Chapter 26

1. (a) The charge that passes through any cross section is the product of the current and time. Since t = 4.0 min = (4.0 min)(60 s/min) = 240 s,

$$q = it = (5.0\text{ A})(240\text{ s}) = 1.2\times 10^3\text{ C}.$$

(b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus,

$$N = q/e = (1200\text{ C})/(1.60 \times 10^{-19}\text{ C}) = 7.5 \times 10^{21}.$$

2. Suppose the charge on the sphere increases by Δq in time Δt. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r},$$

where r is the radius of the sphere. This means $\Delta q = 4\pi\varepsilon_0 r\,\Delta V$. Now, $\Delta q = (i_{\text{in}} - i_{\text{out}})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\begin{aligned}\Delta t &= \frac{\Delta q}{i_{\text{in}} - i_{\text{out}}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{\text{in}} - i_{\text{out}}} = \frac{(0.10\text{ m})(1000\text{ V})}{(8.99\times 10^9\text{ F/m})(1.0000020\text{ A} - 1.0000000\text{ A})}\\ &= 5.6\times 10^{-3}\text{ s}.\end{aligned}$$

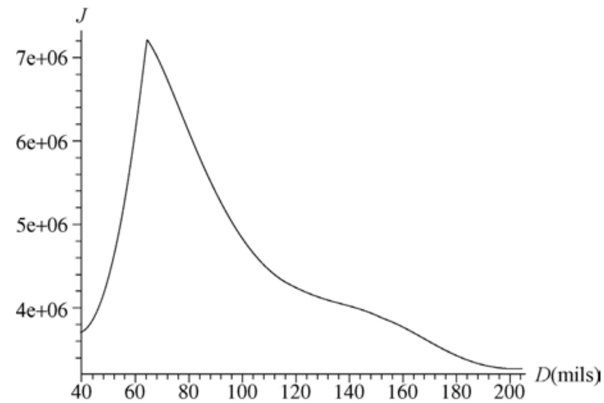
3. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100\times 10^{-6}\text{ A}}{(30\text{ m/s})(50\times 10^{-2}\text{ m})} = 6.7\times 10^{-6}\text{ C/m}^2.$$

4. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2}.$$

For example, the gauge 14 wire with D = 64 mil = 0.0016 m is found to have a (maximum safe) current density of $J = 7.2 \times 10^6$ A/m^2. In fact, this is the wire with the largest value of J allowed by the given data. The values of J in SI units are plotted below as a function of their diameters in mils.



5. (a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14}\ \text{m}^{-3}$, the charge is

$$q = 2e = 2(1.60 \times 10^{-19}\ \text{C}) = 3.20 \times 10^{-19}\ \text{C},$$

and the drift speed is 1.0×10^5 m/s. Thus,

$$J = \left(2\times10^{14}\ /\ \text{m}\right)\left(3.2\times10^{-19}\ \text{C}\right)\left(1.0\times10^{5}\ \text{m}/\text{s}\right) = 6.4\ \text{A}/\text{m}^2.$$

(b) Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

6. (a) Circular area depends, of course, on r^2, so the horizontal axis of the graph in Fig. 26-23(b) is effectively the same as the area (enclosed at variable radius values), except for a factor of π. The fact that the current increases linearly in the graph means that $i/A = J$ = constant. Thus, the answer is “yes, the current density is uniform.”

(b) We find $i/(\pi r^2) = (0.005\ \text{A})/(\pi \times 4 \times 10^{-6}\ \text{m}^2) = 398 \approx 4.0 \times 10^2\ \text{A/m}^2$.

7. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is

$$J = i/A = i/\pi r^2,$$

so

$$r=\sqrt{\frac{i}{\pi J}}=\sqrt{\frac{0.50\ \text{A}}{\pi\left(440\times 10^{4}\ \text{A/m}^{2}\right)}}=1.9\times 10^{-4}\ \text{m}.$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4}$ m) = 3.8×10^{-4} m.

