\max}}{Z_{\min}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{\max})^2}}{R}\right)^2 = 5.$$

We solve for $L_{\max}$:

$$L_{\max} = \frac{2R}{\omega} = \frac{2(120\,\text{V})^2/1000\,\text{W}}{2\pi(60.0\,\text{Hz})} = 7.64\times10^{-2}\,\text{H}.$$

(b) Yes, one could use a variable resistor.

(c) Now we must let

$$\left(\frac{R_{\max}+R_{\text{bulb}}}{R_{\text{bulb}}}\right)^2 = 5,$$

or

$$R_{\max} = \left(\sqrt{5}-1\right)R_{\text{bulb}} = \left(\sqrt{5}-1\right)\frac{(120\,\text{V})^2}{1000\,\text{W}} = 17.8\ \Omega.$$

(d) This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

57. We shall use

$$P_{\text{avg}} = \frac{\varepsilon_m^2 R}{2Z^2} = \frac{\varepsilon_m^2 R}{2\left[R^2+\left(\omega_d L - 1/\omega_d C\right)^2\right]}.$$

where $Z=\sqrt{R^2+\left(\omega_d L-1/\omega_d C\right)^2}$ is the impedance.

(a) Considered as a function of $C$, $P_{\text{avg}}$ has its largest value when the factor $R^2+\left(\omega_d L-1/\omega_d C\right)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2(60.0\,\text{Hz})^2\left(60.0\times10^{-3}\ \text{H}\right)} = 1.17\times10^{-4}\ \text{F}.$$

The circuit is then at resonance.

(b) In this case, we want $Z^2$ to be as large as possible. The impedance becomes large without bound as $C$ becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}} = \frac{\varepsilon_m^2}{2R},$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}} = \frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)} = 90.0\,\text{W}.$$

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle $\phi$ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0,$$

which implies $\phi = 0°$.

(e) At maximum power, the power factor is cos $\phi$ = cos 0° = 1.

(f) The minimum average power is $P_{\text{avg}} = 0$ (as it would be for an open switch).

(g) On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that $\phi = -90°$.

(h) At minimum power, the power factor is cos $\phi$ = cos(–90°) = 0.

58. This circuit contains no reactances, so $\varepsilon_{\text{rms}} = I_{\text{rms}}R_{\text{total}}$. Using Eq. 31-71, we find the average dissipated power in resistor $R$ is

$$P_R = I_{\text{rms}}^2 R = \left(\frac{\varepsilon_m}{r+R}\right)^2 R.$$

In order to maximize $P_R$ we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\varepsilon_m^2\left[(r+R)^2 - 2(r+R)R\right]}{(r+R)^4} = \frac{\varepsilon_m^2(r-R)}{(r+R)^3} = 0 \quad \Rightarrow \quad R = r$$

59. (a) The rms current is

$$\begin{aligned} I_{\text{rms}} &= \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}} \\ &= \frac{75.0\,\text{V}}{\sqrt{(15.0\Omega)^2 + \left\{2\pi(550\,\text{Hz})(25.0\,\text{mH}) - 1/\left[2\pi(550\,\text{Hz})(4.70\,\mu\text{F})\right]\right\}^2}} \\ &= 2.59\,\text{A}. \end{aligned}$$

(b) The rms voltage across $R$ is

$$V_{ab} = I_{\text{rms}}R = (2.59\,\text{A})(15.0\,\Omega) = 38.8\,\text{V}.$$

(c) The phase constant is related to the reactance difference by tan $\phi = (X_L - X_C)/R$. We have

$$\tan\phi = \tan(-42.0°) = -0.900,$$

a negative number. Therefore, $X_L - X_C$ is negative, which leads to $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance $X_L$ would be the same as $X_C$, tan $\phi$ would be zero, and $\phi$ would be zero. Since $\phi$ is not zero, we conclude the circuit is not in resonance.

(e) Since tan $\phi$ is negative and finite, neither the capacitive reactance nor the resistance are zero. This means the box must contain a capacitor and a resistor.

(f) The inductive reactance may be zero, so there need not be an inductor.

(g) Yes, there is a resistor.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\varepsilon_m I\cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\,\text{W}.$$

(i) The answers above depend on the frequency only through the phase constant $\phi$, which is given. If values were given for $R$, $L$ and $C$ then the value of the frequency would also be needed to compute the power factor.

