Easy:

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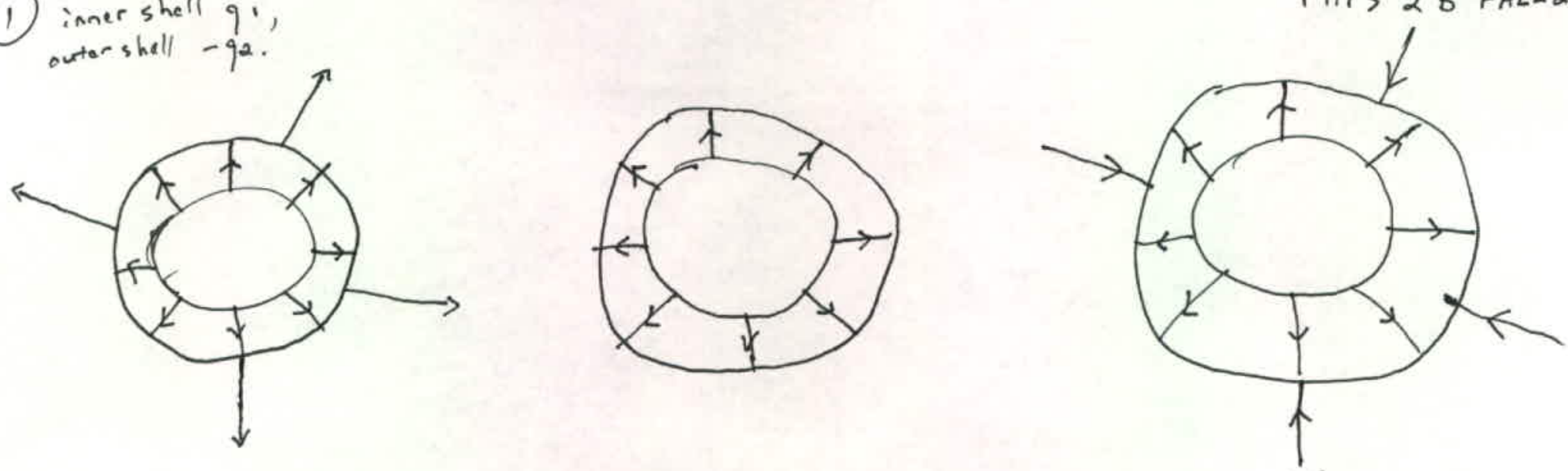
PHYS 2B FALL 2012

(1) inner shell q_1,
outer shell $-q_2$.



$q_1 > q_2$ $\quad$ $q_1 = q_2$ $\quad$ $q_1 < q_2$

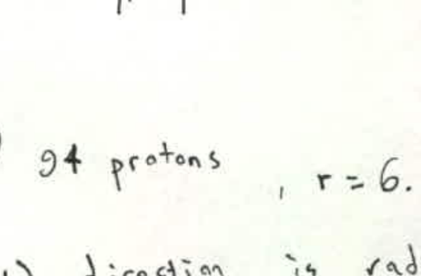
(3) 94 protons , $r = 6.64$ fm

b) direction is radially outward due to protons being positive.

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = \frac{(8.99\times10^9\ N\cdot m^2/C^2)\cdot 94\cdot(1.60\times10^{-19}\ C)}{(6.64\times10^{-15}\ m)^2}$$

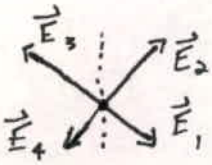
a) $E = 3.07\times10^{21}$ N/C

(7)



$a = 5.00$ cm $\quad$ $q_3 = 20.0$ nC
$q_1 = +10.0$ nC $\quad$ $q_4 = -10.0$ nC
$q_2 = -20.0$ nC

At the center of the square, we expect something like this:



We notice that the x-components exactly cancel with each other since (r) is the same for all $\vec{E}$'s so only the direction and magnitude of $|q|$ differentiates them - giving cancellation.

Furthermore, since all $\vec{E}$ vectors are at 45° angles from vertical, we can say:

$$E_y = (E_2 + E_3 - E_1 - E_4)\cos(45°)$$

$$E_y = \frac{1}{4\pi\epsilon_0}\cdot\frac{1}{r^2}\left(|-20.0\,nC| + |20.0\,nC| - |10.0\,nC| - |-10.0\,nC|\right)\cdot\cos(45°)$$

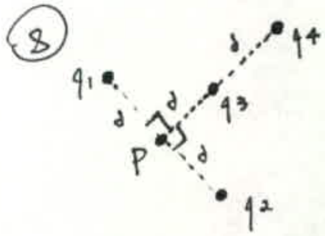
$$E_y = \frac{1}{4\pi\epsilon_0}\,\frac{(20.0\,nC)}{r^2}\cdot\cos(45°)$$

