



If images take too long to load, please inform [web guy](#)

Page #3 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

BIT-80

Since $\partial \mathcal{L} / \partial t = 0$, this is conserved quantity. It is not Newtonian energy: more accurate than $E + V$. note, $\dot{r} = \frac{dr}{dt}$, not $\frac{dr}{dt}$

At late times: $r \rightarrow 0$

$$\frac{1}{2} \dot{r}^2 = \underbrace{\frac{c^2}{2} \left(\frac{E}{mc^2} \right)^2}_0 - \frac{c^2}{2} + \frac{(GM/c^2)c^2}{r}$$

$$\frac{1}{2} \dot{r}^2 = \frac{GM}{r} \quad \left\{ \begin{array}{l} \text{similar to } K + V = 0 \end{array} \right.$$

But... fully relativistic. (or not t): We shall return to this soln. later on.

Time-Like Geodesics

Consider nature of time-like geodesics: $K = c^2$ with $L \neq 0$

$$\frac{1}{2} \dot{r}^2 + V(r) = \text{const} = c^2$$

$$V(r) = \frac{1}{2} \left(1 - \frac{2GM/c^2}{r} \right) \left(c^2 + \frac{L^2}{r^2} \right) \quad \left\{ \begin{array}{l} \text{angular momentum} \\ \text{per unit mass} \end{array} \right. \quad \text{effective potential}$$

$$= \frac{c^2}{2} - \frac{GM}{r} + \frac{L^2}{2r^2} - \frac{GM L^2 / c^2}{r^3}$$

Find extrema of this potential; i.e., roots of $\frac{\partial V}{\partial r} = 0$.

$$\frac{\partial V}{\partial r} = \frac{GM}{r^2} - \frac{L^2}{r^3} + \frac{3GM L^2 / c^2}{r^4}$$

$$= r^{-4} \left[GM r^2 - L^2 r + 3GM L^2 / c^2 \right] = 0$$

[<-previous page](#)

[next page->](#)



If images take too long to load, please inform [web guy](#)

Page #4 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

Solve quadratic equation

$$A = GM, B = -L^2 \quad \left[\begin{array}{l} BH-91 \\ E = 3GM^2/c^2 \end{array} \right]$$

Roots are:

$$R_{\pm} = \frac{L^2 \pm \sqrt{L^4 - 12(GM)^2 L^2 / c^2}}{2GM}$$

$$= \frac{L^2 \pm L^2 \sqrt{1 - \frac{12(GM)^2}{c^2 L^2}}}{2GM}$$

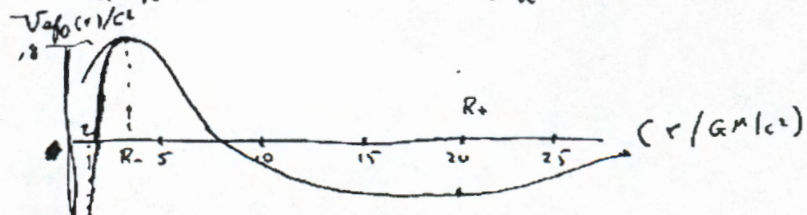
$$R_{\pm} = \frac{L^2}{2GM} \left[1 \pm \sqrt{1 - \frac{12(GM)^2}{c^2 L^2}} \right] \quad (5)$$

• Case I: $L^2 > 12(GM)^2/c^2$, $\sqrt{\quad}$ is real, ~~not~~ exist of $r(r)$

R_+ is a minimum, R_- is a maximum of $r(r)$. (Example) below is $L^2 = 24(GM)^2/c^2$ $\left(\frac{\partial V}{\partial r} \right)_{R_{\pm}}$

$$R_{\pm} = \frac{12GM}{c^2} \left[1 \pm \sqrt{1 - \frac{1}{2}} \right] = \frac{12GM}{c^2} \left(1 \pm \sqrt{\frac{1}{2}} \right)$$

$$\frac{R_+}{GM/c^2} = 12(1.7) = 20 \quad ; \quad \frac{R_-}{GM/c^2} = 12(0.7) = 8.4$$



