Chapter 28

1. (a) Equation 28-3 leads to

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50\times10^{-17}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.60\times10^{-3}\,\text{T}\right)\sin 23.0^\circ} = 4.00\times10^{5}\ \text{m/s}.$$

(b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)\left(4.00\times10^{5}\ \text{m/s}\right)^2 = 1.34\times10^{-16}\ \text{J}\ ,$$

which is equivalent to $K = (1.34 \times 10^{-16}$ J) / $(1.60 \times 10^{-19}$ J/eV) = 835 eV.

2. The force associated with the magnetic field must point in the $\hat{\text{j}}$ direction in order to cancel the force of gravity in the $-\hat{\text{j}}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{\text{k}}$ direction (since $\hat{\text{i}}\times\left(-\hat{\text{k}}\right)=\hat{\text{j}}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 28-3 (with $\phi = 90°$). Therefore, with $m = 1.0\times10^{-2}$ kg, $v = 2.0\times10^{4}$ m/s, and $q = 8.0\times10^{-5}\,\text{C}$, we find

$$\vec{B} = B_z\hat{\text{k}} = -\left(\frac{mg}{qv}\right)\hat{\text{k}} = (-0.061\ \text{T})\hat{\text{k}}\ .$$

3. (a) The force on the electron is

$$\begin{aligned}\vec{F}_B &= q\vec{v}\times\vec{B} = q\left(v_x\hat{\text{i}}+v_y\hat{\text{j}}\right)\times\left(B_x\hat{\text{i}}+B_y\vec{j}\right) = q\left(v_xB_y - v_yB_x\right)\hat{\text{k}}\\ &= \left(-1.6\times10^{-19}\,\text{C}\right)\left[\left(2.0\times10^{6}\ \text{m/s}\right)\left(-0.15\ \text{T}\right)-\left(3.0\times10^{6}\ \text{m/s}\right)\left(0.030\ \text{T}\right)\right]\\ &= \left(6.2\times10^{-14}\ \text{N}\right)\hat{\text{k}}.\end{aligned}$$

Thus, the magnitude of $\vec{F}_B$ is 6.2×10^{14} N, and $\vec{F}_B$ points in the positive z direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction, namely, $\vec{F}_B = -\left(6.2\times10^{-14}\ \text{N}\right)\hat{\text{k}}.$

4. (a) We use Eq. 28-3:

$$F_B = |q|\, vB \sin\phi = (+\,3.2 \times 10^{-19}\ \text{C})\ (550\ \text{m/s})\ (0.045\ \text{T})\ (\sin 52°) = 6.2 \times 10^{-18}\ \text{N}.$$

(b) The acceleration is

$$a = F_B/m = (6.2 \times 10^{-18}\ \text{N}) / (6.6 \times 10^{-27}\ \text{kg}) = 9.5 \times 10^{8}\ \text{m/s}^2.$$

(c) Since it is perpendicular to $\vec{v}, \vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

5. Using Eq. 28-2 and Eq. 3-30, we obtain

$$\vec{F} = q\left(v_x B_y - v_y B_x\right)\hat{\text{k}} = q\left(v_x\left(3B_x\right) - v_y B_x\right)\hat{\text{k}}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\,\hat{\text{k}}$ where $F_z = 6.4 \times 10^{-19}$ N, then we are led to the condition

$$q\left(3v_x - v_y\right)B_x = F_z \quad \Rightarrow \quad B_x = \frac{F_z}{q\left(3v_x - v_y\right)}.$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6 \times 10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4\times10^{-19}\ \text{N}}{(-1.6\times10^{-19}\,\text{C})[3(2.0\ \text{m/s}) - 4.0\ \text{m}]} = -2.0\ \text{T}.$$

6. The magnetic force on the proton is

$$\vec{F} = q\vec{v} \times \vec{B}$$

where $q = +e$. Using Eq. 3-30 this becomes

$$(4 \times 10^{-17})\hat{\text{i}} + (2 \times 10^{-17})\hat{\text{j}} = e[(0.03v_y + 40)\hat{\text{i}} + (20 - 0.03v_x)\hat{\text{j}} - (0.02v_x + 0.01v_y)\hat{\text{k}}]$$

with SI units understood. Equating corresponding components, we find

(a) $v_x = -3.5\times10^{3}$ m/s, and

(b) $v_y = 7.0\times10^{3}$ m/s.

7. We apply $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = m_e\vec{a}$ to solve for $\vec{E}$:

$$\begin{aligned}\vec{E} &= \frac{m_e\vec{a}}{q} + \vec{B}\times\vec{v} \\ &= \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.00\times10^{12}\ \text{m/s}^2\right)\hat{\text{i}}}{-1.60\times10^{-19}\,\text{C}} + (400\,\mu\text{T})\hat{\text{i}}\times\left[(12.0\,\text{km/s})\hat{\text{j}} + (15.0\,\text{km/s})\hat{\text{k}}\right] \\ &= \left(-11.4\hat{\text{i}} - 6.00\hat{\text{j}} + 4.80\hat{\text{k}}\right)\text{V/m}.\end{aligned}$$

8. Letting $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) = 0$, we get

$$vB\sin\phi = E\ .$$

We note that (for given values of the fields) this gives a minimum value for speed whenever the sin ϕ factor is at its maximum value (which is 1, corresponding to ϕ = 90°). So

$$v_{\min} = \frac{E}{B} = \frac{1.50\times10^3\ \text{V/m}}{0.400\ \text{T}} = 3.75\times10^3\ \text{m/s}\,.$$

9. Straight-line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) = 0$. Note that $\vec{v}\perp\vec{B}$ so $\left|\vec{v}\times\vec{B}\right| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$B = \frac{E}{v} = \frac{E}{\sqrt{2K/m_e}} = \frac{100\ \text{V}/(20\times10^{-3}\text{m})}{\sqrt{2\left(1.0\times10^3\text{V}\right)\left(1.60\times10^{-19}\text{C}\right)/\left(9.11\times10^{-31}\text{kg}\right)}} = 2.67\times10^{-4}\text{T}.$$

In unit-vector notation, $\vec{B} = -(2.67\times10^{-4}\ \text{T})\hat{\text{k}}$.