$r = a\cdot\frac{\sqrt{2}}{2}$,
$$E_y = \frac{(8.99\times10^9\ N\cdot m^2/C^2)(20.0\times10^{-9}\ C)\left(\frac{\sqrt{2}}{2}\right)}{\left(5.00\times10^{-2}\,m\cdot\left(\frac{\sqrt{2}}{2}\right)\right)^2}$$

$$E_y = 1.02\times10^5\ N/C$$

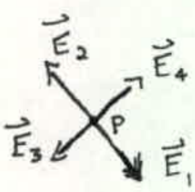
so $\vec{E} = (1.02\times10^5\ N/C)\,\hat{j}$. We can see from the $\vec{E}$ field picture that the resultant vector will be upwards.

(8)



$q_1 = q_2 = +5e$, $q_3 = +3e$, $q_4 = -12e$
$d = 5.0\ \mu m$

$\vec{E}$ at point P?



We can see that $\vec{E}_1$ and $\vec{E}_2$ have the opposite directions but equal magnitudes (same distance same $|q|$) so upon addition they cancel to 0.

That leaves $\vec{E}_3$ and $\vec{E}_4$. They lie upon the same line and also have opposite directions. Check magnitudes.

$$E_3 = \frac{1}{4\pi\epsilon_0}\,\frac{(3e)}{d^2},\quad E_4 = \frac{1}{4\pi\epsilon_0}\,\frac{|-12e|}{(2d)^2}$$

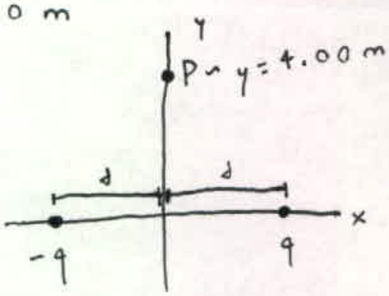
$$E_3 = \frac{3}{4\pi\epsilon_0}\,\frac{e}{d^2},\quad E_4 = \frac{3}{4\pi\epsilon_0}\,\frac{e}{d^2}$$

zero vector.

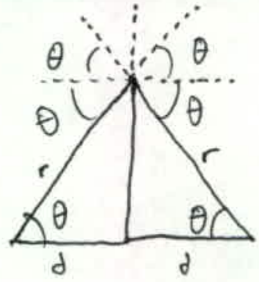
$E_3 = E_4$. Again, complete cancellation. So $\vec{E} = \vec{0}$ at P.

(9) $-q = -3.20 \times 10^{-19}\ C$, $q = 3.20 \times 10^{-19}\ C$

$d = 3.00\ m$



$\vec{E}$ at P?



We can see that the y-component of $\vec{E}$ at P must be 0 because $\vec{E}_q$ and $\vec{E}_{-q}$ have the same magnitudes (same $|q|$ and r) but each one opposes the other across the x-axis.

($\vec{E}_q$ is θ north of $(-x)$ and $\vec{E}_{-q}$ is θ south of $(-x)$)

That leaves E_x. We notice $\vec{E}$ goes to $(-x)$.

$$E_x = (-)\left(E_q \cos\theta + E_{-q}\cos\theta \right)$$

$$E_x = -\left(E_q + E_{-q} \right)\cos\theta \qquad \cos\theta = \frac{d}{r} = \frac{d}{\sqrt{d^2+y^2}}$$

$$E_x = -\frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r^2}\left(|q| + |-q| \right)\cos\theta$$

$$E_x = -\frac{1}{4\pi\epsilon_0} \frac{(2q)}{\left(\sqrt{d^2+y^2}\right)^2} \cdot \frac{d}{\sqrt{d^2+y^2}}$$

$$E_x = -\frac{(8.99 \times 10^9\ N{\cdot}m^2/C^2) \cdot 2(3.20 \times 10^{-19}\ C) \cdot (3.00\ m)}{\left((3.00\ m)^2 + (4.00\ m)^2 \right)^{3/2}}$$

$$E_x = -1.38 \times 10^{-10}\ N/C$$

so

a) $E = 1.38 \times 10^{-10}\ N/C$ ~ magnitude

b) $180°$ clockwise from $(+x)$ ~ direction.

(22) a) 

$q = -300e$

$\ell = 2\pi r \left(\frac{\theta}{360}\right) = 2\pi(4.00\ cm)\left(\frac{40}{360}\right) = \frac{8\pi}{9} \times 10^{-2}\ m$

$\lambda = \frac{q}{\ell} = \frac{-300e}{\left(\frac{8\pi}{9}\times 10^{-2}\ m\right)} = -1.72 \times 10^{-15}\ C/m$

b) 

$q = -300e$

$A = \pi r^2 = \pi(2.00 \times 10^{-2}\ m)^2 = 4\pi \times 10^{-4}\ m^2$

$\sigma = \frac{q}{A} = \frac{-300e}{(4\pi \times 10^{-4}\ m^2)} = 3.82 \times 10^{-14}\ C/m^2$

c) 

$q = -300e$

$A = 4\pi r^2 = 4\pi(2.00 \times 10^{-2}\ m)^2 = 16\pi \times 10^{-4}\ m^2$ ~ 4 times last part, same charge.

