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PHYS 2B FALL 2012

Chapter 25

Easy:

(2)

$Q = CV$

$Q = (25 \times 10^{-6}\ F)(120\ V) = 3 \times 10^{-3}\ C$

(6) $A = 1.00\ m^2$

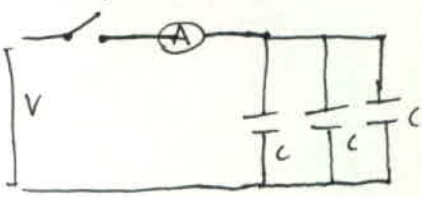
a) $C = \frac{\epsilon_0 A}{d} \Rightarrow d = \frac{\epsilon_0 A}{C} = \frac{(8.85 \times 10^{-12}\ F/m)(1.00\ m^2)}{1.00\ F}$

$d = 8.85 \times 10^{-12}\ m$

b) Don't see why not. pm devices are currently built.

(9) $C = 25.0\ \mu F$

$V = 4200\ V$



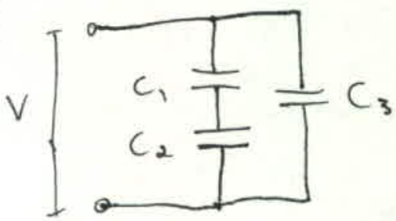
charge that passes through (A) is Q_{total}

$Q_{total} = C_{eq} V$

$Q_{total} = (C + C + C)V = 3CV$

$Q_{total} = 3(25.0 \times 10^{-6}\ F)(4200\ V) = 0.315\ C$

(10)



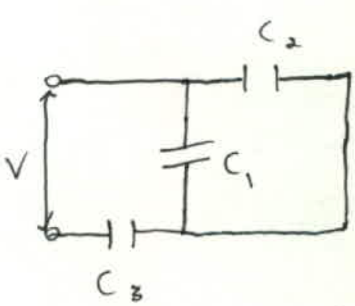
$C_1 = 10.0\ \mu F,\ C_2 = 5.00\ \mu F,\ C_3 = 4.00\ \mu F$

$\frac{1}{10} + \frac{1}{5} = \frac{1}{C_{12}}$

$C_{12} = \frac{10}{3}\ \mu F \quad C_{eq} = C_{12} + C_3 = \frac{10}{3} + 4$

$C_{eq} = 7.33\ \mu F$

(11)



$C_1 = 10.0\ \mu F,\ C_2 = 5.00\ \mu F,\ C_3 = 4.00\ \mu F$

$C_{12} = C_1 + C_2 = 15\ \mu F$

$\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{15} + \frac{1}{4}$

$C_{eq} = \frac{60}{19}\ \mu F = 3.16\ \mu F$

(32) $A = 40\ cm^2 \quad d = 1.0\ mm \quad V = 600\ V$

a) $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.85 \times 10^{-12}\ F/m)(40\ cm^2)}{(1.0 \times 10^{-3}\ m)} \dfrac{(1\ m)^2}{(100\ cm)^2} = 3.54 \times 10^{-11}\ F$

b) $Q = CV = 2.124 \times 10^{-8}\ C$

c) $U = \frac{1}{2} C V^2 = \frac{1}{2}(3.54 \times 10^{-11}\ F)(600\ V)^2 = 6.372 \times 10^{-6}\ J$

d) $V = E d$

$E = \dfrac{V}{d} = \dfrac{600\ V}{(1.0 \times 10^{-3}\ m)} = 6.0 \times 10^5\ N/C$

e) $u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2}(8.85 \times 10^{-12}\ C^2/N \cdot m^2)(6.0 \times 10^5\ N/C)^2$

$u = 1.593\ N/m^2 = 1.593\ J/m^3$

(42) $C = 50\ pF \qquad A = 0.35\ m^2$

a) $C = \dfrac{\epsilon_0 A}{d} \Rightarrow d = \dfrac{\epsilon_0 A}{C} = \dfrac{(8.85\ pF/m)(0.35\ m^2)}{50\ pF}$

