

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation : $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse : $v \rightarrow -v$

Velocity transformation : $u_x' = \frac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse : $v \rightarrow -v$

Spacetime interval : $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift : $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$

Momentum : $\vec{p} = \gamma m \vec{u}$; Energy : $E = \gamma mc^2$; Kinetic energy : $K = (\gamma - 1)mc^2$

Rest energy : $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron : $m_e = 0.511 \text{ MeV}/c^2$; Proton : $m_p = 938.26 \text{ MeV}/c^2$; Neutron : $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit : $1 u = 931.5 \text{ MeV}/c^2$; electron volt : $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law : $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution : $P(E) = C e^{-E/(k_B T)}$

Planck's law : $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons : $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV \AA}$; $k_B = (1/11,600) \text{ eV/K}$

Photoelectric effect : $eV_s = K_{max} = hf - \phi$, ϕ = work function; Bragg equation : $n\lambda = 2d \sin \theta$

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$; $\frac{h}{m_e c} = 0.0243 \text{ \AA}$

Coulomb force : $F = \frac{kq_1 q_2}{r^2}$; Coulomb energy : $U = \frac{kq_1 q_2}{r}$; Coulomb potential : $V = \frac{kq}{r}$

Force in electric and magnetic fields (Lorentz force) : $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering : $\Delta n = C \frac{Z^2}{K_\alpha^2} \frac{1}{\sin^4(\phi/2)}$; $ke^2 = 14.4 \text{ eV \AA}$

Justify all your answers to all problems. Write clearly.

Problem 1 (10 points)

A black body is at temperature $T_0 = 3000 \text{ }^\circ\text{K}$.

(a) At what wavelength λ_0 does it emit maximum power? Give your answer in \AA .

If the temperature of the body is now reduced to $T_1 = 1500 \text{ }^\circ\text{K}$:

(b) By what factor does the power emitted at the wavelength λ_0 found in (a) change?

Does it increase or decrease?

(c) By what factor does the total power emitted at all wavelengths change? Increase or decrease?

(d) By what factor does the power emitted at wavelength $\lambda_1 = 1,000,000 \lambda_0$ change?

Increase or decrease?

(e) Make a qualitative plot of the power emitted versus wavelength at temperatures T_0 and T_1 , indicating which temperature corresponds to which curve and the positions of the maxima.

Problem 2 (10 points)

A light source emits radiation with wavelengths in the entire range 4000Å to 4500Å. The radiation is incident on a metal, and photoelectrons are ejected with maximum kinetic energy 0.5eV.

- (a) Find the work function for this metal. Give your answer in eV.
- (b) Assume the light source is moving away from this metal at speed v . Above a certain value of v , the number of photoelectrons ejected starts to decrease. Find the value of that speed v , assuming that each incident photon that has enough energy to eject a photoelectron will do so. Give your answer as v/c . Hint: Doppler effect.

Problem 3 (10 points)

A monochromatic (i.e. only one wavelength) X-ray beam is incident on a target. A scattering event produces an electron with kinetic energy 50eV, and the scattered photon has wavelength 2Å .

- (a) At which angle was the photon scattered, relative to the incident beam direction? Give your answer in degrees.
- (b) Find the entire range of possible wavelengths for the scattered photons. Give the answer in Å.
- (c) Find the entire range of possible kinetic energies for the scattered electrons. Give the answer in eV.

Justify all your answers to all problems. Write clearly.