Chapter 23

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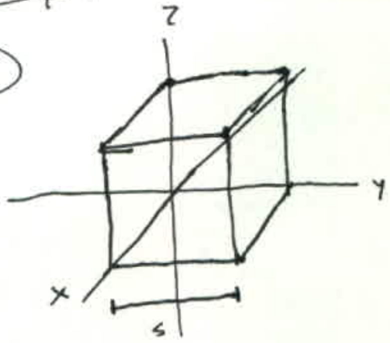
PHYS 2B FALL 2012

Easy:

(3)



cube

$s = 1.40$ m

Flux through right face.
right face has $\hat{A} = \hat{j}$

All Uniform so I can just multiply $\vec{E} \cdot \vec{A}$.

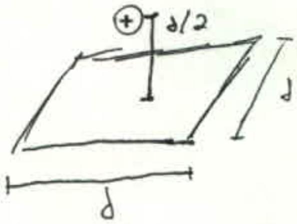
a) $\Phi = \vec{E} \cdot \vec{A} = 6.00\hat{i} \cdot A\hat{j} = 0$

b) $\Phi = \vec{E} \cdot \vec{A} = -2.00\hat{j} \cdot A\hat{j} = (-2.00 \text{ N/C})(1.40 \text{ m})^2 = -3.92 \text{ N}\cdot\text{m}^2/\text{C}$

c) $\Phi = \vec{E} \cdot \vec{A} = (-3.00\hat{i} + 4.00\hat{k}) \cdot A\hat{j} = 0$

d) total flux through the whole cube is 0 for each field. (due to uniform field)

(5)



Flux through square from proton.
Think of this square as one face of a cube enclosing the proton.

Then $\Phi = \frac{1}{6}\oint \vec{E} \cdot d\vec{A} = \frac{q}{6\epsilon_0} = \frac{e}{6\epsilon_0} = \frac{(4\pi)(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19} \text{ C})}{6}$

$\Phi = 3.01 \times 10^{-9} \text{ N}\cdot\text{m}^2/\text{C}$

one face is $\frac{1}{6}$th of the total flux through the cube

(this lets you skip that sphere step I showed to some)

(6) cube from problem 3. $s = 3.0$ m $\vec{E}$ is parallel to $\pm\hat{k}$ everywhere.

$\vec{E} = -34\hat{k}$ N/C top face, $\vec{E} = +20\hat{k}$ N/C bottom face.

$\frac{q}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A} = -34 \cdot A(\hat{k} \cdot \hat{k}) + 20 \cdot A(\hat{k} \cdot -\hat{k}) = -34s^2 - 20s^2 = -54s^2$

$$\frac{q}{\epsilon_0} = -54\, s^2 \qquad q = -54\, s^2 \epsilon_0 = -54^{N/C}(3.0\ m)^2 (8.85 \times 10^{-12}\ C^2/N\cdot m^2)$$

$$q = -4.3 \times 10^{-9}\ C$$

(17) a) conducting sphere

$$\sigma = 8.1\ \mu C/m^2, \quad d = 1.2\ m$$

$$r = d/2 = 0.60\ m$$

$$\sigma = \frac{q}{A} \Rightarrow q = \sigma A = \sigma(4\pi r^2)$$

$$q = (8.1 \times 10^{-6}\ C/m^2)(4\pi)(0.60\ m)^2 = 3.7 \times 10^{-5}\ C$$

b) $$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} = \frac{(3.7 \times 10^{-5}\ C)}{(8.85 \times 10^{-12}\ C^2/N\cdot m^2)} = 4.1 \times 10^6\ N\cdot m^2/C$$

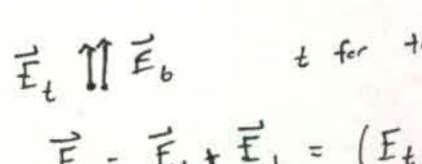
(25) infinite line.

$E = 4.5 \times 10^4$ N/C at 2.0 m away. Find λ.