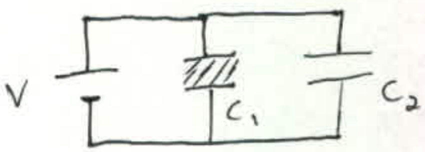
$d = 0.062\ m$

b) $K = 5.6$

$C_{new} = K\, C_{old} = 5.6\,(50\ pF)$

$C_{new} = 280\ pF$

(46)



C_1 has $K = 3.00 \qquad A = 5.00 \times 10^{-3}\ m^2$

$d = 2.00\ mm$

$V = 12.0\ V$

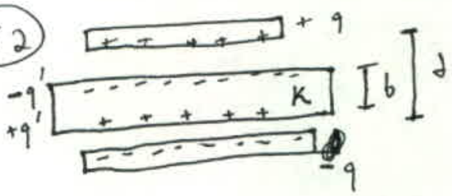
$C_2 = \dfrac{\epsilon_0 A}{d} \qquad C_1 = K C_2 = 3 C_2$

$C_{eq} = C_1 + C_2 = 3C_2 + C_2 = 4 C_2$

$Q_{total} = C_{eq} V = 4 C_2 V = 4 \epsilon_0 A V / d = \dfrac{4(8.85 \times 10^{-12}\ F/m)(5.00 \times 10^{-3}\ m^2)(12.0\ V)}{2.00 \times 10^{-3}\ m} =$

$Q_{total} = 1.06 \times 10^{-9}\ C$

(52)



$b = 0.780\ cm$, $A = 115\ cm^2$, $d = 1.24\ cm$, $k = 2.61$

constant voltage.
$V_0 = 85.5\ V$

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

$$E_1 = \frac{E_0}{k}$$

a) $V = E_0 (d-b) + E_1 b$ voltage stays the same before and after.

$$V = \frac{q}{\epsilon_0 A}(d-b) + \frac{q}{\epsilon_0 A k} b = \frac{q}{\epsilon_0 A}\left(d - b + \frac{b}{k}\right)$$

$$C = \frac{q}{V} = \epsilon_0 A \left(d - b + \frac{b}{k}\right)^{-1} = \epsilon_0 A \left(d + b\left(\frac{1}{k} - 1\right)\right)^{-1}$$

$$C = \frac{(8.85 \times 10^{-12}\ F/m)(115 \times 10^{-4}\ m^2)}{((1.24 \times 10^{-2}\ m) + (0.780 \times 10^{-2}\ m)\left(\frac{1}{2.61} - 1\right))}$$

$$C = 1.34 \times 10^{-11}\ F$$

b) $q = CV = (1.34 \times 10^{-11}\ F)(85.5\ V) = 1.15 \times 10^{-9}\ C$

c) $$E_0 = \frac{q}{\epsilon_0 A} = \frac{(1.15 \times 10^{-9}\ C)}{(8.85 \times 10^{-12}\ C^2/Nm^2)(115 \times 10^{-4}\ m^2)} = 1.13 \times 10^4\ N/C$$

d) $$E_1 = \frac{E_0}{k} = \frac{(1.13 \times 10^4\ N/C)}{2.61} = 4.32 \times 10^3\ N/C$$

Medium:

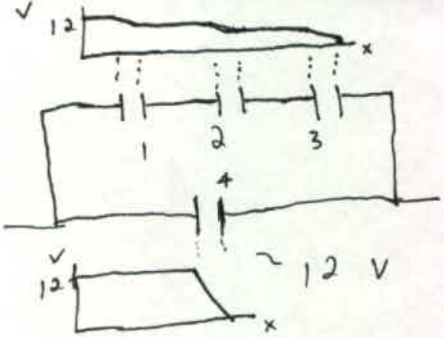
(12) $C = 6.0\ \mu F$ 10 V battery
2 parallel . 1 of capacitors is squeezed to 50% of d.

a) $$C' = \frac{\epsilon_0 A}{(d/2)} = 2\frac{\epsilon_0 A}{d} = 2C = 12\ \mu F$$

so an additional $V(C' - C) = (6.0\ \mu F)(10\ V) = 6.0 \times 10^{-5}\ C$
is transferred to the capacitors by the battery.

b) same as a), $6.0 \times 10^{-5}\ C$.

