PHYSICS 4E QUIZ 4 (open book) SPRING QUARTER 2012
PROF. HIRSCH June 4th

Problem 1 (10 pts)
An electron in a three-dimensional cubic well of side length L has ground state energy 1eV.
(a) Find the value of L, in A.
(b) Find the 5 lowest energy levels for an electron in this well, give their quantum numbers and their energies in eV.
(c) What is the maximum number of electrons that can be accommodated in these energy levels? Take into account the degeneracy of the levels and the electron spin, and assume electrons don't interact with each other.
Use $\hbar^2/m_e = 7.62 eVA^2$

Problem 2 (10 pts)
An electron in a hydrogen-like ion has wavefunction
$\Psi(r,\vartheta,\phi) = Cr^2 e^{-2r/a_0} \sin^2\vartheta\, e^{-2i\phi}$
with C a constant.
(a) Give the values of n, ℓ, m_ℓ, and the ionic charge Z.
(b) Find the most probable value for r, give your answer in terms of a_0. Compare with the radius of the orbit in the Bohr atom for that n.
(c) Find the average r, <r>. Give your answer in terms of a_0. Use that: $\int_0^\infty dr r^p e^{-\lambda r} = \frac{p!}{\lambda^{p+1}}$.
Is the result equal, smaller or larger than the most probable r?
Note that it is not necessary to find C to answer this, if you write <r> as the ratio of two integrals.

Problem 3 (10 pts)
The spin-orbit splitting between the $2^2P_{1/2}$ and $2^2P_{3/2}$ levels of hydrogen is 4.5×10^{-5}eV. Estimate the spin-orbit splitting for the same levels of the He^+ ion (Z=2) as follows:
(a) Compare the speeds of the electron in the 2p levels of H and He^+ (Use Bohr atom). Which is larger, by what factor?
(b) Compare the period of the orbits (time it takes for 1 revolution) for both cases. Which is larger, by what factor?
(c) In the rest frame of the electron, the nucleus is revolving in a circular orbit around the electron. This results in a current i (i=dq/dt). Compare the magnitude of the currents for both cases. Which is larger, by what factor?
(d) Using the above and the fact that the magnetic field at the center of a loop of radius R carrying a current i is $B=\mu_0 i/(2R)$, find the ratio of the spin orbit splittings for He^+ and H.
(e) Give your obtained estimate of the magnitude of the spin-orbit splitting between the $2^2P_{1/2}$ and $2^2P_{3/2}$ levels of He^+ in eV.

Justify all your answers to all problems