[<-previous page](#)

[next page->](#)



If images take too long to load, please inform [web guy](#)

Page #5 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

$B_{11}-B_{21}$

Recap: Geodesic motion in Schwarzschild spacetime

- Timelike geodesics of finite-mass test particles: solutions to

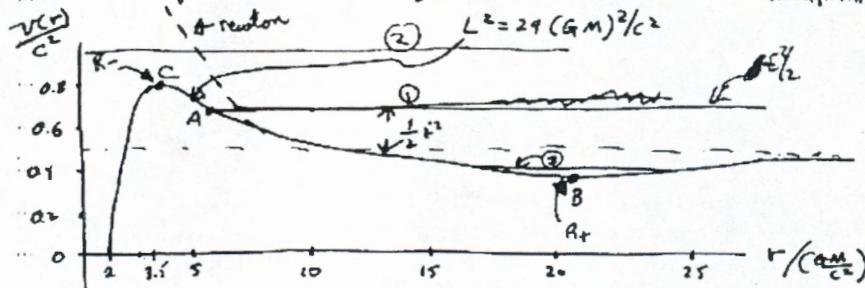
$$\frac{1}{2} \dot{r}^2 + V(r) = \frac{c^2 E^2}{2} \quad (E = E/mc^2)$$

$$\text{where } V(r) = \frac{1}{2} \left(1 - \frac{2GM}{rc^2} \right) \left(c^2 + \frac{L^2}{r^2} \right)$$

- ~~Minimum~~ Orbit classification

Recall: if $L^2 > 12(GM)^2/c^2$

at $R_{\pm} = \frac{L^2}{2GM} \left[1 \pm \sqrt{1 - \frac{12(GM)^2}{c^2 L^2}} \right]$
 has maxima, minima



- In above figure, solid horizontal lines are fixed values $\frac{c^2 E^2}{2}$

- orbit only exists at r where $\frac{E^2}{2} \geq \frac{V(r)}{c^2}$

- ① Unbound orbit with turning point at A

- ② Capture orbit. Unique to GR

[<-previous page](#)

[next page->](#)



If images take too long to load, please inform [web guy](#)

Page #6 of Lecture #11

Lecture #11:

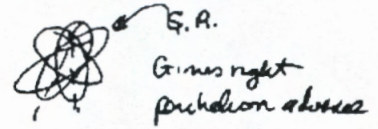
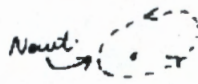
- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

BH-829

③ Bound Orbit : Turning points A_1, A_2



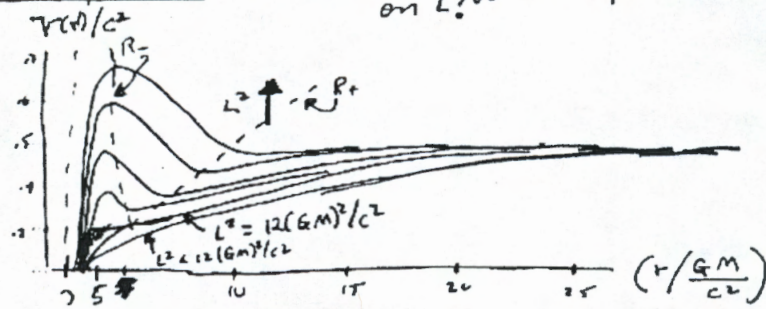
④ Stable circular orbit



⑤ Unstable circular orbit



Dependence on L^2 : How does effective potential depend on L^2 ?



$$\text{Recall } R_{\pm} = \frac{L^2}{2GM} \left[1 \pm \sqrt{1 - \frac{12(GM)^2}{c^2 L^2}} \right]$$

$$R_+ = R_- \text{ if } L^2 = 12(GM)^2/c^2$$

If $L^2 < 12(GM)^2/c^2$

Only possible orbits are capture orbits: there are no extremal points on $V(r)$. These are required for bound and ~~with~~ unbound orbits, and don't result in capture