(18)



~ 12 V

2 V, then 5 V, then 5 V

?

$12 - 5 - 2 = 5$ V

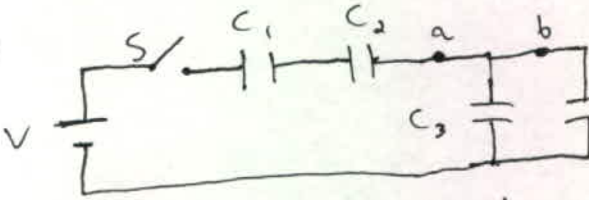
$C_3 = 0.80\ \mu F$

$$Q = C_3 V_3 = (0.80\ \mu F)(5\ V) = 4.0\ \mu C$$

a) $Q = C_1 V_1 \Rightarrow C_1 = \dfrac{Q}{V_1} = \dfrac{4.0\ \mu C}{(2\ V)} = 2.0\ \mu F$

b) $C_2 = \dfrac{Q}{V_2} = \dfrac{4.0\ \mu C}{5\ V} = 0.80\ \mu F$

(19)



$V = 9.0$ V

$C_2 = 3.0\ \mu F$, $C_4 = 4.0\ \mu F$

12 μC passes through a, 8.0 μC passes through b.

a) C_1?
b) C_3?

$Q_1 = Q_2 = 12\ \mu C$

$Q_4 = 8.0\ \mu C$, $Q_3 = 12 - 8\ \mu C = 4.0\ \mu C$

$V = \dfrac{Q}{C}$, $V_3 = V_4 = \dfrac{Q_4}{C_4} = \dfrac{8.0\ \mu C}{4.0\ \mu F} = 2.0$ V

$V_2 = \dfrac{Q_2}{C_2} = \dfrac{12\ \mu C}{3.0\ \mu F} = 4.0 V$

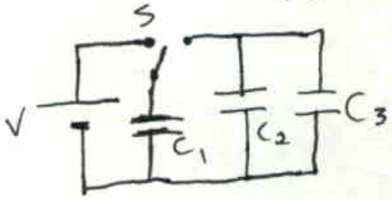
$V_1 + V_2 + V_3 = V$

$V_1 = V - V_2 - V_3 = 9 - 4 - 2 = 3.0$ V

a) $C_1 = \dfrac{Q_1}{V_1} = \dfrac{12\ \mu C}{3.0\ V} = 4.0\ \mu F$

b) $C_3 = \dfrac{Q_3}{V_3} = \dfrac{4.0\ \mu C}{2.0\ V} = 2.0\ \mu F$

(22)



$V = 10$ V, $C_1 = 10\ \mu F$, $C_2 = C_3 = 20\ \mu F$

- First complete circuit only includes V and C_1.

so $Q_1 = C_1 V = 100\ \mu C$

when the switch is thrown right, this charge becomes the total charge for the new circuit that includes only C_1, C_2, C_3.

$$Q_{total} = C_{eq} V$$

$$V = \frac{Q_{total}}{C_{eq}} = \frac{100\ \mu C}{C_1 + C_2 + C_3} = \frac{100\ \mu C}{50\ \mu F} = 2\ V$$

so new charge on C_1 is then $Q_1 = C_1 V = (10\ \mu F)(2\ V) = 20\ \mu C$

(37) $A = 8.50\ cm^2$, $d = 3.00\ mm$, $V = 6.00\ V$

pulled apart without discharge separated from battery. to $d = 8.00\ mm$

$$C_{old} = \frac{\epsilon_0 A}{d_{old}} = \frac{(8.85 \times 10^{-12}\ F/m)(8.50 \times 10^{-4}\ m^2)}{(3.00 \times 10^{-3}\ m)} = 2.51 \times 10^{-12}\ F$$

$$C_{new} = \frac{C_{old}}{8/3} = 9.40 \times 10^{-13}\ F$$

$$Q = C_{old} V_{old} = (2.51 \times 10^{-12} F)(6.00\ V) = 1.50 \times 10^{-11}\ C$$

