Solutions to Quiz 2, Physics 1b (Levine)

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All three versions of this quiz have the same questions, just rearranged. These solutions will follow the order of version A, but I've included the numberings for versions B and C so you can match them to the problems on your quiz.

Answers

Version A: 1-b   2-c   3-c   4-c   5-d   6-d   7-d   8-b

Version B: 1-d   2-c   3-b   4-c   5-c   6-d   7-b   8-d

Version C: 1-c   2-d   3-d   4-c   5-b   6-d   7-b   8-c

Problem A1/B7/C7

First off, let's remind ourselves of the relationship between current and drift velocity:

$$I = nqv_dA \tag{1}$$

I is the current, n is the number of electrons per unit volume, q is of course the charge of an electron, v_d is the drift velocity (the velocity of the moving electrons), and A is the cross-sectional area of the wire. Given the values for all the other quantities, we can solve for v_d. We can easily find the area from the radius: $A = \pi r^2$. The main problem here is that n is not given in the form (number of electrons)/(volume). Instead, we are given the density of gold and the atomic weight, so we have to set up a unit conversion to find n:

$$\frac{19.3 \text{ kg}}{1 \text{ m}^3} \cdot \frac{1 \text{ amu}}{1.66 \times 10^{-27}\text{kg}} \cdot \frac{1 \text{ atom}}{197 \text{ amu}} \cdot \frac{1 \text{ electron}}{1 \text{ atom}} = \frac{n \text{ electrons}}{1 \text{ m}^3}$$

I calculate n to be $\sim 5.902 \times 10^{25}$ electrons per cubic meter. Plugging that value into (1), we can find $v_d = 3.741 \times 10^{-4}$ m/s (*choice b*).

Problem A2/B2/C4

The main fact we need to know for this problem is the relation between resistance R and resistivity ρ:

$$R = \rho \frac{l}{A} \tag{2}$$

Here, l is the length of our wire and A is the cross-sectional area. Since our aluminum and copper wires have equal resistance, we can set up the following equation:

$$\rho_a \frac{5L}{A_a} = \rho_c \frac{L}{A_c}$$

We want to find the ratio between r_c, the radius of the copper wire, and r_a, the radius of the aluminum wire. Remember that the area of a circle is proportional to the radius *squared*:

$$\frac{r_c}{r_a} = \sqrt{\frac{A_c}{A_a}} = \sqrt{\frac{\rho_c \cdot L}{\rho_a \cdot 5L}}$$

We can cancel an L on the top and bottom. Plugging in the given values, we calculate a ratio of 0.35 (*choice c*).

Problem A3/B4/C1

This problem requires the application of equation (2) and some basic geometry. We start with the fact that $R = 8.5\,\Omega = \rho \frac{l}{A}$. We can divide both sides by ρ, and say that $\frac{l}{A} = 5 \times 10^8$ m^{-1}. We also know that volume is length times surface area: $V = l \cdot A = 10^{-4}$ m^3. We now have two equations that relate l and A, so we can eliminate A and solve for l:

$$\left(\frac{l}{A}\right) \cdot (lA) = l^2, \quad \sqrt{l^2} = l$$

Evaluating the square root, we find $l = 223.6$ m, or $\sim 2.2 \times 10^2$ m (*choice c*).

Problem A4/B5/C8

This problem requires us to understand electric power. Electric power is related to voltage, current and resistance in the following way:

$$P = IV = I^2 R = \frac{V^2}{R} \tag{3}$$

Notice that these three definitions are equivalent, based on the fact that $V = IR$. We can use whichever form is most convenient for our situation. In this case, we know our toaster draws 1100 W of power at a voltage of 110 V. If we divide P/V, we find that the current I drawn by the toaster is 10 A. We can then divide V/I to get $R = 11\,\Omega$ (*choice c*).

Problem A5/B1/C3

This problem requires us to know a little more about the physical meaning of power. In general, power is defined as the rate of change in energy over time:

$$P = \frac{dE}{dt}, \text{ or } \frac{\Delta E}{\Delta t} \tag{4}$$

One watt of power is defined as one joule per second. We can use equation (3) to calculate that the electric eel outputs 520 W of power, or 520 J/s. Thus in 2 seconds the electric eel deposits an energy in its victim equal to 520 W × 2 sec = 1040 J (*choice d*).