[<-previous page](#)

[next page->](#)



If images take too long to load, please inform [web guy](#)

Page #7 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

BH-83

Restrictions on R_+, R_-

$$R_+ = \frac{L^2}{2GM} \left[1 + \sqrt{1 - \frac{12(GM)^2}{c^2 L^2}} \right]$$

R_+ only exists for $L^2 \geq 12(GM)^2/c^2$

$\therefore R_+ \geq \frac{12(GM)^2/c^2}{2GM} [1+0]$; $L = 12(GM)^2/c^2$ gives smallest R_+ . In that case $R_+ = R_-$

$$R_+ \geq 3GM/c^2 \quad (6)$$

Thus, in GR no stable orbits exist at $R < \frac{6GM}{c^2}$

(Now look at unstable orbit)

$$R_- = \frac{L^2}{2GM} \left[1 - \sqrt{1 - \frac{12(GM)^2}{c^2 L^2}} \right]$$

$$= \frac{(L^2/2GM)}{(GM/c^2)} \times \frac{GM}{c^2} \left[1 - \sqrt{1 - \frac{6(GM/c^2)}{(L^2/2GM)}} \right]$$

↓ divide by 2GM

$$R_- = x \cdot r_g \left[1 - \sqrt{1 - \frac{6}{x}} \right] \quad \left\{ \begin{array}{l} r_g = GM/c^2 \\ x = (L^2/2GM)/(GM/c^2) \end{array} \right.$$

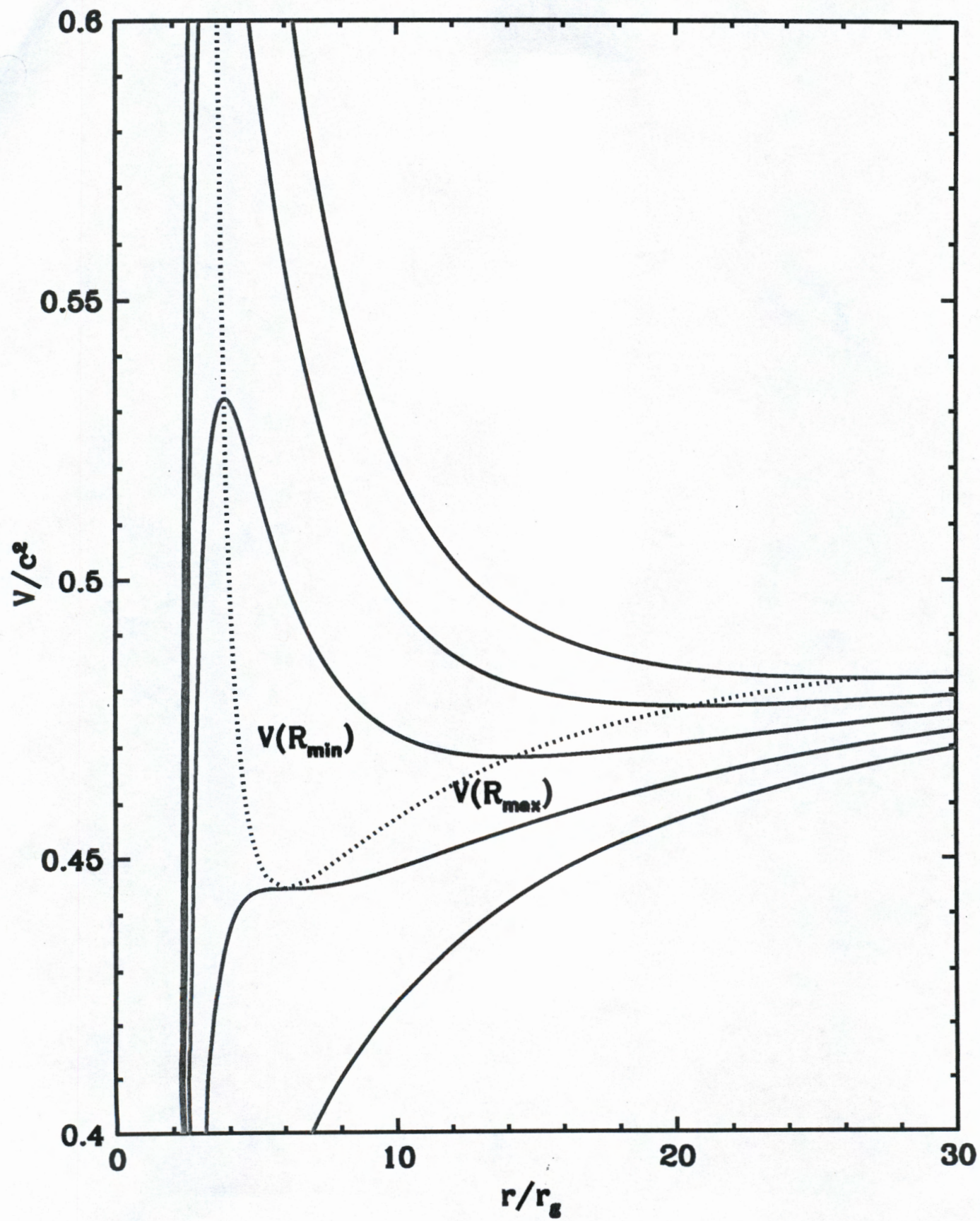
$L \downarrow$ when $x = 6$ $R_- = (R_-)_{\text{min}} = 6GM/c^2$ (largest R_-)