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \Rightarrow \lambda = E\cdot 2\pi\epsilon_0 r = (4.5 \times 10^4\ N/C)(2\pi)(8.85 \times 10^{-12}\ C^2/N\cdot m^2)\cdot(2.0\ m)$$

$$\lambda = 5.0 \times 10^{-10}\ C/m$$

(36)



$\sigma = 1.77 \times 10^{-22}$ C/m² on each nonconducting sheet

a) $\vec{E}_t \upuparrows \vec{E}_b$ t for top, b for bottom

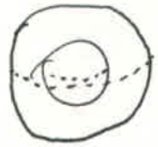
above

$$\vec{E} = \vec{E}_t + \vec{E}_b = (E_t + E_b)\hat{j} = \left(\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}\right)\hat{j} = \frac{\sigma}{\epsilon_0}\hat{j} = \frac{(1.77 \times 10^{-22}\ C/m^2)}{(8.85 \times 10^{-12}\ C^2/Nm^2)}\hat{j}$$

$$\vec{E} = (2.00 \times 10^{-11}\ N/C)\hat{j}$$

b) between $\uparrow\vec{E}_b\ \downarrow\vec{E}_t$ $$\vec{E} = \vec{E}_t + \vec{E}_b = (E_t - E_b)\hat{j} = \left(\frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0}\right)\hat{j} = 0$$

c) below $\vec{E}_b \downdownarrows \vec{E}_t$ $$\vec{E} = \vec{E}_t + \vec{E}_b = (-E_t - E_b)\hat{j} = -\frac{\sigma}{\epsilon_0}\hat{j} = -(2.00 \times 10^{-11}\ N/C)\hat{j}$$

(45) 

$q_{in} = 4.00 \times 10^{-8}\ C$, $q_{out} = 2.00 \times 10^{-8}\ C$

$r_{in} = 10.0\ cm$ $\quad r_{out} = 15.0\ cm$

a) E at $r = 12.0\ cm$ and b) $r = 20.0\ cm$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$EA = \frac{q_{enc}}{\epsilon_0} \qquad E = \frac{q_{enc}}{\epsilon_0 A} = \frac{q_{enc}}{4\pi\epsilon_0 r^2}$$

a) $$E = \frac{\overset{q_{in}}{(4.00 \times 10^{-8}\ C)}(8.99 \times 10^{9}\ N m^2/C^2)}{(12.0 \times 10^{-2} m)^2} = 2.50 \times 10^{4}\ N/C$$

b) $$E = \frac{(\overset{q_{in}}{4.00 \times 10^{-8}} \overset{+}{+} \overset{q_{out}}{2.00 \times 10^{-8}}\ C)(8.99 \times 10^{9}\ N m^2/C^2)}{(20.0 \times 10^{-2}\ m)^2} = 1.35 \times 10^{4}\ N/C$$

(46) $\Phi = -750\ N \cdot m^2/C$ 10.0 cm radius Gaussian sphere.

a) radius is doubled, same flux $\Phi = -750\ N \cdot m^2/C$

because Gauss's law says $\Phi = \frac{q_{enc}}{\epsilon_0}$.

b)

$$\Phi = \frac{q_{enc}}{\epsilon_0} \Rightarrow q_{enc} = \Phi \epsilon_0 = (-750\ N \cdot m^2/C)(8.85 \times 10^{-12}\ C^2/Nm^2)$$

$$q_{enc} = -6.64 \times 10^{-9}\ C$$

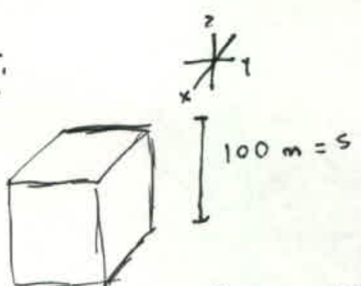
(68) A die is just a cube. inward for odd N, outward for even N.

$$\frac{q}{\epsilon_0} = \Phi = (-1 + 2 - 3 + 4 - 5 + 6)(10^3\ N \cdot m^2/C) = 3 \times 10^3\ N \cdot m^2/C$$

$$q = (3 \times 10^3\ N \cdot m^2/C)(8.85 \times 10^{-12}\ C^2/Nm^2)$$

$$q = 2.66 \times 10^{-8}\ C$$

Medium:

(13) 

top face at 300 m $\quad \vec{E}_t = (60.0\ N/C)\,\hat{k} = -E_t \hat{k}$

bottom face at 200 m $\quad \vec{E}_b = -(100.\ N/C)\,\hat{k} = -E_b \hat{k}$

$$\frac{q}{\epsilon_0} = \Phi = \oint \vec{E}\, d\vec{A} = \vec{E}_t \cdot A\hat{k} + \vec{E}_b \cdot (-A\hat{k})$$

$$q = \epsilon_0 \left(-E_t A + E_b A \right) = \epsilon_0 A \left(E_b - E_t \right) = \epsilon_0 s^2 (E_b - E_t)$$

$$q = (8.85 \times 10^{-12}\ C^2/Nm^2)(100\ m)^2 (100\ N/C - 60.0\ N/C)$$

$$q = 3.54 \times 10^{-6}\ C$$

(15) Another cube. $+q$ at corner.

a) multiple of q/ϵ_0 that gives the flux through each cube face forming the corner

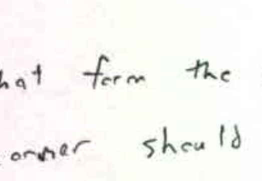
b) each other cube face.

