Physics 4E Quiz 1 Solutions Spring 2012

Problem 1

(a) $$\frac{hc}{\lambda_m k T} = 4.965 \Rightarrow T = \frac{hc}{\lambda_m \cdot 4.965 k} = \frac{12{,}400}{5{,}100 \cdot \frac{1}{11{,}600} \cdot 4.965} = 5680°K$$

$$\boxed{T = 5680°K}$$

(b) $R(\lambda)$ is proportional to $u(\lambda) = \frac{8\pi}{\lambda^5} \frac{hc}{e^{hc/\lambda kT} - 1}$

$$\frac{R(\lambda_2)}{R(\lambda_1)} = \left(\frac{\lambda_1}{\lambda_2}\right)^5 \frac{e^{hc/\lambda_1 kT} - 1}{e^{hc/\lambda_2 kT} - 1}$$

$\lambda_1 = 5{,}100\text{Å}$, $hc/\lambda_1 kT = 4.965$

$\lambda_2 = 10{,}200\text{Å} = 2\lambda_1 \Rightarrow \frac{hc}{\lambda_2 kT} = \frac{1}{2} \frac{hc}{\lambda_1 kT} = \frac{1}{2} \cdot 4.965 = 2.483$

$$\Rightarrow \frac{R(\lambda_2)}{R(\lambda_1)} = \left(\frac{\lambda_1}{2\lambda_1}\right)^5 \frac{e^{4.965} - 1}{e^{2.483} - 1} = \frac{1}{32} \cdot \frac{142.31}{10.97} = 0.405$$

Since $R(\lambda_1)\, d\lambda_1 = 1W$ and $d\lambda_2 = d\lambda_1 = 1\text{Å}$, $\boxed{R(\lambda_2)\, d\lambda_2 = 0.405 W} =$

= power emitted in wavelength range $10{,}200\text{Å}$ to $10{,}201\text{Å}$.

(c) Each photon has energy $\frac{hc}{\lambda_1} = \frac{12{,}400}{5100}\, eV = 2.43\, eV$

Power emitted $= \frac{\text{energy}}{\text{time}} = 1W = \frac{1\,J}{s} = \frac{N_{photons} \cdot 2.43\, eV}{s} \Rightarrow$

$$N_{photons} = \frac{1\,J}{2.43\, eV} = \frac{1\,J}{2.43 \times 1.6 \times 10^{-19} J} = \boxed{2.57 \times 10^{18} \frac{\text{photons}}{\text{second}}}$$

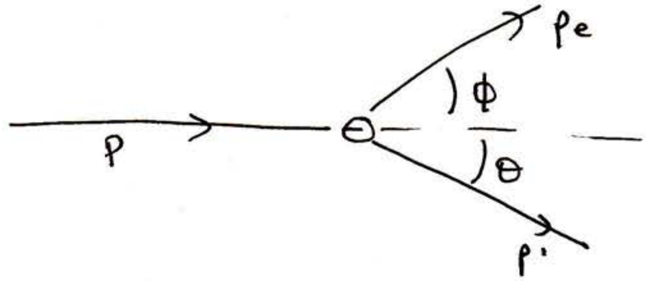
Problem 2

$$\lambda' = \lambda + \lambda_c(1-\cos\theta) \quad ; \quad \lambda_c = 0.0243\,\mathring{A} = \frac{h}{m_e c}$$

Given $\lambda' = \frac{3}{2}\lambda \Rightarrow \frac{1}{2}\lambda = \lambda_c(1-\cos\theta) \Rightarrow$

$$\Rightarrow \lambda = 2\lambda_c(1-\cos\theta) \Rightarrow \quad \boxed{0 \le \lambda \le 4\lambda_c = 0.0972\,\mathring{A}}$$