Lim $x \rightarrow \infty$ $R_- = x r_g [1 - (1 - \frac{3}{x})] \approx 3R_+$ (smallest R_-)

$$\therefore 3GM/c^2 \leq R_- \leq 6GM/c^2 \quad (7)$$

[<-previous page](#)

[next page->](#)



These timelike orbits are unstable (occur at maximum of $V(r)$)
And none exist at $r \leq 3GM/c^2$. Thus increasing L
(centrifugal reaction) does not prevent capture by BH when
you are close to Schwarzschild radius.

Energy:

Recall: $\frac{1}{2} \dot{r}^2 + V(r) = \frac{c^2 E^2}{2}$; $V(r) = \frac{1}{2} (1 - \frac{2GM}{c^2 r}) (c^2 + \frac{L^2}{r^2})$

Recall: $\frac{\partial V}{\partial r} = r^{-4} [GM r^2 - L^2 r + 3GM L^2 / c^2]$

Necessary condition for circular orbit: $\frac{\partial V}{\partial r} = 0 \Rightarrow GM r^2 - L^2 r + \frac{3GM L^2}{c^2} = 0$

Rewrite as: $L^2 [r - \frac{3GM}{c^2}] = GM r^2$

\therefore $L^2 = \frac{GM r^2}{r - \frac{3GM}{c^2}}$ (3)

From circular orbit condition: $\frac{1}{2} \dot{r}^2 = 0 \Rightarrow V(r) = \frac{c^2 E^2}{2}$

$o_n = \frac{c^2 E^2}{2} = \frac{1}{2} (1 - \frac{2GM}{c^2 r}) (c^2 + \frac{L^2}{r^2})$
 $= \frac{c^2}{2} (1 - \frac{2GM}{c^2 r}) (1 + \frac{L^2}{c^2 r^2})$
 $= \frac{c^2}{2} (\frac{r - 2GM/c^2}{r}) (1 + \frac{L^2}{c^2 r^2})$

Now substitute eq (3) for $\frac{L^2}{c^2 r^2}$

$E^2 = (\frac{r - 2GM/c^2}{r}) (1 + \frac{GM/c^2}{r - 3GM/c^2})$

$\therefore E^2 = (\frac{r - 2GM/c^2}{r}) (\frac{r - 3GM/c^2 + GM/c^2}{r - 3GM/c^2})$

o_n $E^2 = \frac{(r - 2GM/c^2)^2}{r(r - 3GM/c^2)}$ (4)



