Physics 2D Quiz 5 Solutions Winter 2012

(a) Normalization: $1 = \int_{-\infty}^{\infty} dx\, |\psi(x)|^2 = D^2 \int_{-\infty}^{\infty} dx\, e^{-2\beta x^2} =$

$$= D^2 \sqrt{\frac{\pi}{2\beta}} \Rightarrow \boxed{D = \left(\frac{2\beta}{\pi}\right)^{1/4}}$$

(b) $\langle x \rangle = 0 \Rightarrow \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\langle x^2 \rangle}$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} dx\, x^2 |\psi(x)|^2 = D^2 \int_{-\infty}^{\infty} dx\, x^2 e^{-2\beta x^2} =$$

$$= D^2 \frac{1}{2\cdot 2\beta} \cdot \sqrt{\frac{\pi}{2\beta}} = \sqrt{\frac{2\beta}{\pi}} \sqrt{\frac{\pi}{2\beta}} \frac{1}{4\beta} = \frac{1}{4\beta} \Rightarrow$$

$$\Delta x = \frac{1}{2\sqrt{\beta}} = \frac{1}{2\sqrt{4\,\text{Å}^{-2}}} = \frac{1}{2\cdot 2}\,\text{Å} = \boxed{0.25\,\text{Å} = \Delta x}$$

(c) $\Delta x \Delta p = \frac{\hbar}{2} \Rightarrow \Delta p = \frac{\hbar}{2\Delta x} = \frac{\hbar}{2\cdot\frac{1}{2\sqrt{\beta}}} = \boxed{\hbar\sqrt{\beta} = \Delta p}$

(d) $K = \frac{p^2}{2m} \Rightarrow \langle K \rangle = \frac{\langle p^2 \rangle}{2m} =$

$$= \frac{(\Delta p)^2}{2m} = \frac{\hbar^2 \beta}{2m} = \frac{(\hbar c)^2}{2mc^2}\beta = \frac{1973^{\cancel{2}}\,\text{eV}^2\,\cancel{\text{Å}^2}\cdot \cancel{4}^{\,2}\,\cancel{\text{Å}^{-2}}}{\cancel{2}\cdot 1973\,\cancel{\text{eV}}} = 2\cdot 1973\,\text{eV}$$

$$\Rightarrow \boxed{\langle K \rangle = 3946\,\text{eV}}$$

Problem 2

(a) $T = e^{-2\alpha \Delta x}$; $\Delta x = 1 \mathring{A}$

$$\alpha = \sqrt{\frac{2m_e}{\hbar^2}(U-E)} = \sqrt{\frac{1}{3.81 eV \mathring{A}^2}\cdot(10eV - 5eV)} = \sqrt{\frac{5}{3.81}} \ \mathring{A}^{-1}$$

$$\Rightarrow \alpha = 1.1456 \mathring{A}^{-1} \Rightarrow \boxed{T = e^{-2.29} = 0.10}$$

so the probability is 10 %

(b) For an electron in the well, probability to tunnel through right barrier:

$$T = e^{-2\alpha\Delta x}, \quad \Delta x = 2 \mathring{A}$$

$$\alpha = \sqrt{\frac{7-5}{3.81}} \ \mathring{A}^{-1} = 0.7245 \mathring{A}^{-1} \Rightarrow 2\alpha \cdot \Delta x = 2.898 \Rightarrow$$

$$\boxed{T = e^{-2.898} = 0.055}$$ so probability is 5.5%

$$\boxed{\text{right / left} = \frac{0.055}{0.1} = 0.55}$$ so about half as likely to go to the right as to the left.

(c) For 3000 electrons coming in from the left: 10 % = 300 go into the well, 2700 are immediately reflected. Of the 300 in the well, ~200 go to the left and ~100 to the right.

So in total, 2900 get reflected, 100 transmitted

$$\Rightarrow \boxed{\begin{aligned} T &= \frac{100}{3000} = \frac{1}{30} = 0.033, \\ R &= \frac{2900}{3000} = 1 - 0.033 = 0.967 \end{aligned}}$$

Problem 3

(a) Radial dependence $= e^{-Zr/na_0} \Rightarrow Z = n \Rightarrow \boxed{n=2}$

$l = 0$ or 1, since there is angular dependence, $\boxed{l=1}$

Since ϕ dependence is $e^{+i m_l \phi} \Rightarrow \boxed{m_l = -1}$

(b) $L_z = m_l \hbar \Rightarrow \boxed{L_z = -\hbar}$

(c) Radial probability is $P(r) = r^2 R^2(r)$

$\Rightarrow P(r) = C \cdot r^2 \cdot r^2 e^{-2r/a_0} = C r^4 e^{-2r/a_0}$

Most probable r:

$P'(r) = 0 = \left(4r^3 - \frac{2}{a_0} r^4\right) C e^{-2r/a_0} \Rightarrow$

$4r^3 = \frac{2}{a_0} r^4 \Rightarrow r = \frac{4a_0}{2} \Rightarrow \boxed{r = 2a_0}$

For the Bohr atom:

$\boxed{r_n = \frac{a_0}{Z} n^2 = \frac{a_0}{2} \cdot 2^2 = 2a_0 \quad \text{same}}$