8. (a) The magnitude of the current density vector is

$$J=\frac{i}{A}=\frac{i}{\pi d^{2}/4}=\frac{4\left(1.2\times 10^{-10}\ \text{A}\right)}{\pi\left(2.5\times 10^{-3}\ \text{m}\right)^{2}}=2.4\times 10^{-5}\ \text{A/m}^{2}.$$

(b) The drift speed of the current-carrying electrons is

$$v_d=\frac{J}{ne}=\frac{2.4\times 10^{-5}\ \text{A}/\text{m}^{2}}{\left(8.47\times 10^{28}/\text{m}^{3}\right)\left(1.60\times 10^{-19}\ \text{C}\right)}=1.8\times 10^{-15}\ \text{m}/\text{s}.$$

9. We note that the radial width $\Delta r = 10$ μm is small enough (compared to $r = 1.20$ mm) that we can make the approximation

$$\int Br2\pi r dr\approx Br2\pi r\Delta r$$

Thus, the enclosed current is $2\pi Br^2\Delta r = 18.1$ μA. Performing the integral gives the same answer.

10. Assuming $\vec{J}$ is directed along the wire (with no radial flow) we integrate, starting with Eq. 26-4,

$$i=\int|\vec{J}|\,dA=\int_{9R/10}^{R}(kr^{2})2\pi r dr=\frac{1}{2}k\pi\left(R^{4}-0.656R^{4}\right)$$

where $k = 3.0 \times 10^8$ and SI units are understood. Therefore, if $R = 0.00200$ m, we obtain $i=2.59\times 10^{-3}\,\text{A}$.

11. (a) The current resulting from this nonuniform current density is

$$i=\int_{\text{cylinder}}J_a\,dA=\frac{J_0}{R}\int_0^R r\cdot 2\pi r dr=\frac{2}{3}\pi R^2 J_0=\frac{2}{3}\pi(3.40\times 10^{-3}\,\text{m})^2(5.50\times 10^4\,\text{A/m}^2)$$
$$=1.33\ \text{A}.$$

(b) In this case,

$$i = \int_{\text{cylinder}} J_b dA = \int_0^R J_0 \left(1 - \frac{r}{R}\right) 2\pi r dr = \frac{1}{3}\pi R^2 J_0 = \frac{1}{3}\pi (3.40\times 10^{-3}\,\text{m})^2 (5.50\times 10^4\,\text{A/m}^2)$$
$$= 0.666\ \text{A}.$$

(c) The result is different from that in part (a) because J_b is higher near the center of the cylinder (where the area is smaller for the same radial interval) and lower outward, resulting in a lower average current density over the cross section and consequently a lower current than that in part (a). So, J_a has its maximum value near the surface of the wire.

12. (a) Since 1 cm^3 = 10^{-6} m^3, the magnitude of the current density vector is

$$J = nev = \left(\frac{8.70}{10^{-6}\ \text{m}^3}\right)\left(1.60\times 10^{-19}\ \text{C}\right)\left(470\times 10^3\ \text{m/s}\right) = 6.54\times 10^{-7}\ \text{A/m}^2.$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a "target" of circular area πR_E^2. The rate of charge transport implied by the influx of protons is

$$i = AJ = \pi R_E^2 J = \pi\left(6.37\times 10^6\ \text{m}\right)^2\left(6.54\times 10^{-7}\ \text{A/m}^2\right) = 8.34\times 10^7\ \text{A}.$$

13. We use $v_d = J/ne = i/Ane$. Thus,

$$t = \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LAne}{i} = \frac{(0.85\,\text{m})\left(0.21\times 10^{-14}\ \text{m}^2\right)\left(8.47\times 10^{28}\ /\text{m}^3\right)\left(1.60\times 10^{-19}\ \text{C}\right)}{300\,\text{A}}$$
$$= 8.1\times 10^2\ \text{s} = 13\,\text{min}.$$

14. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50 \times 10^{-3}\ \text{A})(2000\ \Omega) = 100$ V.

