

**Problem 1** (10 pts)

Test of the correspondence principle for the infinite well.

An electron is in an infinite square well of length  $L$ , in the  $n$ -th state, with  $n > 1$ .

(a) Find an expression for the frequency of the photon emitted when the electron makes a transition to the  $n-1$  th state. Give your answer in terms of  $\hbar$ ,  $m_e$ ,  $L$  and  $n$ , with  $m_e$  the electron mass.

(b) Assuming the electron is moving back and forth in this well with momentum  $p$  given by the classical relation  $E = p^2/2m_e$  with  $E$  the energy, find the classical frequency of motion (i.e. the inverse of how long it takes for the electron to go back and forth once), when the electron is in the  $n$ -th state.

(c) Compare the answers of (a) and (b) and explain why they are the same or different.

**Problem 2** (10 pts)

An electron is in the first excited state of a harmonic oscillator potential. Its classical amplitude of oscillation in that state is  $5A$ . The wavefunction for that state is

$$\psi(x) = Cxe^{-m_e\omega x^2/2\hbar} \text{ (note that there is a typo in the book Eq. 6-58 second line)}$$

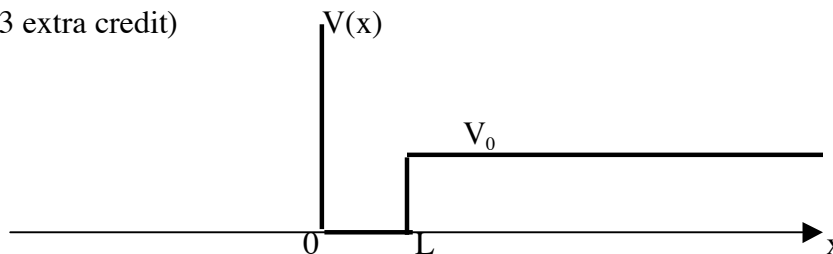
(a) Find its total energy, in eV.

(b) At what distance from the origin is this electron most likely to be found according to quantum mechanics? Give your answer in  $A$  (Hint: it is not the same as the classical amplitude. The maximum of a function is where its derivative is zero).

(c) How much more likely is it to find this electron at the distance from the origin found in (b) than at the classical amplitude? Give the ratio of probabilities.

Use  $\hbar^2/m_e = 7.62eVA^2$

**Problem 3** (10 pts+3 extra credit)



An electron is in the potential shown in the figure.  $V(x)$  is infinite for  $x < 0$ . It is in the lowest energy state. Its wavefunction in the regions (i)  $0 < x < L$  and (ii)  $x > L$  is given by

(i)  $\psi(x) = A\sin(kx) + B\cos(kx)$  and (ii)  $\psi(x) = Ce^{-\alpha x} + De^{\alpha x}$ , with  $\alpha > 0$ .

(a) State which of the coefficients in the wavefunctions given above are zero and why.

(b) The energy of this electron is  $3eV$ , and  $V_0$  is  $7eV$ . Find the values of  $k$  and  $\alpha$  in  $A^{-1}$ .

(c) Make a qualitative plot of the wavefunction  $\psi(x)$ . Indicate where  $x=0$  and  $x=L$  are in your graph.

(d) For extra credit: find the condition that determines the value of the energy  $E_1$  as function of  $V_0$  and  $L$ . Show in a graph how to find this value graphically by the intersection of two curves. Is there always a solution or is there no solution for given  $L$  if  $V_0$  is very small?

**Justify all your answers to all problems**