1-5 This is a case of dilation. $T = \gamma T'$ in this problem with the proper time $T' = T_0$

$$T = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} T_0 \Rightarrow \frac{v}{c} = \left[1 - \left(\frac{T_0}{T}\right)^2\right]^{1/2};$$

in this case $T = 2T_0$, $v = \left\{1 - \left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2} = \left[1 - \left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v = 0.866c$.

1-6 This is a case of length contraction. $L = \frac{L'}{\gamma}$ in this problem the proper length $L' = L_0$,

$L = \left[1 - \frac{v^2}{c^2}\right]^{-1/2} L_0 \Rightarrow v = c\left[1 - \left(\frac{L}{L_0}\right)^2\right]^{1/2}$; in this case $L = \frac{L_0}{2}$,

$v = \left\{1 - \left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2} = \left[1 - \left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v = 0.866c$.

1-7 The problem is solved by using time dilation. This is also a case of $v << c$ so the binomial expansion is used $\Delta t = \gamma\, \Delta t' \cong \left[1 + \frac{v^2}{2c^2}\right]\Delta t'$, $\Delta t - \Delta t' = \frac{v^2 \Delta t'}{2c^2}$; $v = \left[\frac{2c^2(\Delta t - \Delta t')}{\Delta t'}\right]^{1/2}$;

$\Delta t = (24\ \text{h/day})(3\,600\ \text{s/h}) = 86\,400\ \text{s}$; $\Delta t = \Delta t' - 1 = 86\,399\ \text{s}$;

$$v = \left[\frac{2(86\,400\ \text{s} - 86\,399\ \text{s})}{86\,399\ \text{s}}\right]^{1/2} = 0.004\,8c = 1.44 \times 10^6\ \text{m/s}.$$

1-8 $L = \frac{L'}{\gamma}$

$\frac{1}{\gamma} = \frac{L}{L'} = \left[1 - \frac{v^2}{c^2}\right]^{1/2}$

$v = c\left[1 - \left(\frac{L}{L'}\right)^2\right]^{1/2} = c\left[1 - \left(\frac{75}{100}\right)^2\right]^{1/2} = 0.661c$

1-10 (a) $\tau = \gamma \tau'$ where $\beta = \frac{v}{c}$ and

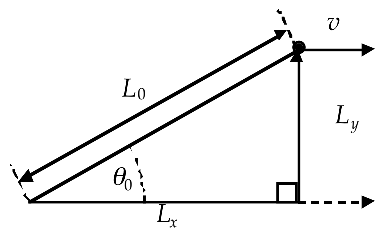
$$\gamma =' \left(1 - \beta^2\right)^{-1/2} = \tau'\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left(2.6 \times 10^{-8}\ \text{s}\right)\left[1 - (0.95)^2\right]^{-1/2} = 8.33 \times 10^{-8}\ \text{s}$$

(b) $d = v\tau = (0.95)(3 \times 10^8)(8.33 \times 10^8\ \text{s}) = 24\ \text{m}$

1-12 (a) 70 beats/min or $\Delta t' = \frac{1}{70}$ min

(b) $\Delta t = \gamma\ \Delta t' = \left[1-(0.9)^2\right]^{-1/2}\left(\frac{1}{70}\right)$ min $= 0.032\ 8$ min/beat or the number of beats per minute $\approx 30.5 \approx 31$.

1-14 (a) Only the x-component of L_0 contacts.



$$L_{x'} = L_0\cos\theta_0 \Rightarrow \frac{L_x\left[L_0\cos\theta_0\right]}{\gamma}$$

$$L_{y'} = L_0\sin\theta_0 \Rightarrow L_y = L_0\sin\theta_0$$

$$L = \left[(L_x)^2 + (L_y)^2\right]^{1/2} = \left[\left(\frac{L_0\cos\theta_0}{\gamma}\right)^2 + (L_0\sin\theta_0)^2\right]^{1/2}$$

$$= L_0\left[\cos^2\theta_0\left(1-\frac{v^2}{c^2}\right) + \sin^2\theta_0\right]^{1/2} = L_0\left[1-\frac{v^2}{c^2}\cos^2\theta_0\right]^{1/2}$$

(b) As seen by the stationary observer, $\tan\theta = \frac{L_y}{L_x} = \frac{L_0\sin\theta_0}{L_0\cos\theta_0/\gamma} = \gamma\tan\theta_0$.