15. The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, then

$$L = (250)2\pi r = (250)(2\pi)(0.12\ \text{m}) = 188.5\ \text{m}.$$

If r_w is the radius of the wire itself, then its cross-sectional area is

$$A = \pi r_w^2 = \pi(0.65\times 10^{-3}\ \text{m})^2 = 1.33\times 10^{-6}\ \text{m}^2.$$

According to Table 26-1, the resistivity of copper is $\rho = 1.69\times10^{-8}\,\Omega\cdot\text{m}$. Thus,

$$R = \frac{\rho L}{A} = \frac{\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(188.5\text{ m}\right)}{1.33\times10^{-6}\text{ m}^2} = 2.4\,\Omega.$$

16. We use $R/L = \rho/A = 0.150\ \Omega/\text{km}$.

(a) For copper $J = i/A = (60.0\text{ A})(0.150\ \Omega/\text{km})/(1.69\times10^{-8}\ \Omega\cdot\text{m}) = 5.32\times10^5\text{ A/m}^2$.

(b) We denote the mass densities as ρ_m. For copper,

$$(m/L)_c = (\rho_m A)_c = (8960\text{ kg/m}^3)\,(1.69\times10^{-8}\ \Omega\cdot\text{m})/(0.150\ \Omega/\text{km}) = 1.01\text{ kg/m}.$$

(c) For aluminum $J = (60.0\text{ A})(0.150\ \Omega/\text{km})/(2.75\times10^{-8}\ \Omega\cdot\text{m}) = 3.27\times10^5\text{ A/m}^2$.

(d) The mass density of aluminum is

$$(m/L)_a = (\rho_m A)_a = (2700\text{ kg/m}^3)(2.75\times10^{-8}\ \Omega\cdot\text{m})/(0.150\ \Omega/\text{km}) = 0.495\text{ kg/m}.$$

17. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0\text{ m})(4.0\text{ A})}{(2.0\text{ V})\left(1.0\times10^{-6}\text{ m}^2\right)} = 2.0\times10^6/\Omega\cdot\text{m}.$$

18. (a) $i = V/R = 23.0\text{ V}/15.0\times10^{-3}\ \Omega = 1.53\times10^3\text{ A}$.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4}\pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4\left(1.53\times10^{-3}\text{ A}\right)}{\pi\left(6.00\times10^{-3}\text{ m}\right)^2} = 5.41\times10^7\text{ A/m}^2.$$

(c) The resistivity is

$$\rho = \frac{RA}{L} = \frac{(15.0\times10^{-3}\Omega)\pi(6.00\times10^{-3}\text{ m})^2}{4(4.00\text{ m})} = 10.6\times10^{-8}\ \Omega\cdot\text{m}.$$

(d) The material is platinum.

19. The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is its cross-sectional area. In this case,

$$A = \pi r^2 = \pi\left(0.50\times10^{-3}\text{ m}\right)^2 = 7.85\times10^{-7}\text{ m}^2.$$

Thus,

$$\rho = \frac{RA}{L} = \frac{\left(50\times10^{-3}\Omega\right)\left(7.85\times10^{-7}\text{ m}^2\right)}{2.0\text{ m}} = 2.0\times10^{-8}\Omega\cdot\text{m}.$$

20. The thickness (diameter) of the wire is denoted by D. We use $R \propto L/A$ (Eq. 26-16) and note that $A = \frac{1}{4}\pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R\left(\frac{D_1}{D_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2\left(\frac{1}{2}\right) = 2R.$$

21. The resistance at operating temperature T is $R = V/i$ = 2.9 V/0.30 A = 9.67 Ω. Thus, from $R - R_0 = R_0\alpha\,(T - T_0)$, we find

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20°\text{C} + \left(\frac{1}{4.5\times10^{-3}/\text{K}}\right)\left(\frac{9.67\,\Omega}{1.1\Omega} - 1\right) = 1.8\times10^3\ °\text{C}.$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. Table 26-1 has been used.