10. (a) The net force on the proton is given by

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B} = \left(1.60\times10^{-19}\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{\text{k}} + \left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\times10^{-3}\text{T}\right)\hat{\text{i}}\right] \\ &= \left(1.44\times10^{-18}\,\text{N}\right)\hat{\text{k}}.\end{aligned}$$

(b) In this case, we have

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B} \\ &= \left(1.60\times10^{-19}\text{C}\right)\left[\left(-4.00\,\text{V/m}\right)\hat{\text{k}} + \left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\ \text{mT}\right)\hat{\text{i}}\right] \\ &= \left(1.60\times10^{-19}\,\text{N}\right)\hat{\text{k}}.\end{aligned}$$

(c) In the final case, we have

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{\text{i}}+\left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\,\text{mT}\right)\hat{\text{i}}\right]\\ &= \left(6.41\times10^{-19}\,\text{N}\right)\hat{\text{i}}+\left(8.01\times10^{-19}\,\text{N}\right)\hat{\text{k}}.\end{aligned}$$

11. Since the total force given by $\vec{F} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v}\times\vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $mv^2/2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = \left(1.2\,\text{T}\right)\sqrt{\frac{2\left(1.60\times10^{-19}\,\text{C}\right)\left(10\times10^{3}\,\text{V}\right)}{\left(9.99\times10^{-27}\,\text{kg}\right)}} = 6.8\times10^{5}\,\text{V/m}.$$

12. (a) The force due to the electric field ($\vec{F} = q\vec{E}$) is distinguished from that associated with the magnetic field ($\vec{F} = q\vec{v}\times\vec{B}$) in that the latter vanishes when the speed is zero and the former is independent of speed. The graph shows that the force (y-component) is negative at $v = 0$ (specifically, its value is -2.0×10^{-19} N there), which (because $q = -e$) implies that the electric field points in the $+y$ direction. Its magnitude is

$$E = \frac{F_{\text{net},y}}{|q|} = \frac{2.0\times10^{-19}\ \text{N}}{1.6\times10^{-19}\,\text{C}} = 1.25\ \text{N/C} = 1.25\ \text{V/m}\,.$$

(b) We are told that the x and z components of the force remain zero throughout the motion, implying that the electron continues to move along the x axis, even though magnetic forces generally cause the paths of charged particles to curve (Fig. 28-11). The exception to this is discussed in Section 28-3, where the forces due to the electric and magnetic fields cancel. This implies (Eq. 28-7) $B = E/v = 2.50 \times 10^{-2}$ T.

For $\vec{F} = q\vec{v}\times\vec{B}$ to be in the opposite direction of $\vec{F} = q\vec{E}$ we must have $\vec{v}\times\vec{B}$ in the opposite direction from $\vec{E}$, which points in the $+y$ direction, as discussed in part (a). Since the velocity is in the $+x$ direction, then (using the right-hand rule) we conclude that the magnetic field must point in the $+z$ direction ($\hat{\text{i}} \times \hat{\text{k}} = -\hat{\text{j}}$). In unit-vector notation, we have $\vec{B} = (2.50\times10^{-2}\ \text{T})\hat{\text{k}}$.

13. We use Eq. 28-12 to solve for V:

$$V = \frac{iB}{nle} = \frac{\left(23\text{A}\right)\left(0.65\,\text{T}\right)}{\left(8.47\times10^{28}/\text{m}^3\right)\left(150\,\mu\text{m}\right)\left(1.6\times10^{-19}\,\text{C}\right)} = 7.4\times10^{-6}\,\text{V}.$$

14. For a free charge q inside the metal strip with velocity $\vec{v}$ we have $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$. We set this force equal to zero and use the relation between (uniform) electric field and potential difference. Thus,

$$v = \frac{E}{B} = \frac{\left|V_x - V_y\right|/d_{xy}}{B} = \frac{\left(3.90 \times 10^{-9}\,\text{V}\right)}{\left(1.20 \times 10^{-3}\,\text{T}\right)\left(0.850 \times 10^{-2}\,\text{m}\right)} = 0.382\,\text{m/s}.$$

15. (a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). With Eq. 28-10, we define the magnitude of the electric field as

$$|\vec{E}| = v|\vec{B}| = \left(20.0\text{ m/s}\right)\left(0.030\text{ T}\right) = 0.600\text{ V/m}.$$

Its direction may be inferred from Figure 28-8; its direction is opposite to that defined by $\vec{v} \times \vec{B}$. In summary,

$$\vec{E} = -(0.600\,\text{V/m})\hat{\text{k}}$$

which insures that $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$ vanishes.

(b) Equation 28-9 yields $V = Ed = (0.600\text{ V/m})(2.00\text{ m}) = 1.20\text{ V}$.

16. We note that $\vec{B}$ must be along the x axis because when the velocity is along that axis there is no induced voltage. Combining Eq. 28-7 and Eq. 28-9 leads to

$$d = \frac{V}{E} = \frac{V}{vB}$$

where one must interpret the symbols carefully to ensure that $\vec{d}, \vec{v},$ and $\vec{B}$ are mutually perpendicular. Thus, when the velocity if parallel to the y axis the absolute value of the voltage (which is considered in the same "direction" as $\vec{d}$) is 0.012 V, and

$$d = d_z = \frac{0.012\text{ V}}{(3.0\text{ m/s})(0.020\text{ T})} = 0.20\text{ m}.$$

On the other hand, when the velocity is parallel to the z axis the absolute value of the appropriate voltage is 0.018 V, and

$$d = d_y = \frac{0.018\text{ V}}{(3.0\text{ m/s})(0.020\text{ T})} = 0.30\text{ m}.$$

Thus, our answers are

(a) d_x = 25 cm (which we arrive at "by elimination," since we already have figured out d_y and d_z),

(b) d_y = 30 cm, and

(c) d_z = 20 cm.