$\sigma = \frac{q}{A} = 9.55 \times 10^{-15}\ C/m^2$

d) 

$q = -300e$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.00 \times 10^{-2}\ m)^3 = \frac{32}{3}\pi \times 10^{-6}\ m^3$

$\rho = \frac{q}{V} = \frac{-300e}{\left(\frac{32\pi}{3} \times 10^{-6}\ m^3\right)} = 1.43 \times 10^{-12}\ C/m^3$

(35) $E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$

$R = 0.600\ m$

$E(z=0) = \frac{\sigma}{2\epsilon_0}$, $E(z) = \frac{1}{2}E(z=0)$ for what z?

$\frac{\cancel{\sigma}}{\cancel{2}\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) = \frac{1}{2}\left(\frac{\cancel{\sigma}}{\cancel{2}\epsilon_0}\right)$

$1 - \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{z}{\sqrt{z^2+R^2}}$

$z^2 + R^2 = 4z^2$

$3z^2 = R^2$

$z = \frac{\sqrt{3}}{3}R = \frac{\sqrt{3}}{3}(0.600\ m) = 0.346\ m$ above z = 0.

(42) $E = 3.0 \times 10^6$ N/C

a) $q = -e \Rightarrow F = |q|E = (e)(3.0\times10^6 \text{ N/C}) = 4.8\times10^{-13}$ N

b) $q = +e \Rightarrow F = |q|E = 4.8\times10^{-13}$ N

(43) $E = 2.00 \times 10^4$ N/C e^- (Ignore gravitation)

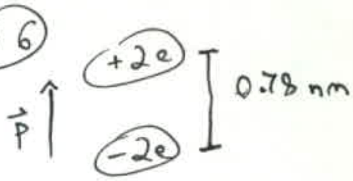
$F = |q|E$, $F = ma$

$ma = |q|E$

$$a = \frac{|q|E}{m} = \frac{(1.60\times10^{-19}\text{ C})(2.00\times10^4 \text{ N/C})}{(9.11\times10^{-31}\text{ kg})}$$

$a = 3.51 \times 10^{15}$ m/s²

(56)



$E = 3.4\times10^6$ N/C , $\tau = pE\sin\theta$, $p = qd$

a) parallel $\tau = pE\sin(0) = 0$

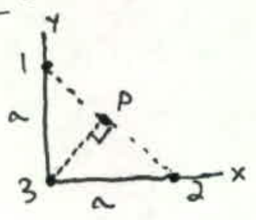
b) perpendicular $\tau = pE\sin(90°) = pE = (2e)(0.78\times10^{-9}\text{ m})(3.4\times10^6 \text{ N/C})$

$\tau = 8.5\times10^{-22}$ N·m

c) antiparallel $\tau = pE(\sin \overset{180°}{(180)}) = 0$

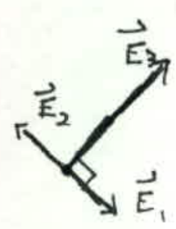
Medium:

(15)



$q_1 = q_2 = +e$, $q_3 = +2e$

$a = 6.00$ µm

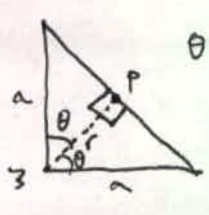


a) magnitude , b) direction of $\vec{E}$ at point P.

$\vec{E}_1$ and $\vec{E}_2$ cancel each other out due to having the same magnitude ($|q|$ and r the same) but opposite directions. That leaves $\vec{E}_3$.

$\vec{E} = \vec{E}_3$.

$$E = E_3 = \frac{1}{4\pi\epsilon_0}\frac{|q_3|}{r^2}$$



$\theta = 45°$

$r = \frac{a}{\sqrt{2}}$

$$E = \frac{1}{4\pi\epsilon_0}\frac{(2e)}{(a/\sqrt{2})^2} = \frac{(8.99\times10^{9}\ N\cdot m^2/C^2)(4)(1.60\times10^{-19}\ C)}{(6.00\times10^{-6}\ m)^2}$$

a) $E = 1.60\times10^{2}\ N/C = 160.\ N/C$

b) $45°$ north of east, or also $45°$ counterclockwise from $(+x)$

(18) $E = \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3} + E_{next}$, what is E_{next}?