22. Let $r = 2.00$ mm be the radius of the kite string and $t = 0.50$ mm be the thickness of the water layer. The cross-sectional area of the layer of water is

$$A = \pi\left[(r+t)^2 - r^2\right] = \pi[(2.50\times10^{-3}\text{ m})^2 - (2.00\times10^{-3}\text{ m})^2] = 7.07\times10^{-6}\text{ m}^2.$$

Using Eq. 26-16, the resistance of the wet string is

$$R = \frac{\rho L}{A} = \frac{(150\,\Omega\cdot\text{m})(800\text{ m})}{7.07\times10^{-6}\text{ m}^2} = 1.698\times10^{10}\,\Omega.$$

The current through the water layer is

$$i = \frac{V}{R} = \frac{1.60\times10^8\text{ V}}{1.698\times10^{10}\,\Omega} = 9.42\times10^{-3}\text{ A}.$$

23. We use $J = E/\rho$, where E is the magnitude of the (uniform) electric field in the wire, J is the magnitude of the current density, and ρ is the resistivity of the material. The electric field is given by $E = V/L$, where V is the potential difference along the wire and L is the length of the wire. Thus $J = V/L\rho$ and

$$\rho = \frac{V}{LJ} = \frac{115 \text{ V}}{(10 \text{ m})(1.4 \times 10^4 \text{ A/m}^2)} = 8.2 \times 10^{-4}\,\Omega \cdot \text{m}.$$

24. (a) Since the material is the same, the resistivity ρ is the same, which implies (by Eq. 26-11) that the electric fields (in the various rods) are directly proportional to their current-densities. Thus, J_1: J_2: J_3 are in the ratio 2.5/4/1.5 (see Fig. 26-24). Now the currents in the rods must be the same (they are "in series") so

$$J_1 A_1 = J_3 A_3\,, \qquad J_2 A_2 = J_3 A_3\ .$$

Since $A = \pi r^2$, this leads (in view of the aforementioned ratios) to

$$4r_2^2 = 1.5r_3^2\,, \qquad 2.5r_1^2 = 1.5r_3^2\ .$$

Thus, with r_3 = 2 mm, the latter relation leads to r_1 = 1.55 mm.

(b) The $4r_2^2 = 1.5r_3^2$ relation leads to r_2 = 1.22 mm.

25. Since the mass density of the material does not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and $A = L_0A_0/L = L_0A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0,$$

where R_0 is the original resistance. Thus, $R = 9(6.0\ \Omega) = 54\ \Omega$.

26. The absolute values of the slopes (for the straight-line segments shown in the graph of Fig. 26-25(b)) are equal to the respective electric field magnitudes. Thus, applying Eq. 26-5 and Eq. 26-13 to the three sections of the resistive strip, we have

$$J_1 = \frac{i}{A} = \sigma_1 E_1 = \sigma_1 (0.50 \times 10^3 \text{ V/m})$$

$$J_2 = \frac{i}{A} = \sigma_2 E_2 = \sigma_2 (4.0 \times 10^3 \text{ V/m})$$

$$J_3 = \frac{i}{A} = \sigma_3 E_3 = \sigma_3 (1.0 \times 10^3 \text{ V/m})\ .$$

We note that the current densities are the same since the values of i and A are the same (see the problem statement) in the three sections, so $J_1 = J_2 = J_3$.

(a) Thus we see that $\sigma_1 = 2\sigma_3 = 2\,(3.00 \times 10^7 (\Omega \cdot \text{m})^{-1}) = 6.00 \times 10^7\ (\Omega \cdot \text{m})^{-1}$.

(b) Similarly, $\sigma_2 = \sigma_3/4 = (3.00 \times 10^7 (\Omega \cdot \text{m})^{-1})/4 = 7.50 \times 10^6 \, (\Omega \cdot \text{m})^{-1}$.