17. (a) Using Eq. 28-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\text{u}} = \frac{2\left(4.50\times10^{-2}\ \text{m}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.20\,\text{T}\right)}{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)} = 2.60\times10^{6}\ \text{m/s}\ .$$

(b) $T = 2\pi r/v = 2\pi(4.50 \times 10^{-2}\ \text{m})/(2.60 \times 10^{6}\ \text{m/s}) = 1.09 \times 10^{-7}$ s.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2}m_\alpha v^2 = \frac{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)\left(2.60\times10^{6}\ \text{m/s}\right)^2}{2\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 1.40\times10^{5}\,\text{eV}\ .$$

(d) $\Delta V = K/q = 1.40 \times 10^{5}\ \text{eV}/2e = 7.00 \times 10^{4}$ V.

18. With the $\vec{B}$ pointing "out of the page," we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent toward the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 28-3 (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e\,|\vec{B}|} = 4.99\times10^{6}\ \text{m/s}.$$

(b) Using either Eq. 28-14 or Eq. 28-16, we find r = 0.00710 m.

(c) Using Eq. 28-17 (in either its first or last form) readily yields $T = 8.93 \times 10^{-9}$ s.

19. Let ξ stand for the ratio ($m/|q|$) we wish to solve for. Then Eq. 28-17 can be written as $T = 2\pi\xi/B$. Noting that the horizontal axis of the graph (Fig. 28-36) is inverse-field (1/B) then we conclude (from our previous expression) that the slope of the line in the graph must be equal to $2\pi\xi$. We estimate that slope is 7.5×10^{-9} T·s, which implies

$$\xi = m/|q| = 1.2\times10^{-9}\ \text{kg/C}.$$

20. Combining Eq. 28-16 with energy conservation ($eV = \frac{1}{2} m_e v^2$ in this particular application) leads to the expression

$$r = \frac{m_e}{e\,B}\sqrt{\frac{2eV}{m_e}}$$

which suggests that the slope of the r versus $\sqrt{V}$ graph should be $\sqrt{2m_e / eB^2}$. From Fig. 28-37, we estimate the slope to be 5×10^{-5} in SI units. Setting this equal to $\sqrt{2m_e / eB^2}$ and solving, we find $B = 6.7 \times 10^{-2}$ T.

21. (a) From $K = \frac{1}{2} m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(1.20\times 10^3 \text{eV}\right)\left(1.60\times 10^{-19} \text{ eV/J}\right)}{9.11\times 10^{-31}\text{kg}}} = 2.05\times 10^7 \text{ m/s}.$$

(b) From $r = m_e v / qB$ we get

$$B = \frac{m_e v}{qr} = \frac{\left(9.11\times 10^{-31}\text{kg}\right)\left(2.05\times 10^7 \text{ m/s}\right)}{\left(1.60\times 10^{-19}\text{C}\right)\left(25.0\times 10^{-2}\text{m}\right)} = 4.67\times 10^{-4} \text{ T}.$$

(c) The “orbital” frequency is

$$f = \frac{v}{2\pi r} = \frac{2.07\times 10^7 \text{ m/s}}{2\pi\left(25.0\times 10^{-2}\text{m}\right)} = 1.31\times 10^7 \text{ Hz}.$$

(d) $T = 1/f = (1.31 \times 10^7 \text{ Hz})^{-1} = 7.63 \times 10^{-8}$ s.

22. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2 / 2$ is the kinetic energy of the particle. Thus, we see that $K = (rqB)^2/2m \propto q^2 m^{-1}$.

(a) $K_\alpha = \left(q_\alpha / q_p\right)^2 \left(m_p / m_\alpha\right) K_p = (2)^2 (1/4) K_p = K_p = 1.0\text{MeV};$

(b) $K_d = \left(q_d/q_p\right)^2 \left(m_p/m_d\right)K_p = (1)^2\left(1/2\right)K_p = 1.0 \text{ MeV}/2 = 0.50 \text{MeV}.$

23. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(1.30\times10^{6}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.350\text{ m}\right)} = 2.11\times10^{-5}\text{ T}.$$

24. (a) The accelerating process may be seen as a conversion of potential energy eV into kinetic energy. Since it starts from rest, $\frac{1}{2}m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2\left(1.60\times10^{-19}\text{C}\right)\left(350\text{ V}\right)}{9.11\times10^{-31}\text{kg}}} = 1.11\times10^{7}\text{ m/s}.$$

(b) Equation 28-16 gives

$$r = \frac{m_e v}{eB} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(1.11\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(200\times10^{-3}\text{T}\right)} = 3.16\times10^{-4}\text{m}.$$

25. (a) The frequency of revolution is

$$f = \frac{Bq}{2\pi m_e} = \frac{\left(35.0\times10^{-6}\text{T}\right)\left(1.60\times10^{-19}\text{C}\right)}{2\pi\left(9.11\times10^{-31}\text{kg}\right)} = 9.78\times10^{5}\text{Hz}.$$

(b) Using Eq. 28-16, we obtain

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_e K}}{qB} = \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(100\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}}{\left(1.60\times10^{-19}\text{ C}\right)\left(35.0\times10^{-6}\text{T}\right)} = 0.964\text{ m}.$$

26. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v}\times\vec{B}$ points leftward, which indeed seems to be the direction it is "pushed''; therefore, $q > 0$ (it is a proton).