(b)



$$\begin{aligned} p_x &: \quad p - p'\cos\theta = p_e\cos\phi \\ p_y &: \quad p'\sin\theta = p_e\sin\phi \end{aligned} \Rightarrow \tan\phi = \frac{p'\sin\theta}{p - p'\cos\theta}$$

If $\phi = \theta \Rightarrow \quad \frac{\cancel{\sin\theta}}{\cos\theta} = \frac{p'\cancel{\sin\theta}}{p - p'\cos\theta} \Rightarrow p - p'\cos\theta = p'\cos\theta \Rightarrow$

$$\Rightarrow 2p'\cos\theta = p \Rightarrow \cos\theta = \frac{p}{2p'}$$

with $p = \frac{h}{\lambda}$, $\lambda' = \frac{3}{2}\lambda \Rightarrow p' = \frac{2}{3}p \Rightarrow \cos\theta = \frac{1}{2\cdot\frac{2}{3}} = \frac{3}{4}$

$\cos\theta = 0.75 \Rightarrow \boxed{\theta = 41.4^\circ}$ (b)

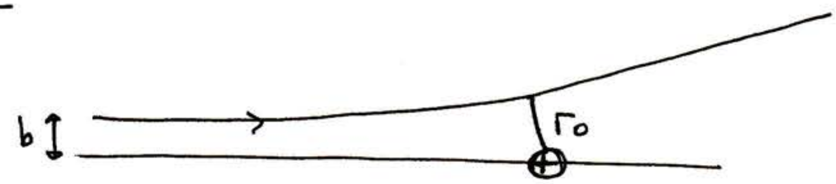
(c) $\lambda = 2\lambda_c(1-\cos\theta) = 2\lambda_c\left(1-\frac{3}{4}\right) = 2\lambda_c\cdot\frac{1}{4} = \lambda_c/2$

$$\lambda = \frac{\lambda_c}{2} = 0.012\,\mathring{A}, \quad \lambda' = \frac{3}{2}\lambda = \frac{3}{4}\lambda_c = 0.018\,\mathring{A}$$

(d) $E^e_{kin} = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 12{,}400\,\text{eV}\left(\frac{1}{0.012} - \frac{1}{0.018}\right) = 3.4\times10^5\,\text{eV}$

$$\boxed{E^e_{kin} = 0.34\,\text{MeV}}$$

Problem 3



r_0 = closest approach for impact parameter $b = 2\times 10^{-4}$ Å

$K_0 = 8$ MeV is kinetic energy far away

$$K_0 = K(r_0) + \frac{kqQ}{r_0} \Rightarrow K(r_0) = K_0 - \frac{kqQ}{r_0} =$$

$$= 8\,\text{MeV} - \frac{ke^2 .2\times 50}{r_0} = 8\,\text{MeV} - \frac{14.4\times 100}{2\times 10^{-4}}\,\text{eV} = 8\,\text{MeV} - 7.2\,\text{MeV}$$

$$\Rightarrow \boxed{K(r_0) = 0.8\,\text{MeV}}$$

(b) Angular momentum conservation: $L = m_\alpha r_0 v(r_0) = m_\alpha b\, v(\infty)$

$$\Rightarrow b = \frac{v(r_0)}{v(\infty)} r_0 = \sqrt{\frac{K(r_0)}{K_0}}\, r_0 = \sqrt{\frac{0.8}{8}}\, r_0 = \sqrt{0.1}\, r_0$$

$$\Rightarrow \boxed{b = 6.3\times 10^{-5}\,\text{Å}}$$

(c) When $b=0$, $K_0 = \frac{kqQ}{r_d} \Rightarrow$

$$\Rightarrow r_d = \frac{kqQ}{K_0} = \frac{ke^2 .\,50\times 2}{K_0} = \frac{14.4\,\text{eVÅ}\,.\,100}{8\times 10^6\,\text{eV}}$$

$\Rightarrow \boxed{r_d = 1.8\times 10^{-4}\,\text{Å}}$ distance of closest approach for impact parameter 0.