27. The resistance of conductor *A* is given by

$$R_A = \frac{\rho L}{\pi r_A^2},$$

where r_A is the radius of the conductor. If r_o is the outside diameter of conductor *B* and r_i is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$, and its resistance is

$$R_B = \frac{\rho L}{\pi\left(r_o^2 - r_i^2\right)}.$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0.$$

28. The cross-sectional area is $A = \pi r^2 = \pi(0.002 \text{ m})^2$. The resistivity from Table 26-1 is $\rho = 1.69 \times 10^{-8}\,\Omega \cdot \text{m}$. Thus, with $L = 3$ m, Ohm's Law leads to $V = iR = i\rho L/A$, or

$$12 \times 10^{-6}\,\text{V} = i\,(1.69 \times 10^{-8}\,\Omega \cdot \text{m})(3.0 \text{ m})/\,\pi(0.002 \text{ m})^2$$

which yields $i = 0.00297$ A or roughly 3.0 mA.

29. First we find the resistance of the copper wire to be

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\ \Omega \cdot \text{m})(0.020 \text{ m})}{\pi (2.0 \times 10^{-3} \text{ m})^2} = 2.69 \times 10^{-5}\,\Omega\,.$$

With potential difference $V = 3.00\,\text{nV}$, the current flowing through the wire is

$$i = \frac{V}{R} = \frac{3.00 \times 10^{-9}\,\text{V}}{2.69 \times 10^{-5}\,\Omega} = 1.115 \times 10^{-4}\,\text{A}\,.$$

Therefore, in 3.00 ms, the amount of charge drifting through a cross section is

$$\Delta Q = i\Delta t = (1.115 \times 10^{-4}\,\text{A})(3.00 \times 10^{-3}\text{ s}) = 3.35 \times 10^{-7}\,\text{C}\,.$$

30. We use $R \propto L/A$. The diameter of a 22-gauge wire is 1/4 that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be

$$R = (1.00\ \Omega)(25 \text{ ft}/1000 \text{ ft})(4)^2 = 0.40\ \Omega.$$

31. (a) The current in each strand is $i = 0.750 \text{ A}/125 = 6.00 \times 10^{-3}$ A.

36. Since the current spreads uniformly over the hemisphere, the current density at any given radius r from the striking point is $J = I/2\pi r^2$. From Eq. 26-10, the magnitude of the electric field at a radial distance r is

$$E = \rho_w J = \frac{\rho_w I}{2\pi r^2},$$

where $\rho_w = 30\,\Omega\cdot\text{m}$ is the resistivity of water. The potential difference between a point at radial distance D and a point at $D+\Delta r$ is

$$\Delta V = -\int_D^{D+\Delta r} E dr = -\int_D^{D+\Delta r} \frac{\rho_w I}{2\pi r^2} dr = \frac{\rho_w I}{2\pi}\left(\frac{1}{D+\Delta r} - \frac{1}{D}\right) = -\frac{\rho_w I}{2\pi}\frac{\Delta r}{D(D+\Delta r)},$$

which implies that the current across the swimmer is

$$i = \frac{|\Delta V|}{R} = \frac{\rho_w I}{2\pi R}\frac{\Delta r}{D(D+\Delta r)}.$$

Substituting the values given, we obtain

$$i = \frac{(30.0\,\Omega\cdot\text{m})(7.80\times10^4\,\text{A})}{2\pi(4.00\times10^3\Omega)}\frac{0.70\text{ m}}{(35.0\text{ m})(35.0\text{ m}+0.70\text{ m})} = 5.22\times10^{-2}\,\text{A}.$$

37. From Eq. 26-25, $\rho \propto \tau^{-1} \propto v_{\text{eff}}$. The connection with v_{eff} is indicated in part (b) of Sample Problem —"Mean free time and mean free distance," which contains useful insight regarding the problem we are working now. According to Chapter 20, $v_{\text{eff}} \propto \sqrt{T}$. Thus, we may conclude that $\rho \propto \sqrt{T}$.