(a) Equation 28-17 becomes $T = 2\pi m_{\text{p}} / e|\vec{B}|$, or

$$2\left(130\times10^{-9}\right) = \frac{2\pi\left(1.67\times10^{-27}\right)}{\left(1.60\times10^{-19}\right)|\vec{B}|}$$

which yields $|\vec{B}| = 0.252\,\text{T}$.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns.

27. (a) We solve for B from $m = B^2qx^2/8V$ (see Sample Problem — "Uniform circular motion of a charged particle in a magnetic field"):

$$B = \sqrt{\frac{8Vm}{qx^2}} .$$

We evaluate this expression using $x = 2.00$ m:

$$B = \sqrt{\frac{8\left(100\times10^{3}\ \text{V}\right)\left(3.92\times10^{-25}\ \text{kg}\right)}{\left(3.20\times10^{-19}\ \text{C}\right)\left(2.00\,\text{m}\right)^2}} = 0.495\,\text{T} .$$

(b) Let N be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where m is the mass of a single ion. M has the value

$$M = \frac{100\times10^{-6}\ \text{kg}}{3600\,\text{s}} = 2.78\times10^{-8}\ \text{kg/s} .$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{\left(3.20\times10^{-19}\ \text{C}\right)\left(2.78\times10^{-8}\ \text{kg/s}\right)}{3.92\times10^{-25}\ \text{kg}} = 2.27\times10^{-2}\ \text{A} .$$

(c) Each ion deposits energy qV in the cup, so the energy deposited in time Δt is given by

$$E = NqV\,\Delta t = \frac{iqV}{q}\Delta t = iV\,\Delta t .$$

For $\Delta t = 1.0$ h,

$$E = \left(2.27\times10^{-2}\ \text{A}\right)\left(100\times10^{3}\ \text{V}\right)\left(3600\,\text{s}\right) = 8.17\times10^{6}\ \text{J} .$$

To obtain the second expression, i/q is substituted for N.

28. Using $F = mv^2/r$ (for the centripetal force) and $K = mv^2/2$, we can easily derive the relation

33. (a) If v is the speed of the positron then $v \sin \phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv \sin \phi = m_e(v \sin \phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB) \sin \phi$. The period is given by

$$T = \frac{2\pi r}{v \sin \phi} = \frac{2\pi m_e}{eB} = \frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.100\text{T}\right)} = 3.58\times10^{-10}\,\text{s}.$$

The equation for r is substituted to obtain the second expression for T.

(b) The pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT \cos \phi$. We use the kinetic energy to find the speed: $K = \frac{1}{2} m_e v^2$ means

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(2.00\times10^3\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}} = 2.65\times10^7\ \text{m/s}\ .$$

Thus,

$$p = \left(2.65\times10^7\ \text{m/s}\right)\left(3.58\times10^{-10}\ \text{s}\right)\cos 89° = 1.66\times10^{-4}\ \text{m}\ .$$

(c) The orbit radius is

$$R = \frac{m_e v \sin \phi}{eB} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.65\times10^7\ \text{m/s}\right)\sin 89°}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.100\ \text{T}\right)} = 1.51\times10^{-3}\ \text{m}\ .$$

34. (a) Equation 3-20 gives $\phi = \cos^{-1}(2/19) = 84°$.

(b) No, the magnetic field can only change the direction of motion of a free (unconstrained) particle, not its speed or its kinetic energy.

(c) No, as reference to Fig. 28-11 should make clear.

(d) We find $v_\perp = v \sin \phi = 61.3$ m/s, so $r = mv_\perp /eB =$ 5.7 nm.

35. (a) By conservation of energy (using qV for the potential energy, which is converted into kinetic form) the kinetic energy gained in each pass is 200 eV.

(b) Multiplying the part (a) result by $n = 100$ gives $\Delta K = n(200\text{ eV}) = 20.0$ keV.

(c) Combining Eq. 28-16 with the kinetic energy relation ($n(200\text{ eV}) = m_p v^2/2$ in this particular application) leads to the expression

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km}\ .$$

For the average energy

$$r = \frac{\sqrt{2\left(8.3\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)\left(3.34\times10^{-27}\ \text{kg}\right)}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.57\ \text{T}\right)} = 0.375\ \text{m}\ .$$

The total distance traveled is about

$$n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2\ \text{m}.$$

38. (a) Using Eq. 28-23 and Eq. 28-18, we find

$$f_{\text{osc}} = \frac{qB}{2\pi m_p} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.20\ \text{T}\right)}{2\pi\left(1.67\times10^{-27}\ \text{kg}\right)} = 1.83\times10^{7}\ \text{Hz}.$$

(b) From $r = m_p v/qB = \sqrt{2m_p k}/qB$ we have

$$K = \frac{\left(rqB\right)^2}{2m_p} = \frac{\left[\left(0.500\text{m}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.20\ \text{T}\right)\right]^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 1.72\times10^{7}\ \text{eV}.$$

39. (a) The magnitude of the magnetic force on the wire is given by $F_B = iLB \sin \phi$, where i is the current in the wire, L is the length of the wire, B is the magnitude of the magnetic field, and ϕ is the angle between the current and the field. In this case $\phi = 70°$. Thus,

$$F_B = \left(5000\ \text{A}\right)\left(100\ \text{m}\right)\left(60.0\times10^{-6}\ \text{T}\right)\sin 70° = 28.2\ \text{N}\ .$$

(b) We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L}\times\vec{B}$ to show that the force is to the west.