If images take too long to load, please inform [web guy](#)

Page #9 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

BH-85

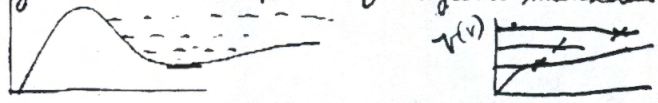
Recall smallest stable circular orbit occurs at $r = 6GM/c^2$

Binding energy: $\frac{mc^2 - E}{mc^2} = 1 - \frac{E}{mc^2}$

From eq. 4 } $E^2 = \frac{(6-2)^2}{6(6-3)} = \frac{(4)^2}{6 \cdot 3} = \frac{16}{18} = \frac{8}{9} \Rightarrow E = \sqrt{\frac{8}{9}}$

$\therefore \frac{B}{m} = 1 - \sqrt{\frac{8}{9}} = 0.0572$

This is maximum fraction of rest-mass energy released when particle at rest at ∞ spirals in to smaller radii and settles into stable circular orbit. Can do this by changing initial E (emitting gravitational radiation). Or if gas viscous transport of angular momentum.



About 6% of mc^2 radiated away in reaching $6GM/c^2$
 Recall fusion reactions release 0.7%. This is one reason why black holes invoked as energy sources to explain enormous energy output from
 X-ray sources
 Quasars
 Double Radio Sources

[<-previous page](#)

[next page->](#)

Before discussing lightlike geodesics, let's examine properties of spacetime near $r = 2GM/c^2$

Coordinate versus Proper time:

Recall effective potential: $V(r) = \frac{1}{2} \left(1 - \frac{2GM/c^2}{r}\right) \left(c^2 + \frac{L^2}{r^2}\right)$

If $L=0$: $\frac{1}{2} \dot{r}^2 + \frac{c^2}{2} \left(1 - \frac{2GM/c^2}{r}\right) = \frac{c^2 E^2}{2}$

$$\left(\frac{dr}{d\tau}\right)^2 = c^2 E^2 - c^2 \left(1 - \frac{2GM/c^2}{r}\right)$$

$$\left(\frac{dr}{cd\tau}\right)^2 = E^2 - 1 + \frac{2GM/c^2}{r}$$

$$\frac{1}{c} \frac{dr}{d\tau} = -\sqrt{E^2 - 1 + \frac{2GM/c^2}{r}} \quad \left\{ \begin{array}{l} \text{inward motion} \end{array} \right\}$$

Consider infall from rest at $r=\infty$. This implies $E=1$

$$\frac{1}{c} \frac{dr}{d\tau} = -\sqrt{\frac{2GM}{c^2}} r^{-1/2}$$

Integrate from $(0, r_1) \rightarrow (\tau, r)$

$$\int_{r_1}^r dr r^{1/2} = -\sqrt{2GM} \int_0^\tau d\tau'$$

$$\frac{2}{3} (r^{3/2} - r_1^{3/2}) = -\sqrt{2GM} \cdot \tau$$

$$r^{3/2} = r_1^{3/2} - \frac{3\sqrt{2}}{2} \sqrt{GM} \cdot \tau$$

Divide by $r_1^{3/2} = (GM/c^2)^{3/2}$

$$\left(\frac{r}{r_1}\right)^{3/2} = \left(\frac{r_1}{r_1}\right)^{3/2} - \left(\frac{3}{\sqrt{2}}\right) \frac{(GM)^{1/2}}{(GM)^{3/2} c^{-3}} \tau$$

$$\boxed{\frac{r}{r_1} = \left[\left(\frac{r_1}{r_1}\right)^{3/2} - \left(\frac{3}{\sqrt{2}}\right) \cdot \frac{\tau}{(GM/c^3)} \right]^{2/3}}$$