Firstly we know that there are 3 faces that form the corner and 3 that do not. The 3 that form the corner should have the same flux, just as the 3 that do not form the corner should have the same flux. By symmetry.

a) $\vec{E}$ is parallel to the corner faces so $\Phi = 0$

b) Take another Gaussian cube that has 2 times the edge length so that $+q$ is at the exact center of the cube. The flux through each face of this new cube is $\frac{1}{6}\frac{q}{\epsilon_0}$. But our original non-corner face only takes up $\frac{1}{4}$ of a new face.

So then each non-corner face of the original cube has $\Phi = \frac{1}{4} \cdot \frac{1}{6}\frac{q}{\epsilon_0} = \frac{1}{24}\frac{q}{\epsilon_0}$



← Each quarter of the larger face gets the same flux by symmetry.

(21) point charge in cavity.
$q = 3.0 \times 10^{-6}\ C$ Conducting shell has net charge $10 \times 10^{-6}\ C$.

a) cavity wall must have charge $-q = -3.0 \times 10^{-6}\ C$ so that the E field inside the conductor is 0.

b) outer surface, $q_{os} + q_{cav} = q_{net}$

$$q_{os} = q_{net} - q_{cav} = 10 \times 10^{-6} - (-3.0 \times 10^{-6})\ C$$

$$q_{os} = 13 \times 10^{-6}\ C = 1.3 \times 10^{-5}\ C$$

fixed negative charge.

(27) $\lambda = -3.6\ nC/m$ thin-walled nonconducting shell.

looking into the wire and shell.

$r = 1.5\ cm$

σ on outside surface. $E = 0$ outside.

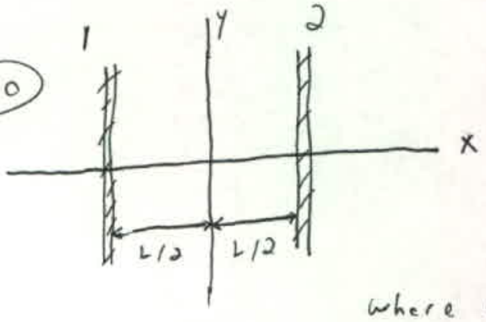
$\sigma \cdot (2\pi r) + \lambda = 0$ for $\hat{r}$

$\sigma \cdot (2\pi r) = -\lambda$

$$\sigma = \frac{-\lambda}{2\pi r} = \frac{(3.6 \times 10^{-9}\ C/m)}{2\pi\,(1.5 \times 10^{-2}\ m)}$$

$$\sigma = 3.8 \times 10^{-8}\ C/m^2$$

(30)



very long parallel lines

$L = 8.0\ cm$. $\lambda_1 = 6.0 \times 10^{-6}\ C/m$

$\lambda_2 = -2.0 \times 10^{-6}\ C/m$

where along the x axis is the E field 0?

Since $\lambda_1 = -\lambda_2$, x cannot be between the lines.

Then since $|\lambda_1| > |\lambda_2|$, we need to be closer to line 2 to get cancellation.

$$E_{line} = \frac{\lambda}{2\pi\epsilon_0 r}$$

x-value to the right of line 2.

$$E_1 = E_2$$

$$\frac{|\lambda_1|}{\cancel{2\pi\epsilon_0}(x + \frac{L}{2})} = \frac{|\lambda_2|}{\cancel{2\pi\epsilon_0}(x - L/2)}$$

$$|\lambda_1|(x - L/2) = |\lambda_2|(x + \tfrac{L}{2})$$

$$(|\lambda_1| - |\lambda_2|)\,x = (|\lambda_1| + |\lambda_2|)\frac{L}{2}$$

$$x = \frac{|\lambda_1| + |\lambda_2|}{|\lambda_1| - |\lambda_2|} \cdot \frac{L}{2} = \frac{(8.0 \times 10^{-6}\ C/m)}{(4.0 \times 10^{-6}\ C/m)} \cdot \frac{(8.0 \times 10^{-2}\ m)}{2}$$

$$x = 8.0 \times 10^{-2}\ m = 8.0\ cm$$

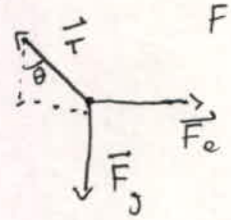
(39)