$$E = \frac{qd}{2\pi\epsilon_0 z^3}\left(1-\left(\frac{d}{2z}\right)^2\right)^{-2} \quad \text{from } (22\text{-}7)$$

$$E = \frac{qd}{2\pi\epsilon_0 z^3}\left[1 + (-2)\left(-\left(\frac{d}{2z}\right)^2\right) + \dots\right]$$

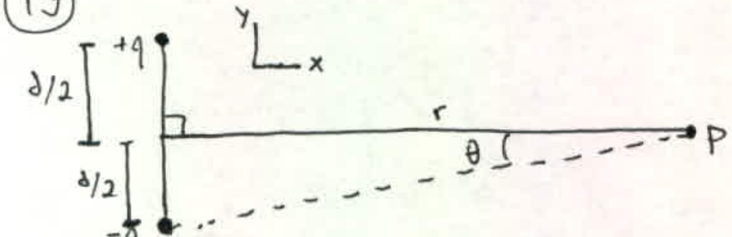
$$E \approx \frac{qd}{2\pi\epsilon_0 z^3}\left[1 + 2\frac{d^2}{4z^2}\right]$$

(22-8) $\qquad E_{next}$ term.

$$E \approx \frac{qd}{2\pi\epsilon_0 z^3} + \frac{qd^3}{4\pi\epsilon_0 z^5}$$

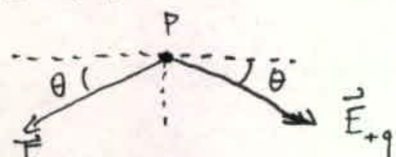
so $E_{next} = \frac{qd^3}{4\pi\epsilon_0 z^5}$

(19)



$r >> d.$

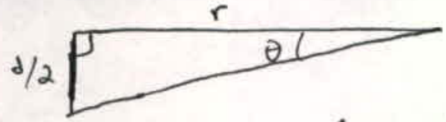
a) magnitude, b) direction. of $\vec{E}$ field at P.



Notice that the x-components will exactly cancel due to both $\vec{E}_{+q}$ and $\vec{E}_{-q}$ having the same magnitude but opposing each other across the y-axis.

That leaves the y-components

$$E_y = -(E_{+q} + E_{-q}) \sin\theta$$



$$\sin\theta = \frac{d/2}{\sqrt{r^2 + (d/2)^2}}$$

$$E = -E_y = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{(r^2 + (d/2)^2)} (|q| + |-q|) \cdot \frac{d/2}{\sqrt{r^2 + (d/2)^2}}$$

a) $E = \frac{1}{4\pi\epsilon_0} \frac{qd}{(r^2 + (d/2)^2)^{3/2}}$ is the exact answer.

$r >> d \Rightarrow \left(\frac{d}{2r}\right)^2 << 1$.

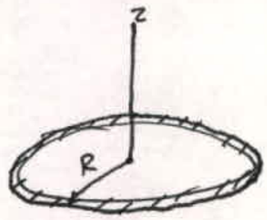
$E = \frac{1}{4\pi\epsilon_0} \frac{qd}{r^3} \left(1 + \left(\frac{d}{2r}\right)^2\right)^{-3/2}$. Then using the binomial expansion...

$$E \approx \frac{1}{4\pi\epsilon_0} \frac{qd}{r^3} \left[1 - \frac{3}{2}\left(\frac{d}{2r}\right)^2\right]$$

a) $E \approx \frac{1}{4\pi\epsilon_0} \frac{qd}{r^3} \left[1 - \frac{3d^2}{8r^2}\right]$ is the approximate answer using $r >> d$.

b) $90°$ clockwise from $(+x)$

(24) Q, radius R.



magnitude of $\vec{E}$ at a) $z = 0$, b) $z = \infty$, c) what positive value z is the magnitude maximum?

d) $R = 2.00$ cm, $Q = 4.00\ \mu C$, evaluate maximum magnitude

$E = \frac{1}{4\pi\epsilon_0} \frac{Qz}{(z^2 + R^2)^{3/2}}$ for a charged ring

a) $E = 0$

b) $E = 0$ because the denominator has a larger power of z.

c) $\frac{dE}{dz} = 0$

$$0 = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{(z^2+R^2)^{3/2}} - \frac{(3/2)(2z) \cdot z}{(z^2+R^2)^{5/2}}\right]$$

$$0 = \frac{z^2 + R^2 - 3z^2}{(z^2+R^2)^{5/2}}$$

$$0 = -2z^2 + R^2$$

$$z^2 = \frac{R^2}{2}$$

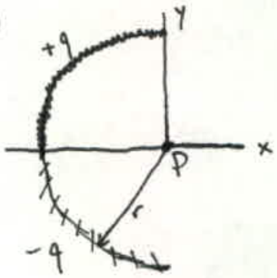
$$z = \frac{R}{\sqrt{2}}$$

d) $$E = \frac{1}{4\pi\epsilon_0} \frac{Q\left(R/\sqrt{2}\right)}{\left(\frac{R^2}{2} + R^2\right)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{3}{2}\right)^{3/2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \frac{3^{3/2}}{4}$$