1-16 For an observer approaching a light source, $\lambda_{\text{ob}} = \left[\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right]\lambda_{\text{source}}$. Setting $\beta = \frac{v}{c}$ and after some algebra we find,

$$\beta = \frac{\lambda_{\text{source}}^2 - \lambda_{\text{obs}}^2}{\lambda_{\text{source}}^2 + \lambda_{\text{obs}}^2} = \frac{(650\text{ nm})^2 - (550\text{ nm})^2}{(650\text{ nm})^2 + (550\text{ nm})^2} = 0.166$$

$$v = 0.166c = \left(4.98\times10^7\ \text{m/s}\right)\left(2.237\ \text{mi/h}\right)\left(\text{m/s}\right)^{-1} = 1.11\times10^8\ \text{mi/h}.$$

1-19 $u_{XA} = -u_{XB}$; $u'_{XA} = 0.7c = \frac{u_{XA} - u_{XB}}{1 - u_{XA}u_{XB}/c^2}$; $0.70c = \frac{2u_{XA}}{1 + (u_{XA}/c)^2}$ or

$0.70u_{XA}^2 - 2cu_{XA} + 0.7c^2 = 0$. Solving this quadratic equation one finds $u_{XA} = 0.41c$ therefore $u_{XB} = -u_{XA} = -0.41c$.

1-21 $u'_X = \frac{u_X - v}{1 - u_X v/c^2} = \frac{0.50c - 0.80c}{1-(0.50c)(0.80c)/c^2} = -0.50c$

1-23 (a) Let event 1 have coordinates $x_1 = y_1 = z_1 = t_1 = 0$ and event 2 have coordinates $x_2 = 100$ mm, $y_2 = z_2 = t_2 = 0$. In S', $x'_1 = \gamma(x_1 - vt_1) = 0$, $y'_1 = y_1 = 0$, $z'_1 = z_1 = 0$, and $t'_1 = \gamma\left[t_1 - \left(\frac{v}{c^2}\right)x_1\right] = 0$, with $\gamma = \left[1-\frac{v^2}{c^2}\right]^{-1/2}$ and so $\gamma = \left[1-(0.70)^2\right]^{-1/2} = 1.40$.
In system S', $x'_2 = \gamma(x_2 - vt_2) = 140$ m, $y'_2 = z'_2 = 0$, and

$$t_2' = \gamma\left[t_2 - \left(\frac{v}{c^2}\right)x_2\right] = \frac{(1.4)(-0.70)(100\text{ m})}{3.00\times10^8\text{ m/s}} = -0.33\ \mu\text{s}.$$

1-31 In this case, the proper time is T_0 (the time measured by the students using a clock at rest relative to them). The dilated time measured by the professor is: $\Delta t = \gamma T_0$ where $\Delta t = T + t$. Here T is the time she waits before sending a signal and t is the time required for the signal to reach the students. Thus we have: $T + t = \gamma T_0$. To determine travel time t, realize that the distance the students will have moved beyond the professor before the signal reaches them is: $d = v(T + t)$. The time required for the signal to travel this distance is: $t = \frac{d}{c} = \frac{v}{c}(T + t)$. Solving for t gives: $t = \left(\frac{v}{c}\right)T\left(1 - \frac{v}{c}\right)^{-1}$. Substituting this into the above equation for $(T + t)$ yields: $T + \left(\frac{v}{c}\right)T\left(1 - \frac{v}{c}\right)^{-1} = \gamma T_0$, or $T\left(1 - \frac{v}{c}\right)^{-1} = \gamma T_0$. Using the expression for γ this becomes: $T = \left(1 - \frac{v}{c}\right)\left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} T_0$, or

$$T = T_0\left(1 - \frac{v}{c}\right)\left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} = T_0\left[\left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right)^{-1}\right]^{1/2}.$$

1-37 Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, event B, as occurring first. We may take the S-frame coordinates of the events as $(x = 0,\ y = 0,\ z = 0,\ t = 0)$ and $(x = 100\text{ m},\ y = 0,\ z = 0,\ t = 0)$. Then the coordinates in S' are given by Equations 1.23 to 1.27. Event A is at $(x' = 0,\ y' = 0,\ z' = 0,\ t' = 0)$. The time of event B is:

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1 - 0.8^2}}\left(0 - \frac{0.8c}{c^2}(100\text{ m})\right) = 1.667\left(\frac{80\text{ m}}{3\times10^8\text{ m/s}}\right) = -4.44\times10^{-7}\text{ s}.$$

The time elapsing before A occurs is 444 ns.