40. The magnetic force on the (straight) wire is

$$F_B = iBL \sin\theta = \left(13.0\text{A}\right)\left(1.50\text{T}\right)\left(1.80\text{m}\right)\left(\sin 35.0°\right) = 20.1\text{N}.$$

41. (a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \quad \Rightarrow \quad i = \frac{mg}{LB} = \frac{(0.0130\,\text{kg})(9.8\,\text{m/s}^2)}{(0.620\,\text{m})(0.440\,\text{T})} = 0.467\,\text{A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

42. (a) From symmetry, we conclude that any *x*-component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the $\hat{\text{k}}$ direction produces on each part of the bent wire a *y*-component of force pointing in the $-\hat{\text{j}}$ direction; each of these components has magnitude

$$|F_y| = i\ell\,|\vec{B}|\sin 30° = (2.0\text{ A})(2.0\text{ m})(4.0\text{ T})\sin 30° = 8\,\text{N}.$$

Therefore, the force on the wire shown in the figure is $(-16\hat{\text{j}})\text{ N}$.

(b) The force exerted on the left half of the bent wire points in the $-\hat{\text{k}}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{\text{k}}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

43. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50$ cm side runs along the +*y* axis, while the $\ell_x = 120$ cm side runs along the +*x* axis. The angle made by the hypotenuse (of length 130 cm) is

$$\theta = \tan^{-1}(50/120) = 22.6°,$$

relative to the 120 cm side. If one measures the angle counterclockwise from the +*x* direction, then the angle for the hypotenuse is 180° – 22.6° = +157°. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the +*z* axis. We take $\vec{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750\,\text{T}$,

$$B_x = -B\cos\theta = -0.0692\,\text{T}\ , \quad B_y = B\sin\theta = 0.0288\,\text{T}.$$

(a) Equation 28-26 produces zero force when $\vec{L}\,||\,\vec{B}$ so there is no force exerted on the hypotenuse of length 130 cm.

(b) On the 50 cm side, the B_x component produces a force $i\ell_y B_x\hat{\text{k}}$, and there is no contribution from the B_y component. Using SI units, the magnitude of the force on the ℓ_y side is therefore

$$(4.00\,\text{A})(0.500\,\text{m})(0.0692\,\text{T}) = 0.138\,\text{N}.$$

(c) On the 120 cm side, the B_y component produces a force $i\ell_x B_y \hat{\mathrm{k}}$, and there is no contribution from the B_x component. The magnitude of the force on the ℓ_x side is also

$$(4.00\,\mathrm{A})(1.20\,\mathrm{m})(0.0288\,\mathrm{T}) = 0.138\,\mathrm{N}.$$

(d) The net force is

$$i\ell_y B_x \hat{\mathrm{k}} + i\ell_x B_y \hat{\mathrm{k}} = 0,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\vec{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$, but $B_y < 0$ and a zero net force would still be the result.

44. Consider an infinitesimal segment of the loop, of length *ds*. The magnetic field is perpendicular to the segment, so the magnetic force on it has magnitude $dF = iB\,ds$. The horizontal component of the force has magnitude

$$dF_h = (iB\cos\theta)ds$$

and points inward toward the center of the loop. The vertical component has magnitude

$$dF_y = (iB\sin\theta)ds$$

and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$\begin{aligned} F_v &= iB\sin\theta\int ds = 2\pi aiB\sin\theta = 2\pi(0.018\text{ m})(4.6\times10^{-3}\text{ A})(3.4\times10^{-3}\text{ T})\sin 20° \\ &= 6.0\times10^{-7}\text{ N}. \end{aligned}$$

We note that i, B, and θ have the same value for every segment and so can be factored from the integral.

45. The magnetic force on the wire is

$$\begin{aligned} \vec{F}_B &= i\vec{L}\times\vec{B} = iL\hat{\mathrm{i}}\times\left(B_y\hat{\mathrm{j}} + B_z\hat{\mathrm{k}}\right) = iL\left(-B_z\hat{\mathrm{j}} + B_y\hat{\mathrm{k}}\right) \\ &= (0.500\mathrm{A})(0.500\mathrm{m})\left[-(0.0100\mathrm{T})\hat{\mathrm{j}} + (0.00300\mathrm{T})\hat{\mathrm{k}}\right] \\ &= \left(-2.50\times10^{-3}\hat{\mathrm{j}} + 0.750\times10^{-3}\hat{\mathrm{k}}\right)\mathrm{N}. \end{aligned}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°.$$

Consequently,

$$B_{\min} = \frac{0.60(1.0\,\text{kg})(9.8\,\text{m/s}^2)}{(50\,\text{A})(1.0\,\text{m})(\cos 31° + 0.60\sin 31°)} = 0.10\,\text{T}.$$

(b) As shown above, the angle is $\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°$.

48. We use $d\vec{F}_B = id\vec{L} \times \vec{B}$, where $d\vec{L} = dx\hat{\text{i}}$ and $\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}}$. Thus,

$$\vec{F}_B = \int id\vec{L} \times \vec{B} = \int_{x_i}^{x_f} idx\hat{\text{i}} \times \left(B_x\hat{\text{i}} + B_y\hat{\text{j}}\right) = i\int_{x_i}^{x_f} B_y dx\hat{\text{k}}$$
$$= (-5.0\text{A})\left(\int_{1.0}^{3.0}(8.0x^2 dx)\ (\text{m}\cdot\text{mT})\right)\hat{\text{k}} = (-0.35\text{N})\hat{\text{k}}.$$

49. The applied field has two components: $B_x > 0$ and $B_z > 0$. Considering each straight segment of the rectangular coil, we note that Eq. 28-26 produces a nonzero force only for the component of $\vec{B}$ that is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight segment of the coil that lies along the y axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the y axis. Consequently, the torque derives completely from the force exerted on the straight segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Figure 28-44 carrying current in the $-y$ direction. Now, the B_z component will produce a force on this straight segment which points in the $-x$ direction (back towards the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $NiLB_x$ that points (by the right-hand rule) in the $+z$ direction. Since the action of this force is perpendicular to the plane of the coil, and is located a distance x away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)(x) = NiLxB\cos\theta = (20)(0.10\text{ A})(0.10\text{ m})(0.050\text{ m})(0.50\text{ T})\cos 30°$$
$$= 0.0043\ \text{N}\cdot\text{m}.$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3\times10^{-3}\ \text{N}\cdot\text{m})\hat{\text{j}}$.