Proper

~~the for~~ therefore: \int time taken to go from
 $r = r_1 \rightarrow r = 0$

$$\tau_{out} = \frac{\sqrt{2}}{3} \left(\frac{GM}{c^3} \right) \left(\frac{r_1}{r_0} \right)^{3/2}$$

$$\text{Suppose } r_1 = 6 \cdot GM/c^2 \Rightarrow \tau_{out} = \frac{\sqrt{2}}{3} \left(\frac{GM}{c^3} \right) (6)^{3/2}$$

$$\text{For } M = 1 \times M_{\odot} = 2 \times 10^{33} \text{ g}, \quad (GM/c^3) = 5 \times 10^{-6} \text{ sec}$$

$$\Rightarrow \tau_{out} = \frac{\sqrt{2}}{3} \times 6^{3/2} \times 5 \times 10^{-6} = 35 \mu\text{s} !$$

Fast in fall!



If images take too long to load, please inform [web guy](#)

Page #11 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

250

Let's relate $t \rightarrow \tau$

Recall: $\frac{dt}{d\tau} = \frac{E}{1 - \frac{2GM}{rc^2}}$, in this case $E=1 \Rightarrow \frac{dt}{d\tau} = \frac{1}{1 - \frac{2GM}{rc^2}}$

Recall $\frac{1}{c} \frac{dr}{d\tau} = - \frac{\sqrt{2GM/c^2}}{r^{1/2}} \Rightarrow \frac{dr}{d\tau} = - \sqrt{2GM} \frac{1}{r^{1/2}}$

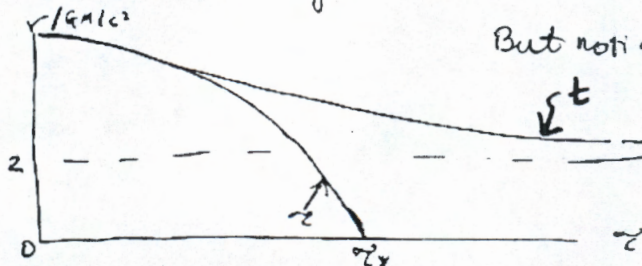
New: $\frac{dt}{dr} = \frac{dt/d\tau}{dr/d\tau} = \frac{1}{1 - \frac{2GM}{rc^2}} \times \left[- \frac{1}{\sqrt{2GM}/r^{1/2}} \right]$

$$\frac{dt}{dr} = - \frac{r^{1/2}}{\sqrt{2GM} \left(1 - \frac{2GM}{rc^2}\right)} \quad \textcircled{B}$$

Integrate this:

$$\textcircled{C} \quad t = t_* + \frac{2GM}{c^3} \left[-\frac{2}{3} \left(\frac{r}{2GM/c^2}\right)^{3/2} - 2 \left(\frac{r}{2GM/c^2}\right)^{1/2} + \ln \left| \frac{\left(\frac{r}{2GM/c^2}\right)^{1/2} + 1}{\left(\frac{r}{2GM/c^2}\right)^{1/2} - 1} \right| \right]$$

t_* fixed by labeling time particle crosses a particular radius r . Notice $t(\tau)$ given by ~~substituting~~ $A \rightarrow C$



But notice, $r = \frac{2GM}{c^2} \Rightarrow t \rightarrow \infty$

[<-previous page](#)

[next page->](#)



If images take too long to load, please inform [web guy](#)

Page #17 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

814-889

Physical Significance of Schwarzschild radius

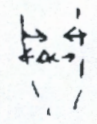
Notice that nothing special happens to clock of infalling observer when $r = r_g = 2GM/c^2$; i.e., proper time clocks keep ticking regularly. Of course distant observer does see weirdness.

Does infalling astronaut get crushed at $r = r_g$?
To answer that one needs to compute tidal accelerations, and those come from curvature of spacetime.