$$E = \frac{(8.99 \times 10^9\ N\cdot m^2/C^2)\,(4.00 \times 10^{-6}\ C)\cdot 3^{3/2}}{(2.00 \times 10^{-2}\ m)^2 \cdot 4}$$

$$E = 1.17 \times 10^8\ N/C$$

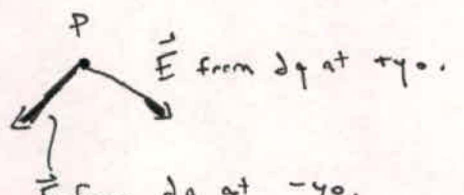
(26)



$r = 5.00$ cm, $+q = 4.50$ pC, $-q = -4.50$ pC.

$\vec{E}$ at P? a) magnitude, b) direction.

we can see that the top and bottom halves will pairwise cancel their x-components. That leaves the y-components.



$$dE_y = dE_{y,+q} + dE_{y,-q}$$ add the pair of y-components together. We can see both contributions are negative from our vector drawing.

$$dE_y = \frac{-1}{4\pi\epsilon_0} \frac{(dq + dq)}{r^2} \sin\theta$$

$$dE_y = \frac{-1}{2\pi\epsilon_0} \frac{dq}{r^2} \sin\theta \qquad dq = \lambda\, ds, \quad \lambda = \frac{q}{(2\pi r/4)}$$ ← a quarter of a circle has charge q.

$$\lambda = \frac{2q}{\pi r}$$

$$dE_y = -\frac{1}{2\pi\epsilon_0} \frac{\lambda\, ds}{r^2} \sin\theta \qquad ds = r\, d\theta$$

$$dE_y = -\frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} \sin\theta\, d\theta$$

$$E = -\int dE_y = +\int_{0^\circ}^{90^\circ} \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\sin\theta\, d\theta$$

, Only need to go from 0° to 90° because we already added together the lower half pair to the upper half.

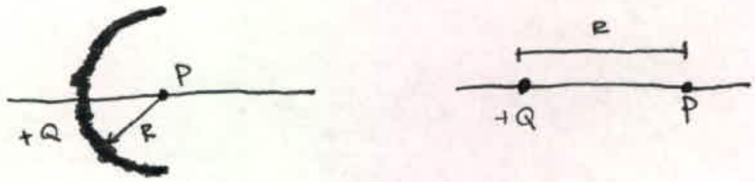
$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\left(-\cos\theta\right)\Big|_{0^\circ}^{90^\circ}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} = \frac{1}{2\pi\epsilon_0}\frac{(2q/\pi r)}{r} = \frac{1}{\pi^2\epsilon_0}\frac{q}{r^2} = \frac{4(8.99\times10^9\ N\cdot m^2/C^2)(4.50\times10^{-12}\ C)}{\pi(5.00\times10^{-2}\ m)^2}$$

a) $E = 2.06\times10^1\ N/C = 20.6\ N/C$

b) 90° clockwise from $(+x)$

(29)



$$\lambda = \frac{Q}{\pi R}$$

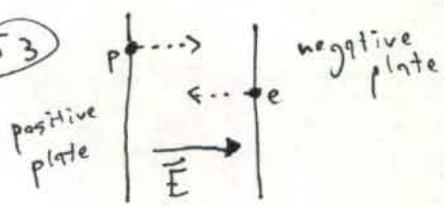
From sample problem,

$$E_{arc} = \frac{\lambda}{4\pi\epsilon_0 R}\int_{-90^\circ}^{90^\circ}\cos\theta\, d\theta = \frac{\lambda}{4\pi\epsilon_0 R}\sin\theta\Big]_{-90^\circ}^{90^\circ} = \frac{\lambda}{2\pi\epsilon_0 R} = \frac{Q}{2\pi^2\epsilon_0 R^2}$$

$$E_{point} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$$

$$\frac{E_{point}}{E_{arc}} = \frac{\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\right)}{\left(\frac{Q}{2\pi^2\epsilon_0 R^2}\right)} = \frac{\pi}{2}$$

(53)



5.0 cm apart.

Let $x=0$ be the positive plate, $x = 0.050$ be the negative plate

$F = |q|E = eE$ for both proton and electron.