Problem A6/B8/C2
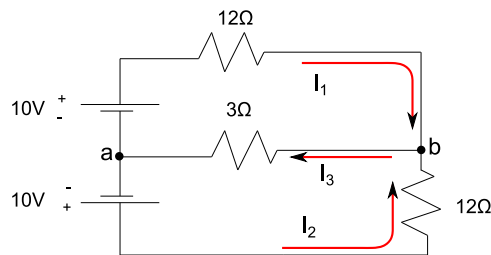
In this problem, we want to find how much current flows through the central resistor (i.e. the $3\,\Omega$ one). To do this, we are going to use Kirchoff's Laws. First, Kirchoff's Voltage Law, which states that if you travel around some closed loop in a circuit, then the net change in voltage over that entire loop is zero:

$$\Delta V_1 + \Delta V_2 + \ldots + \Delta V_n = 0 \tag{5}$$

Let's apply this to the upper and lower loops, labeled 1 and 2 in the diagram. We'll start at node a, and work our way around in the direction of the current flow:

$$\text{loop 1: } 10V - (I_1 \cdot 12\,\Omega) - (I_3 \cdot 3\,\Omega) = 0$$
$$\text{loop 2: } 10V - (I_2 \cdot 12\,\Omega) - (I_3 \cdot 3\,\Omega) = 0$$

Next, let's use Kirchoff's Current Law, which states that the sum of all currents coming into a junction equals the sum of all currents going out of a junction:

$$I_{in} = I_{out}, \quad \text{or} \quad \sum I = 0 \tag{6}$$

We can apply this law at node b to see that $I_1 + I_2 = I_3$. In fact, we can do even better than that: Perhaps you noticed from looking at the diagram that

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loop 1 and loop 2 have the exact same characteristics. Both loops have a 10 V source feeding current across a 12 Ω resistor before reaching node b. Also look at the two loop equations above - they're identical! The inescapable conclusion is that $I_1 = I_2$. We can then say $I_3 = 2I_1$, and we're ready to solve for the value of I_3. Take what we know about the various currents, and plug them back in to one of our loop equations (it doesn't matter which):

$$10V - (I_1 \cdot 12\,\Omega) - (2I_1 \cdot 3\,\Omega) = 0$$

Solving this equation, we find $I_1 = 0.555A$, so $I_3 = 2I_1 = 1.11A$ (*choice d*).

Problem A7/B6/C6
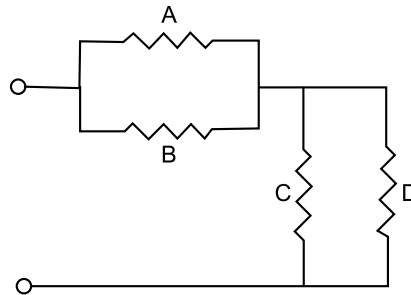
This problem requires us to know how to add resistors in series and in parallel. Remember that the rules for resistors are opposite to the rules for capacitors. To add resistors in series, simply add their resistances directly:

$$R_{equiv} = R_1 + R_2 + \ldots + R_n \tag{7}$$

To add resistors in parallel, we add their inverses:

$$\frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \tag{8}$$

Now, let's take a look at our circuit. I have labeled the resistors A, B, C, D. According to the problem statement, all four resistors have $R = 100\,\Omega$. We can add A and B in parallel to get $\left(\frac{1}{100} + \frac{1}{100}\right)^{-1} = 50\,\Omega$. Resistors C and D are also in parallel, so we can repeat the process to get $50\,\Omega$ again. We're left with two resistors, each of $50\,\Omega$, in series; they add directly to make $100\,\Omega$ (*choice d*).

Problem A8/B3/C5
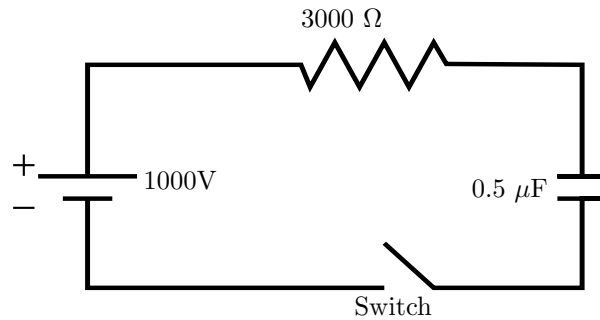
Our setup looks something like the picture above. If the capacitor is initially uncharged, that means it also has no initial voltage difference across it (because $Q = CV$). Thus, when the switch is closed, Kirchoff's Voltage Law (5) gives:

$$1000V - I \cdot 3000\,\Omega = 0$$

All we have to do is divide, to find $I = 0.33$ A (*choice b*).

Further Reading:

(Source: xkcd.com/643/)