38. The slope of the graph is $P = 5.0 \times 10^{-4}$ W. Using this in the $P = V^2/R$ relation leads to $V = 0.10$ Vs.

39. Eq. 26-26 gives the rate of thermal energy production:

$$P = iV = (10.0\,\text{A})(120\,\text{V}) = 1.20\text{ kW}.$$

Dividing this into the 180 kJ necessary to cook the three hotdogs leads to the result $t = 150$ s.

40. The resistance is $R = P/i^2 = (100\text{ W})/(3.00\text{ A})^2 = 11.1\ \Omega$.

41. (a) Electrical energy is converted to heat at a rate given by $P = V^2/R,$ where V is the potential difference across the heater and R is the resistance of the heater. Thus,

$$P = \frac{(120\ \text{V})^2}{14\ \Omega} = 1.0 \times 10^3\ \text{W} = 1.0\ \text{kW}.$$

(b) The cost is given by $(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW}\cdot\text{h}) = \text{US\$}0.25$.

42. (a) Referring to Fig. 26-32, the electric field would point down (toward the bottom of the page) in the strip, which means the current density vector would point down, too (by Eq. 26-11). This implies (since electrons are negatively charged) that the conduction electrons would be "drifting" upward in the strip.

(b) Equation 24-6 immediately gives 12 eV, or (using $e = 1.60 \times 10^{-19}$ C) 1.9×10^{-18} J for the work done by the field (which equals, in magnitude, the potential energy change of the electron).

(c) Since the electrons don't (on average) gain kinetic energy as a result of this work done, it is generally dissipated as heat. The answer is as in part (b): 12 eV or 1.9×10^{-18} J.

43. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50\ \text{V}}{3.00\ \text{V}}\right)^2 (0.540\ \text{W}) = 0.135\ \text{W}.$$

44. Since $P = iV$, the charge is

$$q = it = Pt/V = (7.0\ \text{W})\ (5.0\ \text{h})\ (3600\ \text{s/h})/9.0\ \text{V} = 1.4 \times 10^4\ \text{C}.$$

45. (a) The power dissipated, the current in the heater, and the potential difference across the heater are related by $P = iV$. Therefore,

$$i = \frac{P}{V} = \frac{1250\ \text{W}}{115\ \text{V}} = 10.9\ \text{A}.$$

(b) Ohm's law states $V = iR$, where R is the resistance of the heater. Thus,

$$R = \frac{V}{i} = \frac{115\ \text{V}}{10.9\ \text{A}} = 10.6\ \Omega.$$

(c) The thermal energy E generated by the heater in time t = 1.0 h = 3600 s is

$$E = Pt = (1250\,\text{W})(3600\,\text{s}) = 4.50 \times 10^6\ \text{J}.$$

46. (a) Using Table 26-1 and Eq. 26-10 (or Eq. 26-11), we have

$$|\vec{E}| = \rho|\vec{J}| = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(\frac{2.00\,\text{A}}{2.00\times10^{-6}\,\text{m}^2}\right) = 1.69\times10^{-2}\,\text{V/m}.$$

(b) Using L = 4.0 m, the resistance is found from Eq. 26-16:

$$R = \rho L/A = 0.0338\ \Omega.$$

The rate of thermal energy generation is found from Eq. 26-27:

$$P = i^2 R = (2.00\ \text{A})^2(0.0338\ \Omega) = 0.135\ \text{W}.$$

Assuming a steady rate, the amount of thermal energy generated in 30 minutes is found to be (0.135 J/s)(30 × 60 s) = 2.43×10^2 J.