$F = m_p a_p$, $F = m_e a_e$ $\qquad x = x_0 + v_0 t + \frac{1}{2}at^2$

$eE = m_p a_p$, $eE = m_e a_e$

$a_p = \frac{eE}{m_p}$, $a_e = \frac{eE}{m_e}$

$$x = \frac{1}{2}\left(\frac{eE}{m_p}\right)t^2 \quad , \quad x = 0.050 - \frac{1}{2}\left(\frac{eE}{m_e}\right)t^2$$

proton goes right, electron goes left.

set them equal to each other since I want to know about when they meet.

$$\frac{1}{2}\left(\frac{eE}{m_p}\right)t^2 = 0.050 - \frac{1}{2}\left(\frac{eE}{m_e}\right)t^2$$

$$\frac{eE}{2}\, t^2\left(\frac{1}{m_p} + \frac{1}{m_e}\right) = 0.050$$

$$t^2 = \frac{0.10}{eE}\left(\frac{1}{m_p} + \frac{1}{m_e}\right)^{-1}$$

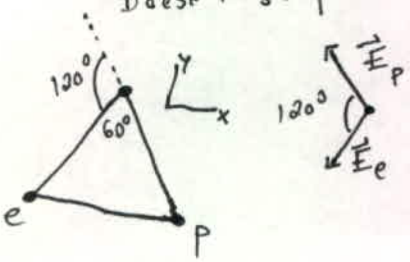
so $x = \frac{1}{2}\left(\frac{eE}{m_p}\right)t^2 = 0.050\left(\frac{\cancel{eE}}{\cancel{eE}}\right)\frac{1}{m_p}\left(\frac{1}{m_p} + \frac{1}{m_e}\right)^{-1}$

$$x = \frac{0.050\, m_e}{m_e + m_p} = \frac{0.050\,(9.11\times10^{-31}\text{ kg})}{(9.11\times10^{-31} + 1.67\times10^{-27})\text{ kg}}$$

$x = 2.7\times10^{-5}$ m from the positive plate.

Doesn't surprise me :).

(66)



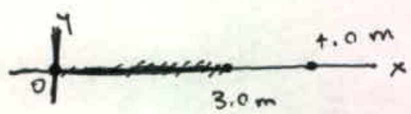
We can see that the y-components cancel exactly. That leaves the x-components, same for both $\vec{E}_e$ and $\vec{E}_p$.

$$E = 2\,|E_{p,x}| = 2E_p\cos 60^\circ = E_p$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{(2.0\times10^{-6}\text{ m})^2}$$

$$E = 3.6\times10^2\ \text{N/C} = 360\ \text{N/C}$$

(67) $\lambda = 9.0$ nC/m



$$dq = \lambda\, dx \qquad r = 4.0 - x$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(4.0-x)^2}$$

$$E = \int dE = \int_0^{3.0} \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(4.0-x)^2} = \frac{\lambda}{4\pi\epsilon_0}\int_0^3 \frac{dx}{(4-x)^2} \qquad \begin{matrix} u = 4-x & x=3 \to u=1 \\ du = -dx & x=0 \to u=4 \end{matrix}$$

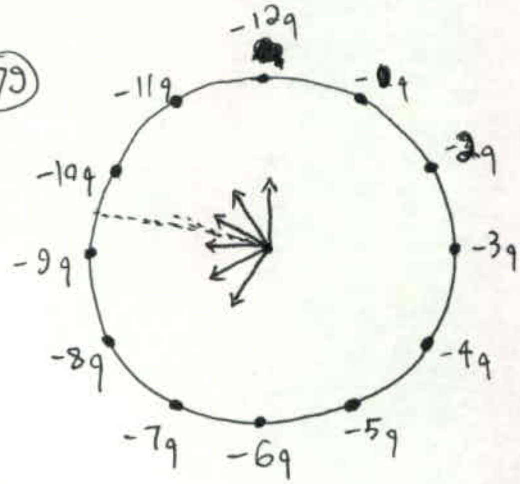
$$E = \frac{\lambda}{4\pi\epsilon_0}\int_4^1 \frac{(-du)}{u^2} = \frac{\lambda}{4\pi\epsilon_0}\int_1^4 \frac{du}{u^2}$$

$$E = \frac{\lambda}{4\pi\epsilon_0}\left(-\frac{1}{u}\right)\Bigg]_1^4 = \frac{\lambda}{4\pi\epsilon_0}\left[-\frac{1}{4}+1\right]$$

$$E = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{3}{4}\right) = \left(\frac{3}{4m}\right)(8.99\times10^9\ N\cdot m^2/C^2)(9.0\times10^{-9}\ C/m)$$

$$E = 61\ N/C$$

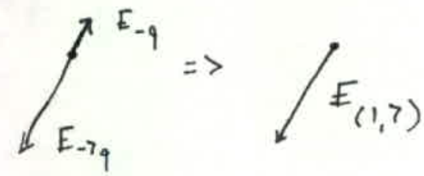
(79)



$7-1 = 8-2 = \dots = 12-6$

choose pairs $(1,7), (2,8), \dots, (6,12)$ that oppose each other.

For any pair, the $\vec{E}_{net}$ at the center follows this trend.