$m = 1.0\ mg$, $q = 2.0 \times 10^{-8}\ C$ $\theta = 30°$.

$$E_{sheet} = \frac{\sigma}{2\epsilon_0}$$

$$F_e = qE_{sheet} = \frac{q\sigma}{2\epsilon_0}$$

FBD.



$$F_g = mg$$

$$T\sin\theta = F_e \quad , \quad T\cos\theta = F_g$$

$$T = \frac{F_e}{\sin\theta} \longrightarrow \frac{F_e \cos\theta}{\sin\theta} = F_g$$

$$\frac{q\sigma}{2\epsilon_0} = mg\tan\theta \longleftarrow F_e = F_g\tan\theta$$

$$\sigma = \frac{2\epsilon_0 mg\tan\theta}{q} = \frac{2(8.85\times10^{-12}\ C^2/Nm^2)(1.0\times10^{-6}\ kg)(9.8\ m/s^2)\tan(30°)}{2.0\times10^{-8}\ C}$$

$$\sigma = 5.0 \times 10^{-9}\ C/m^2$$

(42) $1.0\ m^2$ metal plates, $5.0\ cm$ apart, $|q|$ same but opposite signs.

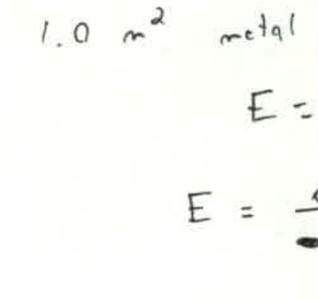
$$E = 55\ N/C$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{|q|/A}{\epsilon_0}$$

$$|q| = \epsilon_0 EA = \cancel{(8.85\times10^{-12}\ C^2/Nm^2)(55\ N/C)(1.0\ m^2)}$$

$$|q| = 4.9\times10^{-10}\ C$$

(49)



$a = 2.00\ cm$ solid sphere nonconducting uniform charge $q_1 = 5.00$ fC

$b = 2.00a$, $c = 2.40a$

shell has net charge $q_2 = -q_1$

magnitude of $\vec{E}$ at

a) $r = 0$, b) $r = a/2.00$, c) $r = a$, d) $r = 1.50a$, e) $r = 2.30a$, f) $r = 3.50a$. Net charge on g) inner and h) outer surfaces of shell.

a) $\overset{r=0}{E=0}$, No enclosed charge.

b) $\overset{r=a/2}{\oint \vec{E} \cdot d\vec{A}} = \frac{q_{enc}}{\epsilon_0}$

$E \cdot A = \frac{q_1 V_{enc}}{\epsilon_0 V_T}$ ~ total volume

$$E = \frac{q_1\left(\cancel{\frac{4}{3}\pi}\left(\frac{a}{2}\right)^{\cancel{3}}\right)}{\epsilon_0\left(4\pi\cancel{\left(\frac{a}{2}\right)^2}\right)\left(\cancel{\frac{4}{3}\pi}\, a^3\right)} = \frac{q_1\left(\frac{a}{2}\right)}{4\pi\epsilon_0 a^3} = \frac{q_1}{2(4\pi\epsilon_0)a^2}$$

$$E = \frac{(5.00\times10^{-15}\ C)(8.99\times10^9\ N\cdot m^2/C^2)}{2(2.00\times10^{-2}\ m)^2} = 5.62\times10^{-2}\ N/C$$

$$E = 0.0562\ N/C$$

c) $r = a$ $\quad E = \frac{q_1}{4\pi\epsilon_0 a^2} = 0.112\ N/C$ - 2 times part (a).

d) $r = 1.50\,a$ ~ between a and b. Sphere of charge acts like point charge.

$$E = \frac{q_1}{4\pi\epsilon_0(1.5a)^2} = \frac{q_1}{2.25(4\pi\epsilon_0)a^2} = 0.0499\ N/C$$

e) $r = 2.30\,a$ ~ between b and c -> in conducting shell.

$E = 0.$

f) $r = 3.50\,a$ ~ outside of conducting shell. No charge enclosed because

$q_1 + q_2 = 0$, so $E = 0$.

g) inner surface net charge.

