PHYSICS 2D QUIZ 4 WINTER QUARTER 2012
PROF. HIRSCH FEBRUARY 27TH, 2012

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma\Delta t' \equiv \gamma\,\Delta t_p$; $L = L_p/\gamma$; $c = 3\times10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \rightarrow -v$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \rightarrow -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1+v/c}\ /\ \sqrt{1-v/c}$

Momentum: $\vec{p} = \gamma m\vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\ MeV/c^2$ Proton: $m_p = 938.26\ MeV/c^2$ Neutron: $m_n = 939.55\ MeV/c^2$

Atomic mass unit: $1\ u = 931.5\ MeV/c^2$; electron volt: $1\text{eV} = 1.6\times10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67\times10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda,T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda,T) = N_\lambda(\lambda)\times\overline{E}(\lambda,T) = \dfrac{8\pi}{\lambda^4}\times\dfrac{hc/\lambda}{e^{hc/\lambda k_BT} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV A$; $k_B = (1/11{,}600)eV/K$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, $\phi \equiv$ work function; Bragg equation: $n\lambda = 2d\sin\vartheta$

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_ec}(1 - \cos\theta)$; $\dfrac{h}{m_ec} = 0.0243A$

Coulomb force: $F = \dfrac{kq_1q_2}{r^2}$; Coulomb energy: $U = \dfrac{kq_1q_2}{r}$; Coulomb potential: $V = \dfrac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Rutherford scattering: $\Delta n = C\dfrac{Z^2}{K_\alpha^2}\dfrac{1}{\sin^4(\phi/2)}$; $ke^2 = 14.4\ eV A$; $\hbar c = 1973\ eV A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097\times10^7\ m^{-1} = \dfrac{1}{911.3A}$

Bohr atom: $E_n = -\dfrac{ke^2Z}{2r_n} = -E_0\dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e(ke^2)}{2\hbar^2} = 13.6eV$; $K = \dfrac{m_ev^2}{2}$; $U = -\dfrac{ke^2Z}{r}$

$hf = E_i - E_f$; $r_n = r_0n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_eke^2} = 0.529A$; $L = m_evr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j\cos(k_jx - \omega_jt)$, or $y(x,t) = \int dk\ a(k)\ e^{i(kx - \omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$; Heisenberg: $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Schrodinger equation: $-\dfrac{\hbar^2}{2\text{m}}\dfrac{\partial^2\Psi}{\partial x^2} + \text{U(x)}\Psi(\text{x,t}) = i\hbar\dfrac{\partial\Psi}{\partial t}$; $\Psi(\text{x,t}) = \psi(\text{x})e^{-i\frac{E}{\hbar}t}$

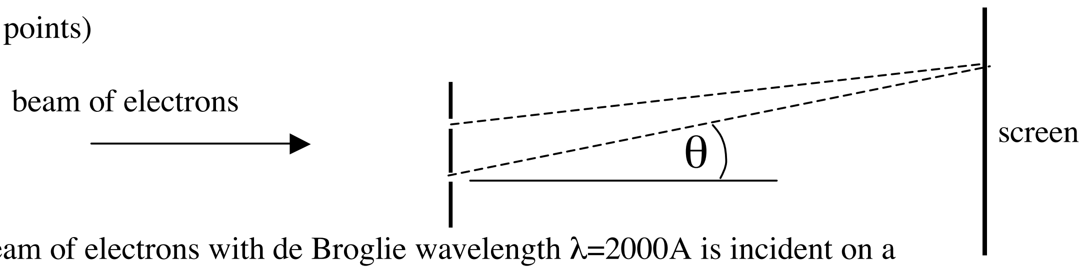
PHYSICS 2D | QUIZ 4 | WINTER QUARTER 2012
PROF. HIRSCH | | FEBRUARY 27TH, 2012

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 3.81 eVA^2$ (electron)

Justify all your answers to all problems. Write clearly.

<u>Problem 1</u> (10 points)



A horizontal beam of electrons with de Broglie wavelength λ=2000A is incident on a double slit, giving rise to an interference pattern on a distant screen. The distance between the slits is 8000A.

(a) Give the speed of the electrons, in m/s.

(b) Give their (non-relativistic) phase velocity and their group velocity, both in m/s.

(c) Find the smallest angle θ for which a minimum in the interference pattern on the screen occurs. Give your answer in degrees.

(d) At that point on the screen (the minimum found in (c)), the electron wavefunction is $\Psi = \Psi_1 + \Psi_2$, where Ψ_1 is the part of the wavefunction that went through the upper slit and Ψ_2 the part of the wavefunction that went through the lower slit, with $\Psi_1 = Ae^{i\phi_1}$ and $\Psi_2 = Ae^{i\phi_2}$ and A a real number. If ϕ_1=π/4 (expressed in radians), what is ϕ_2 in radians?

<u>Problem 2</u> (10 points)

An electron is in a one-dimensional box of length 4A.

(a) Find the minimum kinetic energy that this electron can have, in eV.

(b) Find the wavelength of the longest wavelength photon that can be absorbed by this electron when it is in the minimum kinetic energy state.

(c) Find the probability that the electron in this state will be found at a distance in the range 2.9A to 3A from the left wall. How does it compare to the classical result?

<u>Problem 3</u> (10 points)

Assume the box in problem 2 is really a finite square well, of length 4A and height 10eV, and the electron is in the lowest energy state.

(a) Give an expression for the wavefunction $\Psi(x)$ valid in the region x>L. Estimate the value of the penetration depth (in A) by using the energy found in problem 2(a).

(b) Find an approximate value for the energy of this electron by using the result for the penetration depth found in (a). Give the answer in eV. Explain why the result found is higher, lower or equal than the value found in problem 2(a).

(c) Find an expression for the probability that the electron will be found in the region $L < x < \infty$. Do not approximate $\Psi(x)$ by a constant, do the integral. Then, express your result in terms of the value of the wave function at the edge of the well, $\Psi(x = L)$. Then, assuming $\Psi(x = L)$=0.1 $A^{-1/2}$, find a numerical value for that probability.

Justify all your answers to all problems. Write clearly.