Yielding 6 $\vec{E}$ vectors of the same magnitude, pointing toward $\{7, 8, 9, 10, 11, 12\}$

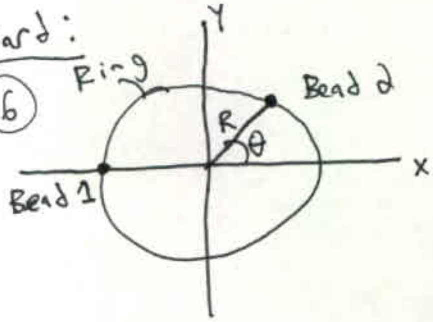
One can see by the dotted line that by pairing $(9,10)$, $(8,11)$, $(7,12)$ the components perpendicular to the dotted line vanish.

That leaves an $\vec{E}$ field pointed towards 9:30 halfway between 9 and 10.

hour hand lines up.

Hard:

(16)



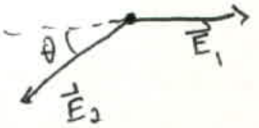
$R = 50.0$ cm

Bead 1 $\to q_1 = 2.00\ \mu C$ fixed

Bead 2 $\to q_2 = 6.00\ \mu C$ movable

$\vec{E}$ at center of ring.

a) positive and b) negative value of θ such that $E = 2.00\times10^5$ N/C



$$E_x = E_1 - E_2\cos\theta \quad , \quad E_y = -E_2\sin\theta$$

$$2.00\times10^5\,\text{N/C} = E = \sqrt{E_x^2 + E_y^2} = \sqrt{(E_1 - E_2\cos\theta)^2 + (-E_2\sin\theta)^2}$$

$$E = \sqrt{E_1^2 - 2E_1E_2\cos\theta + E_2^2\cos^2\theta + E_2^2\sin^2\theta}$$

$$E = \sqrt{E_1^2 - 2E_1E_2\cos\theta + E_2^2}$$

$$E^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\theta$$

$$2E_1E_2\cos\theta = E_1^2 + E_2^2 - E^2$$

$$\cos\theta = \frac{E_1^2 + E_2^2 - E^2}{2E_1E_2}$$

since $q_2 = 3q_1$,
$E_2 = 3E_1$

$$\cos\theta = \frac{E_1^2 + 9E_1^2 - E^2}{6E_1^2}$$

$$\cos\theta = \frac{10}{6} - \frac{E^2}{6E_1^2} = \frac{10}{6} - \frac{1}{6}\left(\frac{E}{E_1}\right)^2$$

$$\cos\theta = \frac{10}{6} - \frac{1}{6}\left(\frac{(2.00\times10^5\,\text{N/C})\cdot(50.0\times10^{-2}\,\text{m})^2}{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C})}\right)^2$$

$$\cos\theta = \frac{10}{6} - \frac{1}{6}(2.78)^2 = 0.378$$

a), b) $\theta = \pm 67.8^\circ$

(31)

$L = 8.15$ cm $\quad -q = -4.23$ fC , $a = 12.0$ cm

a) λ? , b) magnitude and c) direction of $\vec{E}$ at P?

d) magnitude if $a = 50$ m , e) rod replaced by particle of charge $-q$.

a) $\lambda = \frac{-q}{L} = \frac{(-4.23 \times 10^{-15}\ C)}{(8.15 \times 10^{-2}\ m)} = -5.19 \times 10^{-14}\ C/m$

b) $E = \frac{|\lambda|}{4\pi\epsilon_0} \int_a^{a+L} \frac{du}{u^2}$ from problem 67 earlier.

$$E = \frac{|\lambda|}{4\pi\epsilon_0} \left(-\frac{1}{u}\right)\Big]_a^{a+L} = \frac{|\lambda|}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{a+L}\right) = \frac{|\lambda|}{4\pi\epsilon_0} \frac{L}{a(a+L)}$$

$$E = \frac{(+5.19 \times 10^{-14}\ C/m)(8.99 \times 10^{9}\ N \cdot m^2/C^2)(8.15 \times 10^{-2}\ m)}{(12.0 \times 10^{-2}\ m)(20.15 \times 10^{-2}\ m)}$$

$$E = 1.57 \times 10^{-3}\ N/C$$

c) 180° clockwise from $(+x)$

d) $a = 50\ m$, since $a \gg L$, $a + L \approx a$. So Then

$$E = \frac{|\lambda|}{4\pi\epsilon_0} \frac{L}{a \cdot a} = \frac{|\lambda|}{4\pi\epsilon_0} \frac{L}{a^2} = \frac{(5.19 \times 10^{-14}\ C/m)(8.99 \times 10^{9}\ N \cdot m^2/C^2) \cdot (8.15 \times 10^{-2}\ m)}{(50\ m)^2}$$