47. (a) From $P = V^2/R = AV^2/\rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60\times10^{-6}\ \text{m}^2)(75.0\ \text{V})^2}{(5.00\times10^{-7}\ \Omega\cdot\text{m})(500\ \text{W})} = 5.85\ \text{m}.$$

(b) Since $L \propto V^2$ the new length should be $L' = L\left(\dfrac{V'}{V}\right)^2 = (5.85\ \text{m})\left(\dfrac{100\ \text{V}}{75.0\ \text{V}}\right)^2 = 10.4\ \text{m}.$

48. The mass of the water over the length is

$$m = \rho AL = (1000\ \text{kg/m}^3)(15\times10^{-5}\ \text{m}^2)(0.12\ \text{m}) = 0.018\ \text{kg},$$

and the energy required to vaporize the water is

$$Q = Lm = (2256\ \text{kJ/kg})(0.018\ \text{kg}) = 4.06\times10^4\ \text{J}.$$

The thermal energy is supplied by joule heating of the resistor:

$$Q = P\Delta t = I^2 R\Delta t.$$

Since the resistance over the length of water is

$$R = \frac{\rho_w L}{A} = \frac{(150\ \Omega\cdot\text{m})(0.120\ \text{m})}{15\times10^{-5}\ \text{m}^2} = 1.2\times10^5\ \Omega,$$

the average current required to vaporize water is

$$I = \sqrt{\frac{Q}{R\Delta t}} = \sqrt{\frac{4.06\times10^4 \text{ J}}{(1.2\times10^5 \Omega)(2.0\times10^{-3} \text{ s})}} = 13.0 \text{ A}.$$

49. (a) Assuming a 31-day month, the monthly cost is

$$(100 \text{ W})(24 \text{ h/day})(31\text{days/month})\,(6 \text{ cents/kW}\cdot\text{h}) = 446 \text{ cents} = \text{US\$4.46}.$$

(b) $R = V^2/P = (120 \text{ V})^2/100 \text{ W} = 144\ \Omega$.

(c) $i = P/V = 100 \text{ W}/120 \text{ V} = 0.833$ A.

50. The slopes of the lines yield P_1 = 8 mW and P_2 = 4 mW. Their sum (by energy conservation) must be equal to that supplied by the battery: P_{batt} = (8 + 4) mW = 12 mW.

51. (a) We use Eq. 26-16 to compute the resistances:

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0\times10^{-6}\ \Omega\cdot\text{m})\frac{1.0 \text{ m}}{\pi(0.00050 \text{ m})^2} = 2.55\ \Omega.$$

The voltage follows from Ohm's law: $|V_1 - V_2| = V_C = iR_C = (2.0 \text{ A})(2.55\ \Omega) = 5.1\text{V}$.

(b) Similarly,

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0\times10^{-6}\ \Omega\cdot\text{m})\frac{1.0 \text{ m}}{\pi(0.00025 \text{ m})^2} = 5.09\ \Omega$$

and $|V_2 - V_3| = V_D = iR_D = (2.0 \text{ A})(5.09\ \Omega) = 10.2\text{ V} \approx 10\text{ V}$.

(c) The power is calculated from Eq. 26-27: $P_C = i^2 R_C = 10\text{ W}$.

(d) Similarly, $P_D = i^2 R_D = 20\text{ W}$.

52. Assuming the current is along the wire (not radial) we find the current from Eq. 26-4:

$$i = \int |\vec{J}|\, dA = \int_0^R kr^2\, 2\pi r dr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}$$

where $k = 2.75 \times 10^{10}$ A/m^4 and R = 0.00300 m. The rate of thermal energy generation is found from Eq. 26-26: $P = iV$ = 210 W. Assuming a steady rate, the thermal energy generated in 40 s is $Q = P\Delta t = (210 \text{ J/s})(3600 \text{ s}) = 7.56 \times 10^5$ J.

53. (a) From $P = V^2/R$ we find $R = V^2/P = (120 \text{ V})^2/500 \text{ W} = 28.8\ \Omega$.

(b) Since $i = P/V$, the rate of electron transport is