An alternative way to do this problem is through the use of Eq. 28-37. We do not show those details here, but note that the magnetic moment vector (a necessary part of Eq. 28-37) has magnitude

$$|\vec{\mu}| = NiA = (20)(0.10\,\text{A})(0.0050\,\text{m}^2)$$

and points in the –z direction. At this point, Eq. 3-30 may be used to obtain the result for the torque vector.

50. We use $\tau_{\max} = |\vec{\mu}\times\vec{B}|_{\max} = \mu B = i\pi r^2 B,$ and note that $i = qf = qv/2\pi r$. So

$$\tau_{\max} = \left(\frac{qv}{2\pi r}\right)\pi r^2 B = \frac{1}{2}qvrB = \frac{1}{2}(1.60\times10^{-19}\,\text{C})(2.19\times10^{6}\,\text{m/s})(5.29\times10^{-11}\,\text{m})(7.10\times10^{-3}\,\text{T})$$
$$= 6.58\times10^{-26}\,\text{N}\cdot\text{m}.$$

51. We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\,\text{A}.$$

52. The insight central to this problem is that for a given length of wire (formed into a rectangle of various possible aspect ratios), the maximum possible area is enclosed when

the ratio of height to width is 1 (that is, when it is a square). The maximum possible value for the width, the problem says, is x = 4 cm (this is when the height is very close to zero, so the total length of wire is effectively 8 cm). Thus, when it takes the shape of a square the value of x must be ¼ of 8 cm; that is, x = 2 cm when it encloses maximum area (which leads to a maximum torque by Eq. 28-35 and Eq. 28-37) of $A = (0.020\text{ m})^2 = 0.00040\text{ m}^2$. Since N = 1 and the torque in this case is given as $4.8 \times 10^{-4}\ \text{N}\cdot\text{m}$, then the aforementioned equations lead immediately to i = 0.0030 A.

53. We replace the current loop of arbitrary shape with an assembly of small adjacent rectangular loops filling the same area that was enclosed by the original loop (as nearly as possible). Each rectangular loop carries a current i flowing in the same sense as the original loop. As the sizes of these rectangles shrink to infinitesimally small values, the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\vec{\tau}$ exerted by $\vec{B}$ on the nth rectangular loop of area ΔA_n is given by $\Delta\tau_n = NiB\sin\theta\Delta A_n$. Thus, for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB\sum_n \Delta A_n = NiAB\sin\theta.$$

54. (a) The kinetic energy gained is due to the potential energy decrease as the dipole swings from a position specified by angle θ to that of being aligned (zero angle) with the field. Thus,

$$K = U_i - U_f = -\mu B\cos\theta - (-\mu B\cos 0°).$$

Therefore, using SI units, the angle is

$$\theta = \cos^{-1}\left(1 - \frac{K}{\mu B}\right) = \cos^{-1}\left(1 - \frac{0.00080}{(0.020)(0.052)}\right) = 77°.$$

(b) Since we are making the assumption that no energy is dissipated in this process, then the dipole will continue its rotation (similar to a pendulum) until it reaches an angle θ = 77° on the other side of the alignment axis.

55. (a) The magnitude of the magnetic moment vector is

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi(7.00\,\text{A})\left[(0.200\,\text{m})^2 + (0.300\,\text{m})^2\right] = 2.86\,\text{A}\cdot\text{m}^2.$$

(b) Now,

$$\mu = \pi r_2^2 i_2 - \pi r_1^2 i_1 = \pi(7.00\,\text{A})\left[(0.300\,\text{m})^2 - (0.200\,\text{m})^2\right] = 1.10\,\text{A}\cdot\text{m}^2.$$

56. (a) $\mu = NAi = \pi r^2 i = \pi(0.150\,\text{m})^2(2.60\,\text{A}) = 0.184\,\text{A}\cdot\text{m}^2.$

(a) By using the right-hand rule, we see that $\vec{\mu}$ is in the –*y* direction. Thus, we have

$$\vec{\mu} = (NiA)(-\hat{\mathrm{j}}) = -(3)(2.00\ \mathrm{A})(4.00\times10^{-3}\ \mathrm{m}^2)\hat{\mathrm{j}} = -(0.0240\ \mathrm{A\cdot m^2})\hat{\mathrm{j}}.$$

The corresponding orientation energy is

$$U = -\vec{\mu}\cdot\vec{B} = -\mu_y B_y = -(-0.0240\ \mathrm{A\cdot m^2})(-3.00\times10^{-3}\ \mathrm{T}) = -7.20\times10^{-5}\ \mathrm{J}.$$

(b) Using the fact that $\hat{\mathrm{j}}\cdot\hat{\mathrm{i}} = 0$, $\hat{\mathrm{j}}\times\hat{\mathrm{j}} = 0$, and $\hat{\mathrm{j}}\times\hat{\mathrm{k}} = \hat{\mathrm{i}}$, the torque on the coil is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = \mu_y B_z\hat{\mathrm{i}} - \mu_y B_x\hat{\mathrm{k}}\\ &= (-0.0240\ \mathrm{A\cdot m^2})(-4.00\times10^{-3}\ \mathrm{T})\hat{\mathrm{i}} - (-0.0240\ \mathrm{A\cdot m^2})(2.00\times10^{-3}\ \mathrm{T})\hat{\mathrm{k}}\\ &= (9.60\times10^{-5}\ \mathrm{N\cdot m})\hat{\mathrm{i}} + (4.80\times10^{-5}\ \mathrm{N\cdot m})\hat{\mathrm{k}}.\end{aligned}$$

Note: The orientation energy is highest when $\vec{\mu}$ is in the opposite direction of $\vec{B}$, and lowest when $\vec{\mu}$ lines up with $\vec{B}$.