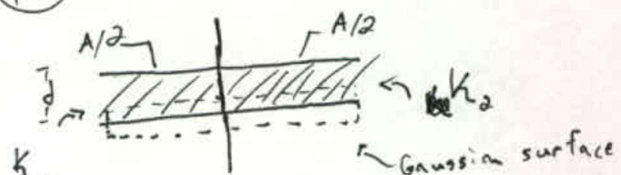
a) $V_{new} = \frac{Q}{C_{new}} = \frac{1.50 \times 10^{-11}\ C}{9.40 \times 10^{-13} F} = 16.0 V$

b) $U_{old} = \frac{Q^2}{2C_{old}} = \frac{(1.50 \times 10^{-11}\ C)^2}{2(2.51 \times 10^{-12}\ F)} = 4.51 \times 10^{-11}\ J$

c) $U_{new} = \frac{Q^2}{2C_{new}} = \frac{Q^2}{2C_{old}}(8/3) = U_{old}(8/3) = 1.20 \times 10^{-10}\ J$

d) $W = U_{new} - U_{old} = 7.52 \times 10^{-11}\ J$

(48) $A = 5.56\ cm^2$, $d = 5.56\ mm$



$K_1 = 7.00$, $K_2 = 12.0$

$$\oint K\vec{E} \cdot d\vec{A} = q/\epsilon_0$$

For Dielectric

$$K_1 E (A/2) + K_2 E (A/2) = q/\epsilon_0$$

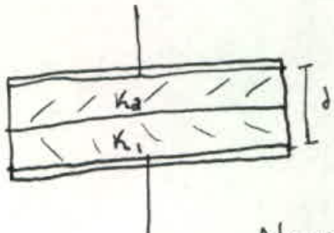
$$\frac{V}{d}\left(\frac{k_1+k_2}{2}\right)A = \frac{q}{\epsilon_0}$$

$$C = \frac{q}{V} = \frac{\epsilon_0 A}{d}\left(\frac{k_1+k_2}{2}\right) = \frac{(8.85\times10^{-12}\ F/m)(5.56\times10^{-4}\ m^2)(7+12)}{2(5.56\times10^{-3}\ m)}$$

parallel inspired

$$C = 8.41\ pF$$

(49)



$A = 7.89\ cm^2$ $d = 4.62$ mm $k_1 = 11.0$, $k_2 = 12.0$

Nonuniform field from bottom plate to top plate.

call E_1 in k_1 and E_2 in k_2.

then $$V = \int_-^+ E\,ds = E_1\left(\frac{d}{2}\right) + E_2\left(\frac{d}{2}\right) = \frac{d}{2}(E_1+E_2)$$

$$\oint k\vec{E}\cdot d\vec{A} = q/\epsilon_0$$

$$k_1E_1A = \frac{q}{\epsilon_0} \quad \text{and} \quad k_2E_2A = \frac{q}{\epsilon_0} \qquad E_2 = \frac{q}{k_2\epsilon_0 A}$$

$$E_1 = \frac{q}{k_1\epsilon_0 A}$$

gaussian surface at bottom gaussian surface at top.

$$V = \frac{d}{2}\left(\frac{q}{k_1\epsilon_0 A} + \frac{q}{k_2\epsilon_0 A}\right) = \frac{qd}{2\epsilon_0 A}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$$

$$C = \frac{q}{V} = \frac{2\epsilon_0 A}{d}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1} = \frac{2(8.85\times10^{-12}\ F/m)(7.89\times10^{-4}\ m^2)}{(4.62\times10^{-3}\ m)\left(\frac{1}{11}+\frac{1}{12}\right)}$$

series inspired

$$C = 17.3\ pF$$

(53) $A = 0.12\ m^2$ $d_0 = 1.2$ cm 120V initially. dielectric of thickness 4.0 mm, $k = 4.8$, symmetrically placed between plates.

a) $$C_0 = \frac{\epsilon_0 A}{d_0} = \frac{(8.85\times10^{-12}\ F/m)(0.12\ m^2)}{(1.2\times10^{-2}\ m)}$$

$$C_0 = 88.5\ pF$$

b) Figure 25-17 is the exact example of this. We will follow similarly.
free charge is the same before and after insertion.