$$E = 1.52 \times 10^{-8}\ N/C$$

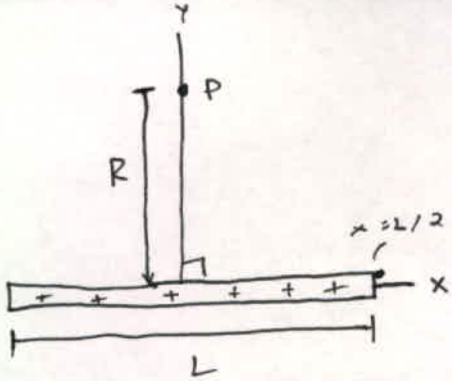
e) $E = \frac{1}{4\pi\epsilon_0} \frac{|-q|}{(a + L/2)^2} \approx \frac{1}{4\pi\epsilon_0} \frac{|-q|}{a^2} = \frac{|\lambda|}{4\pi\epsilon_0} \frac{L}{a^2}$ Again, $a \gg L$, $a + \frac{L}{2} \approx a$.

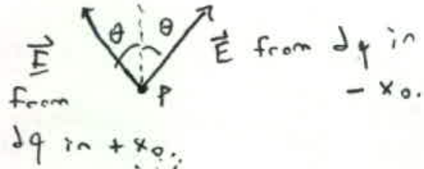
$$E = 1.52 \times 10^{-8}\ N/C \text{ same as d)}$$

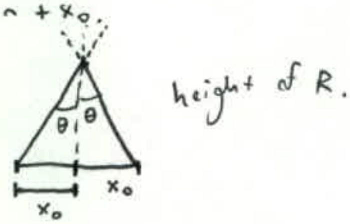
(32) $q = 7.81\ pC$

$L = 14.5\ cm$

$R = 6.00\ cm$







a) magnitude and b) direction of $\vec{E}$ at P?

we can see that the x-components cancel by pairwise cancellation (for dq at x_0 take dq at $-x_0$ and the 2 will add together to just a y-component).

$$dq = \left(\frac{q}{L}\right) dx$$

$$dE = \underset{\text{due to the pair.}}{(2)} \cdot \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \cos\theta$$

$$\cos\theta = \frac{R}{\sqrt{x^2 + R^2}}, \quad r = \sqrt{x^2 + R^2}$$

$$dE = \frac{1}{2\pi\epsilon_0} \left(\frac{q}{L}\right) \frac{R\,dx}{(x^2 + R^2)^{3/2}}$$

$$-\sin\theta\, d\theta = \frac{R(2x\,dx)(-\frac{1}{2})}{(x^2+R^2)^{3/2}}$$

$$E = \int dE = \int_0^{L/2} \frac{1}{2\pi\epsilon_0} \left(\frac{q}{L}\right) \cdot R \frac{dx}{(x^2+R^2)^{3/2}}$$

$$\frac{R\,dx}{(x^2+R^2)^{3/2}} = \frac{\sin\theta\, d\theta}{x}$$

$$E = \frac{1}{2\pi\epsilon_0}\left(\frac{q}{L}\right) R \int_0^{L/2} \frac{dx}{(x^2+R^2)^{3/2}}$$

$$\sin\theta = \frac{x}{\sqrt{x^2+R^2}} = \frac{x}{R}\cos\theta$$

$$\frac{R\,dx}{(x^2+R^2)^{3/2}} = \frac{\cancel{x}}{R\cancel{x}} \cos\theta\, d\theta$$

$$E = \frac{1}{2\pi\epsilon_0}\left(\frac{q}{L}\right) \int_0^{\theta_m} \frac{\cos\theta}{R}\, d\theta$$

$$\frac{R\,dx}{(x^2+R^2)^{3/2}} = \frac{\cos\theta}{R}\, d\theta$$

$$E = \frac{2}{4\pi\epsilon_0}\left(\frac{q}{L}\right)\left(\frac{1}{R}\right)(\sin\theta_m - \sin(0))$$

$x = 0 \rightarrow \theta = 0$

$x = L/2 \rightarrow \theta_m = \sin^{-1}\left(\frac{L/2}{\sqrt{R^2+(L/2)^2}}\right)$

$$E = \frac{2}{4\pi\epsilon_0}\left(\frac{q}{RL}\right)\left(\frac{L/2}{\sqrt{R^2+(L/2)^2}}\right)$$

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{R}\right)\left(\frac{1}{\sqrt{R^2+(L/2)^2}}\right)$$

$$E = \frac{(8.99 \times 10^9\ N\cdot m^2/C^2)(7.81 \times 10^{-12}\ C)}{(6.00 \times 10^{-2}\ m)\left((6.00\times 10^{-2})^2 + \left(\frac{14.5\times 10^{-2}}{2}\right)^2\right)^{1/2} m}$$

$$E = 1.24 \times 10^1\ N/C = 12.4\ N/C$$

b) $90°$ counterclockwise from $(+x)$