62. We use Eq. 31-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\,\text{V})\left(\frac{500}{50}\right) = 1.00\times10^3\ \text{V}.$$

63. (a) The stepped-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\text{V})\left(\frac{10}{500}\right) = 2.4\,\text{V}.$$

(b) By Ohm's law, the current in the secondary is $I_s = \dfrac{V_s}{R_s} = \dfrac{2.4\,\text{V}}{15\Omega} = 0.16\,\text{A}.$

We find the primary current from Eq. 31-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\text{A})\left(\frac{10}{500}\right) = 3.2\times10^{-3}\ \text{A}.$$

(c) As shown above, the current in the secondary is $I_s = 0.16\,\text{A}$.

64. For step-up transformer:

(a) The smallest value of the ratio $V_s/V_p$ is achieved by using $T_2T_3$ as primary and $T_1T_3$ as secondary coil: $V_{13}/V_{23}$ = (800 + 200)/800 = 1.25.

(b) The second smallest value of the ratio $V_s/V_p$ is achieved by using $T_1T_2$ as primary and $T_2T_3$ as secondary coil: $V_{23}/V_{13}$ = 800/200 = 4.00.

(c) The largest value of the ratio $V_s/V_p$ is achieved by using $T_1T_2$ as primary and $T_1T_3$ as secondary coil: $V_{13}/V_{12}$ = (800 + 200)/200 = 5.00.

For the step-down transformer, we simply exchange the primary and secondary coils in each of the three cases above.

(d) The smallest value of the ratio $V_s/V_p$ is 1/5.00 = 0.200.

(e) The second smallest value of the ratio $V_s/V_p$ is 1/4.00 = 0.250.

(f) The largest value of the ratio $V_s/V_p$ is 1/1.25 = 0.800.

65. (a) The rms current in the cable is $I_{\text{rms}} = P/V_t = 250\times10^3\,\text{W}/\left(80\times10^3\,\text{V}\right) = 3.125\,\text{A}$. Therefore, the rms voltage drop is $\Delta V = I_{\text{rms}}R = (3.125\,\text{A})(2)(0.30\,\Omega) = 1.9\,\text{V}$.

(b) The rate of energy dissipation is $P_d = I_{\text{rms}}^2 R = (3.125\,\text{A})(2)(0.60\,\Omega) = 5.9\,\text{W}$.

(c) Now $I_{\text{rms}} = 250\times10^3\,\text{W}/\left(8.0\times10^3\,\text{V}\right) = 31.25\,\text{A}$, so $\Delta V = (31.25\,\text{A})(0.60\,\Omega) = 19\,\text{V}$.

(d) $P_d = (3.125\,\text{A})^2(0.60\,\Omega) = 5.9\times10^2\,\text{W}$.

(e) $I_{\text{rms}} = 250\times10^3\,\text{W}/\left(0.80\times10^3\,\text{V}\right) = 312.5\,\text{A}$, so $\Delta V = (312.5\,\text{A})(0.60\,\Omega) = 1.9\times10^2\,\text{V}$.

(f) $P_d = (312.5\,\text{A})^2(0.60\,\Omega) = 5.9\times10^4\,\text{W}$.

66. (a) The amplifier is connected across the primary windings of a transformer and the resistor $R$ is connected across the secondary windings.

$$f = \frac{X_L}{2\pi L} = \frac{1.30\times10^3\,\Omega}{2\pi\left(45.0\times10^{-3}\,\text{H}\right)} = 4.60\times10^3\,\text{Hz}.$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi fC)^{-1}$:

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi\left(4.60\times10^3\,\text{Hz}\right)\left(1.30\times10^3\,\Omega\right)} = 2.66\times10^{-8}\,\text{F}.$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when $f$ is doubled, $X_L$ doubles and $X_C$ reduces by half. Thus,

$$X_L = 2(1.30\times10^3\ \Omega) = 2.60\times10^3\ \Omega\ .$$

(d) $X_C = 1.30\times10^3\ \Omega/2 = 6.50\times10^2\ \Omega$.

71. (a) The impedance is $Z = (80.0\text{ V})/(1.25\text{ A}) = 64.0\ \Omega$.

(b) We can write $\cos\phi = R/Z$. Therefore,

$$R = (64.0\ \Omega)\cos(0.650\text{ rad}) = 50.9\ \Omega.$$

(c) Since the current leads the emf, the circuit is capacitive.

72. (a) From Eq. 31-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L/1.50)}{(V_L/2.00)}\right)$$

which becomes $\tan^{-1}(2/3) = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 31-60, we have

$$\varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(9.98\text{ V})^2 + (20.0\text{ V} - 13.3\text{ V})^2} = 12.0\text{ V}\ .$$

73. (a) From Eq. 31-4, we have $L = (\omega^2C)^{-1} = ((2\pi f)^2C)^{-1} = 2.41\ \mu$H.

(b) The total energy is the maximum energy on either device (see Fig. 31-4). Thus, we have $U_{\max} = \frac{1}{2}LI^2 = 21.4$ pJ.