c), d) $$Q = C_0V_0 = (88.5\ pF)(120\ V) = 10.6\ nC$$

$$V = \int_-^+ E\,ds = E_0(d-b) + E_1 b = E_0\left(d-b+\frac{b}{k}\right), \quad E_0 = \frac{Q}{\epsilon_0 A}$$

$$V = \frac{Q}{\epsilon_0 A}\left(d - b + \frac{b}{k}\right) \qquad b = 4.0 \text{ mm}$$

$$C = \frac{Q}{V} = \epsilon_0 A\left(d-b+\frac{b}{k}\right)^{-1} = \frac{(8.85\times10^{-12}\ F/m)(0.12\ m^2)}{\left(8.0\ \text{mm} + \frac{4.0\ \text{mm}}{4.8}\right)\times10^{-3}}$$

b) $C = 120$ pF

charge mentioned before.

e) $$E_0 = \frac{Q}{\epsilon_0 A} = \frac{(10.6\times10^{-9}\ C)}{(8.85\times10^{-12}\ C^2/Nm^2)(0.12\ m^2)} = 10.\times10^3\ N/C = 10.\ kN/C$$

f) $$E_1 = \frac{E_0}{k} = 2.08\ kN/C$$

g) $$V = \frac{Q}{C} = \frac{(10.6\times10^{-9}\ C)}{(120\times10^{-12}\ F)} = 88.3\ V$$

h) $$W = U_F - U_i = \frac{Q^2}{2C} - \frac{Q^2}{2C_0} = \frac{Q^2}{2}\left(\frac{1}{C}-\frac{1}{C_0}\right) = \frac{(10.6\times10^{-9}\ C)^2}{2}\cdot\left(\frac{1}{(120\times10^{-12}\ F)} - \frac{1}{(88.5\times10^{-12}\ F)}\right)$$

$$W = -168\ nJ$$

(62) For all arrangements, $U = 75\ \mu J,\ 100\ \mu J,\ 300\ \mu J,\ 400\ \mu J$
least to greatest.

Each individually, series, parallel.

since $U = \frac{1}{2}C_{eq}V^2$ for all cases, we know that...

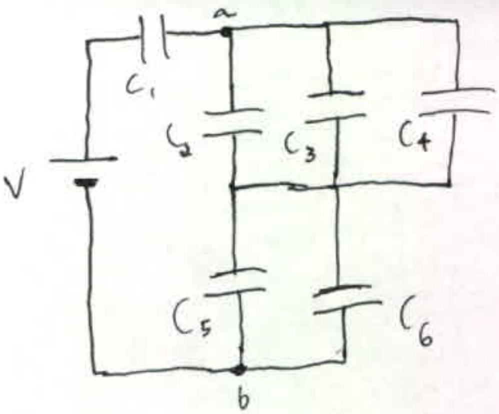
Firstly, $C_{eq,series} < C_1$ and $C_{eq,series} < C_2$. Also the parallel C_{eq} is greater than either C_1 or C_2. So looking at $U = \frac{1}{2}C_{eq}V^2$ we know that the 2 middle energy values are the values of the individual cases, $100\ \mu J$ and $300\ \mu J$

so $U_1 = \frac{1}{2}C_1V^2 \qquad U_2 = \frac{1}{2}C_2V^2$

$$C_1 = \frac{2U_1}{V^2} \qquad C_2 = \frac{2U_2}{V^2}$$

a), b) $$C_1 = \frac{2(100\times10^{-6}\ J)}{(10\ V)^2} = 2\ \mu F,\quad C_2 = \frac{2(300\times10^{-6}\ J)}{(10\ V)^2} = 6\ \mu F$$

(64)



$V = 12\ V, \quad C_1 = C_5 = C_6 = 6.0\ \mu F,$

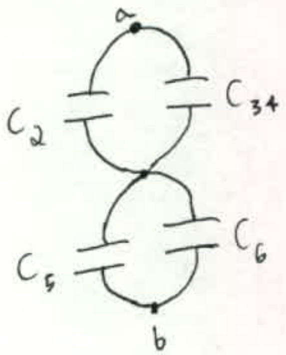
$C_2 = C_3 = C_4 = 4.0\ \mu F$

First C_3 and C_4 are parallel.