62. Looking at the point in the graph (Fig. 28-50(b)) corresponding to i_2 = 0 (which means that coil 2 has no magnetic moment) we are led to conclude that the magnetic moment of coil 1 must be $\mu_1 = 2.0\times10^{-5}\ \mathrm{A\cdot m^2}$. Looking at the point where the line crosses the axis (at i_2 = 5.0 mA) we conclude (since the magnetic moments cancel there) that the magnitude of coil 2's moment must also be $\mu_2 = 2.0\times10^{-5}\ \mathrm{A\cdot m^2}$ when $i_2 = 0.0050\ \mathrm{A}$, which means (Eq. 28-35)

$$N_2A_2 = \frac{\mu_2}{i_2} = \frac{2.0\times10^{-5}\ \mathrm{A\cdot m^2}}{0.0050\ \mathrm{A}} = 4.0\times10^{-3}\ \mathrm{m}^2.$$

Now the problem has us consider the direction of coil 2's current changed so that the net moment is the sum of two (positive) contributions, from coil 1 and coil 2, specifically for the case that i_2 = 0.007 A. We find that total moment is

$$\mu = (2.0\times10^{-5}\ \mathrm{A\cdot m^2}) + (N_2A_2\, i_2) = 4.8\times10^{-5}\ \mathrm{A\cdot m^2}.$$

63. The magnetic dipole moment is $\vec{\mu} = \mu\left(0.60\,\hat{\mathrm{i}} - 0.80\,\hat{\mathrm{j}}\right)$, where

$$\mu = NiA = Ni\pi r^2 = 1(0.20\ \mathrm{A})\pi(0.080\ \mathrm{m})^2 = 4.02\times10^{-4}\ \mathrm{A\cdot m^2}.$$

Here *i* is the current in the loop, *N* is the number of turns, *A* is the area of the loop, and *r* is its radius.

(a) The torque is

$$\begin{aligned}\vec{\tau} = \vec{\mu}\times\vec{B} &= \mu\left(0.60\hat{\mathrm{i}} - 0.80\hat{\mathrm{j}}\right)\times\left(0.25\hat{\mathrm{i}} + 0.30\hat{\mathrm{k}}\right)\\ &= \mu\left[(0.60)(0.30)\left(\hat{\mathrm{i}}\times\hat{\mathrm{k}}\right) - (0.80)(0.25)\left(\hat{\mathrm{j}}\times\hat{\mathrm{i}}\right) - (0.80)(0.30)\left(\hat{\mathrm{j}}\times\hat{\mathrm{k}}\right)\right]\\ &= \mu\left[-0.18\hat{\mathrm{j}} + 0.20\hat{\mathrm{k}} - 0.24\hat{\mathrm{i}}\right].\end{aligned}$$

Here $\hat{\mathrm{i}}\times\hat{\mathrm{k}} = -\hat{\mathrm{j}}, \hat{\mathrm{j}}\times\hat{\mathrm{i}} = -\hat{\mathrm{k}},$ and $\hat{\mathrm{j}}\times\hat{\mathrm{k}} = \hat{\mathrm{i}}$ are used. We also use $\hat{\mathrm{i}}\times\hat{\mathrm{i}} = 0$. Now, we substitute the value for μ to obtain

$$\vec{\tau} = \left(-9.7\times10^{-4}\hat{\mathrm{i}} - 7.2\times10^{-4}\hat{\mathrm{j}} + 8.0\times10^{-4}\hat{\mathrm{k}}\right)\mathrm{N\cdot m}.$$

(b) The orientation energy of the dipole is given by

$$\begin{aligned}U = -\vec{\mu}\cdot\vec{B} &= -\mu\left(0.60\hat{\mathrm{i}} - 0.80\hat{\mathrm{j}}\right)\cdot\left(0.25\hat{\mathrm{i}} + 0.30\hat{\mathrm{k}}\right)\\ &= -\mu(0.60)(0.25) = -0.15\mu = -6.0\times10^{-4}\,\mathrm{J}.\end{aligned}$$

Here $\hat{\mathrm{i}}\cdot\hat{\mathrm{i}} = 1, \hat{\mathrm{i}}\cdot\hat{\mathrm{k}} = 0, \hat{\mathrm{j}}\cdot\hat{\mathrm{i}} = 0,$ and $\hat{\mathrm{j}}\cdot\hat{\mathrm{k}} = 0$ are used.

64. Eq. 28-39 gives $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi$, so at $\phi = 0$ (corresponding to the lowest point on the graph in Fig. 28-51) the mechanical energy is

$$K + U = K_o + (-\mu B) = 6.7\times10^{-4}\,\mathrm{J} + (-5\times10^{-4}\,\mathrm{J}) = 1.7\times10^{-4}\,\mathrm{J}.$$

The turning point occurs where $K = 0$, which implies $U_{turn} = 1.7\times10^{-4}$ J. So the angle where this takes place is given by

$$\phi = -\cos^{-1}\left(\frac{1.7\times10^{-4}\ \mathrm{J}}{\mu B}\right) = 110°$$

where we have used the fact (see above) that $\mu B = 5\times10^{-4}$ J.

65. If N closed loops are formed from the wire of length L, the circumference of each loop is L/N, the radius of each loop is $R = L/2\pi N$, and the area of each loop is $A = \pi R^2 = \pi\left(L/2\pi N\right)^2 = L^2/4\pi N^2$.

(a) For maximum torque, we orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular (i.e., at a $90°$ angle) to the field.

(b) The magnitude of the torque is then

$$\frac{E}{E_c}=\frac{B}{ne\rho}=\frac{0.65\text{ T}}{\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\text{C}\right)\left(1.69\times10^{-8}\Omega\cdot\text{m}\right)}=2.84\times10^{-3}.$$

79. (a) Since $K = qV$ we have $K_p = \frac{1}{2}K_\alpha\left(\text{as } q_\alpha = 2K_p\right)$, or $K_p / K_\alpha = 0.50$.