Formula for curvature:

$$\frac{d^2(\Delta x)}{dt^2} \propto R_{\mu\nu\rho\sigma}$$

Tidal acceleration
curvature tensor



R 's are physical objects: Numerical values do not depend on choice of coordinates (unlike metric which is not directly measurable)

$R_{\mu\nu\rho\sigma} \propto M/r^3$ (as it should be since it is tidal acceleration)

• $r = r_g$, R is lg. but not ∞ . Nothing special happens at $r = r_g$. Of course stretching would be intense.

• $r = 0$, $R \rightarrow \infty$: Tidal stretching blows up and astronaut is kaput. A singularity! [next page->](#)

[<-previous page](#)



If images take too long to load, please inform [web guy](#)

Page #18 of Lecture #11

Lecture #11:

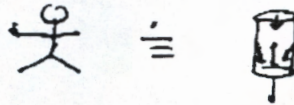
- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

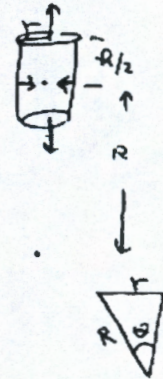
Tidal ripping and pressure at $r = R$

BHBB



In rest frame of astronaut we have:

- (a) ripped from head to toe in vertical
- (b) squeezed at the side



$$\Delta F_{\parallel} = (\partial F_{\parallel} / \partial z) R/2 = \frac{GMm}{R^3} \cdot \frac{R}{2}$$

$$\Delta F_{\perp} = \frac{GMm}{R^2} \cdot \frac{r}{R} = \frac{GMm}{R^2} \cdot r$$

Suppose data is

$$M = 1 M_{\odot} = 2 \times 10^{33} \text{g}$$

$$m = 80 \text{ kg} = 8 \times 10^4 \text{g}$$

$$R = r_g = 3 \text{ km} = 3 \times 10^5 \text{ cm} = 2 GM/c^2$$

$$R = 2 \text{ m} = 2 \times 10^2 \text{ cm}$$

$$r = 10 \text{ cm}$$

Pressure: $P_{\perp} = \frac{\Delta F_{\perp}}{A_{\perp}} = \frac{\Delta F_{\perp}}{2\pi r R} = \frac{GMm \cdot r}{R^3} \times \frac{1}{2\pi r R} = \frac{GMm}{R^3} \cdot \frac{1}{2\pi R}$

$$P'_{\parallel} = \frac{\Delta F_{\parallel}}{A'_{\parallel}} = \frac{\Delta F_{\parallel}}{\pi r^2} = \frac{GMm}{R^3} \cdot \frac{R}{2} \times \frac{1}{\pi r^2} = \frac{GMm}{R^3} \times \frac{1}{2\pi} \cdot \frac{R}{r^2}$$

$$P'_{\parallel} / P_{\perp} = \frac{R/r^2}{1/R} = \left(\frac{R}{r}\right)^2 = (20)^2 = 400$$

$$P'_{\parallel} = \frac{6.7 \times 10^{-8} \times 2 \times 10^{33} \times (8 \times 10^4)}{(3 \times 10^5)^3} \times \frac{2 \times 10^2}{2 \times 9 \times (10)^2} = 1.3 \times 10^{14} \text{ dynes cm}^{-2}$$

[<-previous page](#)

[next page->](#)



If images take too long to load, please inform [web.guy](#)

Page #19 of Lecture #11

Lecture #11:

- [Page 1](#)
- [Page 2](#)
- [Page 3](#)
- [Page 4](#)
- [Page 5](#)
- [Page 6](#)
- [Page 7](#)
- [Page 8](#)
- [Page 9](#)
- [Page 10](#)
- [Page 11](#)
- [Page 12](#)
- [Page 13](#)
- [Page 14](#)
- [Page 15](#)
- [Page 16](#)
- [Page 17](#)
- [Page 18](#)
- [Page 19](#)
- [Page 20](#)
- [Page 21](#)
- [Page 22](#)
- [Page 23](#)
- [Page 24](#)

[<-previous page](#)

[next page->](#)