Redraw circuit from point a to point b.

$$C_{34} = C_3 + C_4 = 8.0\ \mu F$$

a)



-> equivalent to what is drawn in the figure.

so then $C_{234} = C_2 + C_{34} = 12\ \mu F$

$C_{56} = C_5 + C_6 = 12\ \mu F$

then $C_{23456} = \frac{12\ \mu F}{2} = 6.0\ \mu F$ when $C_{234} = C_{56}$ because C_{234} and C_{56} are in series.

Then C_1 is in series with ↑, so we get

$$C_{eq} = \frac{6.0\ \mu F}{2} = 3.0\ \mu F$$

so $q_{net} = C_{eq} V = (3.0 \times 10^{-6}\ F)(12\ V) = 36\ \mu C$

b) We know that C_1 has q_{net}, so $V_1 = \frac{q_{net}}{C_1} = \frac{36\ \mu C}{6.0\ \mu F} = 6.0 V = V_{23456}$

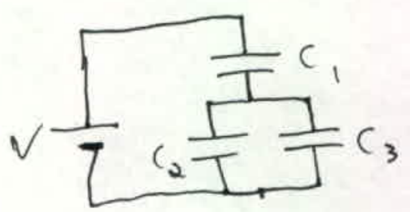
We know that since $C_{234} = C_{56}$ that V is split evenly between them, so $V_{234} = \frac{6.0\ V}{2} = 3.0\ V.$

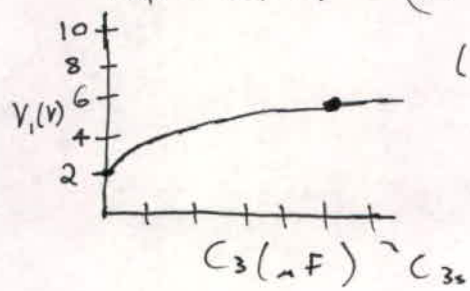
Then 2, 3, and 4 are in parallel so $V_4 = 3.0\ V.$

Therefore

$$Q_4 = C_4 V_4 = (4.0\ \mu F)(3.0\ V) = 12\ \mu C$$

Hard:
(26)





$(0,2), (\frac{5}{6} C_{3s}, 6)$ are 2 points.

$C_{3s} = 12.0\ \mu F$

$C_3 \to \infty \Rightarrow V_1 \to 10\ V.$

a) $V = V_1 + V_3$ For $C_3 \to \infty$, $C_2 + C_3 \to \infty$.

Then $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{1}{C_1} + \frac{1}{\infty} \approx \frac{1}{C_1}$

so $C_{eq} \approx C_1$ when $C_3 \to \infty$.

$Q_{net} = C_{eq} V = C_1 V$

then we have $Q_1 = C_1 V_1$. But $Q_1 = Q_{net}$ always because it's in series with the battery.

so

$$C_1 V_1 = C_{eq} V = C_1 V$$

so $V_1 = V$ when $C_3 \to \infty$.

so $V = 10\ V$.

b)

when $C_3 = 0$, $V_1 = 2\ V$

c)

$V_1 + V_2 = V \Rightarrow V_2 = 8\ V$

$Q_1 = Q_2$ so $C_1 V_1 = C_2 V_2$

$2 C_1 = 8 C_2$

$C_1 = 4 C_2$

when $C_3 = \frac{5}{6} C_{3s} = 10.0\ \mu F$,
$V_1 = 6\ V$

then $V_2 = V_3 = 4\ V$

$Q_2 + Q_3 = Q_1$

$C_2 V_2 + C_3 V_3 = C_1 V_1$

$4 C_2 + 4 C_3 = 6 C_1$

$C_2 + 10.0\ \mu F = 1.5 C_1$

$C_2 = 1.5 C_1 - 10.0\ \mu F$

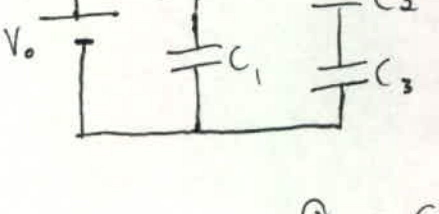
$C_1 = 4(1.5 C_1 - 10.0\ \mu F)$

$5 C_1 = 40.0\ \mu F$

$C_1 = 8.0\ \mu F$

then $C_2 = \frac{C_1}{4} = 2.0\ \mu F$

(28)