(b) Similarly, $q_\alpha = 2K_d$, $K_d / K_\alpha = 0.50$.

(c) Since $r = \sqrt{2mK}\big/qB \propto \sqrt{mK}\big/q$, we have

$$r_d=\sqrt{\frac{m_dK_d}{m_pK_p}}\frac{q_pr_p}{q_d}=\sqrt{\frac{\left(2.00\text{u}\right)K_p}{\left(1.00\text{u}\right)K_p}}r_p=10\sqrt{2}\text{ cm}=14\text{ cm}.$$

(d) Similarly, for the alpha particle, we have

$$r_\alpha=\sqrt{\frac{m_\alpha K_\alpha}{m_pK_p}}\frac{q_pr_p}{q_\alpha}=\sqrt{\frac{\left(4.00\text{u}\right)K_\alpha}{\left(1.00\text{u}\right)\left(K_\alpha/2\right)}}\frac{er_p}{2e}=10\sqrt{2}\text{ cm}=14\text{ cm}.$$

80. (a) The largest value of force occurs if the velocity vector is perpendicular to the field. Using Eq. 28-3,

$$\begin{aligned}F_{B,\max} &= |q|\, vB \sin(90°) = ev\,B = (1.60\times10^{-19}\text{ C})\,(7.20\times10^{6}\text{ m/s})\,(83.0\times10^{-3}\text{ T})\\ &= 9.56\times10^{-14}\text{ N}.\end{aligned}$$

(b) The smallest value occurs if they are parallel: $F_{B,\min} = |q|\, vB \sin(0) = 0$.

(c) By Newton's second law, $a = F_B/m_e = |q|\, vB \sin\theta/m_e$, so the angle θ between $\vec{v}$ and $\vec{B}$ is

$$\theta=\sin^{-1}\left(\frac{m_ea}{|q|vB}\right)=\sin^{-1}\left[\frac{\left(9.11\times10^{-31}\text{kg}\right)\left(4.90\times10^{14}\text{ m/s}^2\right)}{\left(1.60\times10^{-16}\text{C}\right)\left(7.20\times10^{6}\text{ m/s}\right)\left(83.0\times10^{-3}\text{T}\right)}\right]=0.267°.$$

81. The contribution to the force by the magnetic field $\left(\vec{B}=B_x\hat{\text{i}}=(-0.020\text{ T})\hat{\text{i}}\right)$ is given by Eq. 28-2:

$$\begin{aligned}\vec{F}_B &= q\vec{v}\times\vec{B}=q\left(\left(17000\hat{\text{i}}\times B_x\hat{\text{i}}\right)+\left(-11000\hat{\text{j}}\times B_x\hat{\text{i}}\right)+\left(7000\hat{\text{k}}\times B_x\hat{\text{i}}\right)\right)\\ &= q\left(-220\hat{\text{k}}-140\hat{\text{j}}\right)\end{aligned}$$

$$\begin{aligned}\vec{F} &= q\vec{v}\times\vec{B} = (+e)\left(\left(v_yB_z - v_zB_y\right)\hat{\mathrm{i}} + \left(v_zB_x - v_xB_z\right)\hat{\mathrm{j}} + \left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}\right)\\ &= \left(1.60\times10^{-19}\right)\left(\left((4)(0.008) - (-6)\,(-0.004)\right)\hat{\mathrm{i}} +\right.\\ &\quad \left.\left((-6)(0.002) - (-2)\,(0.008)\right)\hat{\mathrm{j}} + \left((-2)(-0.004) - (4)(0.002)\right)\hat{\mathrm{k}}\right)\\ &= \left(1.28\times10^{-21}\right)\hat{\mathrm{i}} + \left(6.41\times10^{-22}\right)\hat{\mathrm{j}}\end{aligned}$$

with SI units understood.

(b) By definition of the cross product, $\vec{v}\perp\vec{F}$. This is easily verified by taking the dot (scalar) product of $\vec{v}$ with the result of part (a), yielding zero, provided care is taken not to introduce any round-off error.

(c) There are several ways to proceed. It may be worthwhile to note, first, that if B_z were 6.00 mT instead of 8.00 mT then the two vectors would be exactly antiparallel. Hence, the angle θ between $\vec{B}$ and $\vec{v}$ is presumably "close" to 180°. Here, we use Eq. 3-20:

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{B}}{|\vec{v}|\,|\vec{B}|}\right) = \cos^{-1}\left(\frac{-68}{\sqrt{56}\sqrt{84}}\right) = 173°\,.$$

86. (a) We are given $\vec{B} = B_x\hat{\mathrm{i}} = (6\times10^{-5}\,\mathrm{T})\hat{\mathrm{i}}$, so that $\vec{v}\times\vec{B} = -v_yB_x\hat{\mathrm{k}}$ where $v_y = 4\times10^4$ m/s. We note that the magnetic force on the electron is $(-e)\left(-v_yB_x\hat{\mathrm{k}}\right)$ and therefore points in the $+\hat{\mathrm{k}}$ direction, at the instant the electron enters the field-filled region. In these terms, Eq. 28-16 becomes

$$r = \frac{m_e v_y}{e\,B_x} = 0.0038\,\mathrm{m}.$$

(b) One revolution takes $T = 2\pi r/v_y = 0.60\ \mu$s, and during that time the "drift" of the electron in the x direction (which is the *pitch* of the helix) is $\Delta x = v_xT = 0.019$ m where $v_x = 32\times10^3$ m/s.

(c) Returning to our observation of force direction made in part (a), we consider how this is perceived by an observer at some point on the $-x$ axis. As the electron moves away from him, he sees it enter the region with positive v_y (which he might call "upward'') but "pushed" in the $+z$ direction (to his right). Hence, he describes the electron's spiral as clockwise.