$q_{in} = -q_1 = -5.00\ fC$ so that $E = 0$ in spherical shell conductor.

h) outer surface net charge

$q_{out} + q_{in} = q_{net}$

$q_{out} = q_{net} - q_{in} = -q_1 - (-q_1) = 0$

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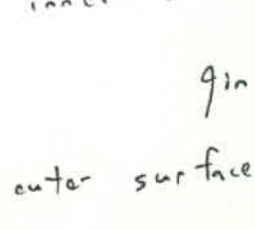


spherical shell uniform charge density $\rho = 1.84\ nC/m^3$

$a = 10.0\ cm$, $b = 2.00a$.

magnitude of $\vec{E}$ at a) $r = 0$, b) $r = a/2.00$, c) $r = a$, d) $r = 1.50a$, e) $r = b$, f) $r = 3.00b$

a) $E = 0$ at $r = 0$

b) $E = 0$ $r = a/2$ No charge enclosed.

c) $E = 0$ $r = a$

Volume of shell is big sphere minus little sphere.

d) $\oint_{r=1.50a} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$

$$E \cdot A = \frac{\rho V_{enc}}{\epsilon_0} \qquad E = \frac{\rho\left(\frac{4}{3}\pi(1.50a)^3 - \frac{4}{3}\pi(a)^3\right)}{\epsilon_0\left(4\pi(1.50a)^2\right)}$$

$$E = \frac{\rho a\left(\frac{4}{3}\pi\right)\left((1.50)^3 - 1\right)}{4\pi\epsilon_0 (1.50)^2} = \frac{\frac{4}{3}\pi\rho a\,(2.375)}{4\pi\epsilon_0(2.25)}$$

$$E = \frac{\frac{4}{3}\pi\,(1.84\times10^{-9}\ C/m^3)(10.0\times10^{-2}\ m)(2.375)(8.99\times10^{9}\ Nm^2/C^2)}{2.25}$$

$E = 7.31$ N/C

e) $r = b$ Now I can just use $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

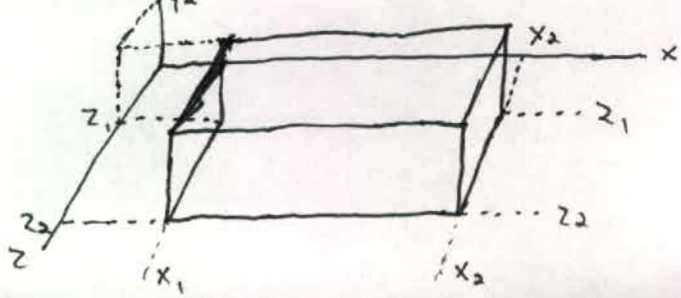
$$E = \frac{1}{4\pi\epsilon_0}\frac{\rho V}{b^2} = \frac{1}{4\pi\epsilon_0}\frac{\rho\left(\frac{4}{3}\pi(2.00a)^3 - \frac{4}{3}\pi a^3\right)}{(2.00a)^2} = \frac{\rho a\left(\frac{4}{3}\pi\right)(7)}{4\pi\epsilon_0(4)}$$

$$E = \frac{\frac{4}{3}\pi\rho a}{4\pi\epsilon_0}\frac{(7)}{(4)} = 12.1\ N/C$$

f) $r = 3.00\,b$. Again $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$. but q is the same from (e)

$$E = \frac{\frac{4}{3}\pi\rho a^3}{4\pi\epsilon_0}\frac{(7)}{(6a)^2} = \frac{\frac{4}{3}\pi\rho a}{4\pi\epsilon_0}\frac{(7)}{(36)} = 1.35\ N/C$$

Hard: 16



$24.0\,\epsilon_0$ C net charge enclosed in box.

$\vec{E} = \left[(10.0 + 2.00x)\hat{i} - 3.00\hat{j} + bz\hat{k}\right]$ N/C

$y_1 = 0$

$y_2 = 1.00$ m, $x_1 = 1.00$ m, $x_2 = 4.00$ m, $z_1 = 1.00$ m, $z_2 = 3.00$ m

What is b?

6 faces of the box to calculate flux for. Call them

$\Phi_{+x}, \Phi_{-x}, \Phi_{+y}, \Phi_{y}, \Phi_{+z}, \Phi_{-z}$. Remember the (-) for all Φ_-.