$V_0 = 12.0\ V$, $C_1 = 4.00\ \mu F$, $C_2 = 6.00\ \mu F$, $C_3 = 3.00\ \mu F$

call Q_0 the charge on C_1 when hooked up alone to the battery

$$q_0 = Q_0 = C_1 V_0 = (4.00\ \mu F)(12.0\ V) = 48.0\ \mu C$$

After the switch flips right,
The potential at the top plate of C_1 = the potential at the top plate of C_2, while the potential at the bottom plate of C_1 = the potential at the bottom plate of C_3. Therefore we need to make an equivalent capacitor with C_2 and C_3 (C_{23}) such that they are in series.

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C_{23} = \frac{C_2C_3}{C_2+C_3}$$

then the Voltage across C_1 = Voltage across C_{23}.

$$V_1 = V_{23}$$

$$\frac{q_1}{C_1} = \frac{q_{23}}{C_{23}} = \frac{q_2}{C_{23}} \quad q_{23} = q_2 = q_3 \text{ since they are in series.}$$

We also have that the total charge q_0 must be conserved, so

$$q_0 = q_1 + q_{23} = q_1 + q_2$$

$$q_1 = -q_2 + q_0$$

$$\frac{-q_2+q_0}{C_1} = \frac{q_2}{C_{23}}$$

$$q_2\left(\frac{1}{C_1} + \frac{1}{C_{23}}\right) = \frac{q_0}{C_1}$$

$$q_2 = \frac{q_0}{C_1}\left(\frac{1}{C_1} + \frac{1}{C_{23}}\right)^{-1}$$

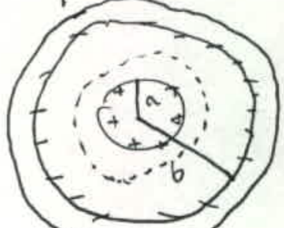
$$q_2 = \frac{48.0\ \mu C}{4.00\ \mu F}\left(\frac{1}{4} + \frac{6+3}{(6)(3)}\right)^{-1} \mu F$$

a), b), c)

$$q_2 = 16.0\ \mu C = q_3$$

$$q_1 = -q_2 + q_0 = 32.0\ \mu C$$

(66) cylindrical capacitor radii a and b. Show that $\frac{1}{2}U$ lies within a cylinder of radius $r = \sqrt{ab}$



$$u = \frac{1}{2}\epsilon_0 E^2$$

$$U(r) = \int \frac{1}{2}\epsilon_0 E^2\, dV = \frac{1}{2}\epsilon_0 \int_0^L\int_0^{2\pi}\int_a^r \left(\frac{q}{2\pi\epsilon_0 L r}\right)^2 r\, dr\, d\theta\, dz$$

$$U(r) = \frac{1}{2}\epsilon_0 \left(\frac{q}{2\pi\epsilon_0 L}\right)^2 (2\pi L)\int_a^r \frac{1}{r}\, dr = \frac{q^2}{4\pi\epsilon_0 L}\ln\left(\frac{r}{a}\right)$$

$$\frac{U(r)}{U(b)} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{\cancel{\frac{q^2}{4\pi\epsilon_0 L}}}{\cancel{\frac{q^2}{4\pi\epsilon_0 L}}}\,\frac{\ln\left(\frac{r}{a}\right)}{\ln\left(\frac{b}{a}\right)}$$

$$\frac{1}{2}\ln\left(\frac{b}{a}\right) = \ln\left(\frac{r}{a}\right)$$

$$\ln(r) = \frac{1}{2}\ln(ab)$$

$$r = \sqrt{ab}$$