BH98C

$$1 \text{ atmosphere} = 10^6 \text{ dynes/cm}^2$$

$$P_{ii} \approx 10^8 \text{ atmospheres} \Rightarrow P_L = \frac{10^8}{10^2} \times 10^5 \text{ atmospheres}$$

Human body can withstand no more than $P = 10^2 \text{ At}$.
 So astronaut would be finished at ~~10^2~~
 $r = r_R > r_g$

for n
$$\frac{P_v(r_R)}{P_v(r_g)} = \left(\frac{r_g}{r_R}\right)^3 = \frac{10^2}{10^8} = \frac{1}{10^6}$$

$$\Rightarrow r_R = 100 \cdot r_g \quad \text{or } r_R = 300 \text{ km (180 mi)}$$

~~Massive~~ Mercuria Black Holes Less lethal

Recall $P_{BH} \propto M/R^3$

$$\therefore P(r_g) \propto M / (2GM/c^2)^3 \propto 1/M^2$$

Suppose $M = 2 \times 10^7 M_{\odot}$ (Mass of BH at center of ~~Andromeda~~ ^{Andromeda} Galaxy)

$$\frac{P(r_g)_{\text{Galaxy}}}{P(r_g)_{\text{MW}}} = \left(\frac{1}{2 \times 10^7}\right)^2 = 2.5 \times 10^{-15}$$

$$\therefore P(r_g) = 2.5 \times 10^{-15} \times 1.3 \times 10^{14} \approx 0.33 \text{ At}$$

At $r = r_g = 2 \times 10^7 \cdot 3 \text{ km} = 6 \times 10^7 \text{ km}$ no problem

But at smaller radius all boats are off

[<-previous page](#)

[next page->](#)

PHYSICS 161: Black Holes in the Cosmos

Instructor: Art Wolfe

Office, Phone, and Email : SERF 423, 534-7435, awolfe@ucsd.edu

Office Hours: Fri. after 10:30 A. M., or by appointment

Term Paper

Option A: Review Paper (10 page paper) **Option B:** Review Talk/ Shorter Paper (5 page paper)

Option C: Calculation with talk/written presentation

Physics and Engineering majors typically do not get enough training in written and oral communication, though this will likely be required in any job in industry or academia. So the written portion of the final should be very carefully checked for clarity, grammar, spelling, etc.

For option A, the idea is a Scientific American level paper clearly explaining to an intelligent lay person some exciting topic in modern astrophysics somewhat related to class work. The goal of the paper is to get the reader interested in the topic, to maintain his/her interest throughout the paper, to explain things at a level they can understand, and at the same time to be scientifically accurate. This is not an easy task.

I will be looking for

- *How well you understood the subject you picked.
- *How you expressed yourself; i.e., could the reader/listener follow?
- *How well you maintained a constant technical level.
- *Grammar, spelling, presentation, etc.

For option B, the goal is similar but the talk will be given to the class, with the written part being your working outline/presentation notes. The talk must be well rehearsed and organized. 15 minutes is a very short time to get across the essence of any topic, so I recommend preparing your material, then giving the talk out loud to yourself keeping track of time, then redoing the content and technical level of the talk to fit in the allowed time.

The paper or talk may be on any topic relevant to the class. But it should be something you are interested in.

Examples:

Quasars, gamma ray bursts, the big bang, the cosmic microwave background radiation, gravitational waves, X-ray binary stars, measuring black hole masses, accretion disks around black holes, black holes at the center of galaxies, effects of such black holes on galaxy formation, Hawking Radiation, black hole at the center of the Milky Way etc.

You can get the information from books or magazines in the library, from your textbooks, from the internet, or from any other sources. I recommend looking through these to find a topic that interests you. When you have picked a topic, you can ask me for recommendations of where to find appropriate information. USE the MARLAR LIBRARY in the SERF building

For option C, you will give a paper or talk as above, but rather than review some subject, you will present the results of a calculation.

- Examples: *Computer solution and graphical presentation of trajectories near a black hole
*Fuller calculations of some class topics

IMPORTANT: Hand in one page outline of what you plan to do. This is required.