Although the E field varies with x and z, because the x variation only lies in the x-component and the z variation in the z-component, our Φ calculations do not require integrals. Just $\Phi = \vec{E} \cdot \vec{A}$ since $\vec{E}$ is constant on each face.

$$\Phi_{+x} = (10 + 2(4))(3-1)(1-0) = 36.0 \text{ Nm}^2/C$$

$$\Phi_{-x} = -(10 + 2(1))(2)(1) = -24.0 \text{ Nm}^2/C$$

$$\Phi_{+y} = (-3)(4-1)(3-1) = -18.0 \text{ Nm}^2/C$$

$$\Phi_{-y} = -(-3)(4-1)(3-1) = 18.0 \text{ Nm}^2/C$$

$$\Phi_{+z} = b(3)(4-1)(1-0) = 9b \text{ Nm}^2/C$$

$$\Phi_{-z} = -b(1)(4-1)(1-0) = -3b \text{ Nm}^2/C$$

$$\frac{q}{\epsilon_0} = \Phi = \Phi_{+x} + \Phi_{-x} + \Phi_{+y} + \Phi_{-y} + \Phi_{+z} + \Phi_{-z}$$

$$24 = 36 - 24 \cancel{-18} + \cancel{18} + 9b - 3b$$

$$24 = 12 + 6b$$

$$b = 2$$

(53) $R = 5.60$ cm solid nonconducting sphere $\rho = (14.1 \text{ pC/m}^3) r/R$

$\rho = \rho_0 r/R$

a) $q(r) = \int \rho \, dV = \frac{\rho_0}{R} \int_0^{2\pi}\int_0^{\pi}\int_0^{r} r \, (r^2 \sin\theta \, dr \, d\theta \, d\phi)$

charge enclosed as a function of r.

$$q(r) = \frac{4\pi\rho_0}{R} \int_0^r r^3 dr = \frac{4\pi\rho_0}{R}\left(\frac{r^4}{4}\right)\Big]_0^r = \frac{\pi\rho_0}{R} r^4$$

total charge $\to$ $q(R) = \frac{\pi\rho_0}{R} R^4 = \pi\rho_0 R^3 = \pi(14.1 \times 10^{-12} \text{ C/m}^3)(5.60 \times 10^{-2} \text{ m})^3$

$q(R) = 7.78 \times 10^{-15}$ C

b) magnitude of $\vec{E}$ at $r=0$, c) $r = R/2.00$, d) $r=R$, e) Graph E versus r.

$E = 0$ at $r=0$.

c) $r = R/2$

$$q(R/2) = \frac{\pi \rho_0}{R}(R/2)^4 = \frac{\pi \rho_0 R^3}{16}$$

function.

$$\Phi = \frac{q}{\epsilon_0} = \oint \vec{E}\cdot d\vec{A} = EA$$

$$E = \frac{q}{\epsilon_0 A} = \frac{\cancel{\pi} \rho_0 R^3/16}{\epsilon_0 (4\cancel{\pi}(R/2)^2)} = \frac{\rho_0}{16\epsilon_0} R$$

$$E = \frac{(14.1\times10^{-12}\, C/m^3)(5.60\times10^{-2}\, m)}{16\,(8.85\times10^{-12}\, C^2/Nm^2)} = 5.58\times10^{-3}\ N/C$$

d) $r = R$

function value $q(R)$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q(R)}{R^2} = \frac{(8.99\times10^9\ Nm^2/C^2)(7.78\times10^{-15}\ C)}{(5.60\times10^{-2}\ m)^2}$$

$$E = 2.23\times10^{-2}\ N/C$$

e)

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} \quad \text{for } r \le R,$$

$$E\cdot A = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{\epsilon_0 A} = \frac{(\cancel{\pi}\rho_0 r^{\cancel{4}\,2}/R)}{\epsilon_0 (4\cancel{\pi}\cancel{r^2})} = \frac{\rho_0 r^2}{4\epsilon_0 R} = \frac{\rho_0}{4\epsilon_0}\frac{r^2}{R}$$

where $\frac{\rho_0}{4\epsilon_0 R}$ is a constant.

For $r \ge R$,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q_{total}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(\pi\rho_0 R^3)}{r^2} = \frac{\rho_0 R^3}{4\epsilon_0}\frac{1}{r^2} = \frac{\rho_0}{4\epsilon_0}\frac{R^3}{r^2}$$

We can see that both match up at $r=R$.

parabola for $r \le R$, then $\frac{1}{r^2}$ for $r \ge R$