4-13. (a) $r_n = \dfrac{n^2 a_0}{Z}$ (Equation 4-18)

$$r_6 = \frac{6^2\left(0.053nm\right)}{1} = 1.91nm$$

(b) $r_6\left(He^+\right) = \dfrac{6^2\left(0.053nm\right)}{2} = 0.95nm$

4-15. $\dfrac{1}{\lambda} = Z^2 R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$ (Equation 4-22)

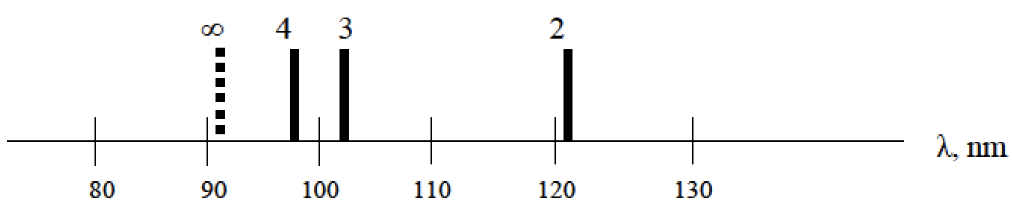
$$\frac{1}{\lambda_{ni}} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right) = R\left(\frac{n_i^2 - 1}{n_i^2}\right)$$

$$\lambda_{ni} = \frac{n_i^2}{R\left(n_i^2 - 1\right)} = \frac{n_i^2}{\left(1.0968\times 10^7 m\right)\left(n_i^2 - 1\right)} = \left(91.17nm\right)\left(\frac{n_i^2}{n_i^2 - 1}\right)$$

$$\lambda_2 = \frac{4}{3}\left(91.17nm\right) = 121.57nm \qquad \lambda_3 = \frac{9}{8}\left(91.17nm\right) = 102.57nm$$

$$\lambda_4 = \frac{16}{15}\left(91.17nm\right) = 97.25nm \qquad \lambda_\infty = 91.17nm$$

None of these are in the visible; all are in the ultraviolet.



4-19. (a) $a_u = \dfrac{\hbar^2}{\mu_\mu k e^2} = \dfrac{\mu_e}{\mu_\mu} \bullet \dfrac{\hbar^2}{\mu_e k e^2} = \dfrac{\mu_e}{\mu_\mu} a_0 = \dfrac{9.11\times 10^{-31} kg}{1.69\times 10^{-28} kg}\left(0.0529nm\right) = 2.56\times 10^{-4} nm$

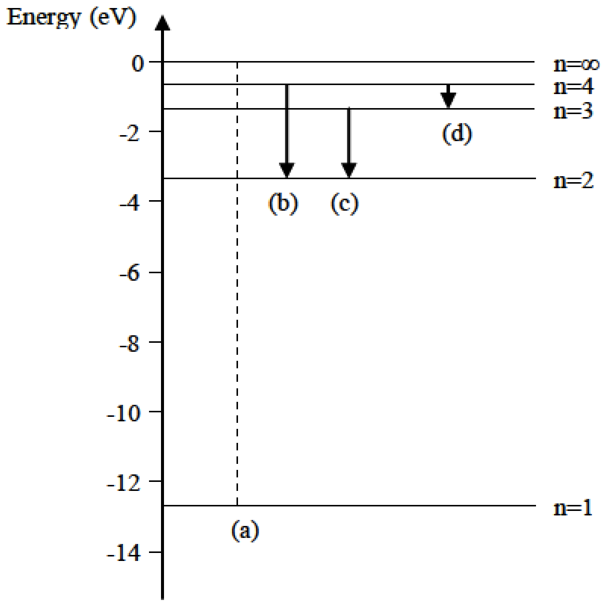
(b) $E_\mu = \dfrac{\mu_\mu k^2 e^4}{2\hbar^2} = \dfrac{\mu_\mu}{\mu_e} \bullet \dfrac{\mu_e k^2 e^4}{2\hbar^2} = \dfrac{\mu_\mu}{\mu_e} \bullet E_0 = \dfrac{1.69\times 10^{-28} kg}{9.11\times 10^{-31} kg}\left(13.6eV\right) = 2520eV$

(c) The shortest wavelength in the Lyman series is the series limit ($n_i = \infty,\ n_f = 1$). The photon energy is equal in magnitude to the ground state energy $-E_\mu$.

$$\lambda_\infty = \frac{hc}{E_\mu} = \frac{1240eV \bullet nm}{2520eV} = 0.492nm$$

(The reduced masses have been used in this solution.)

4-21.



(a) Lyman limit, (b) H_β line, (c) H_α line, (d) longest wavelength line of Paschen series

4-24. (a) The reduced mass correction to the Rydberg constant is important in this case.

$$R = R_\infty\left(\frac{1}{1+m/M}\right) = R_\infty\left(\frac{1}{2}\right) = 5.4869\times10^6 m^{-1} \quad \text{(from Equation 4-26)}$$

$$E_n = -hcR/n^2 \quad \text{(from Equations 4-23 and 4-24)}$$

$$E_1 = -(1240eV \bullet nm)(5.4869\times10^6 m^{-1})(10^{-9} m/nm)/(1)^2 = -6.804eV$$

Similarly, $E_2 = -1.701eV$ and $E_3 = -0.756eV$

(b) Lyman α is the $n = 2 \rightarrow n = 1$ transition.

$$\frac{hc}{\lambda} = E_2 - E_1 \quad \rightarrow \quad \lambda_\alpha = \frac{hc}{E_2 - E_1} = \frac{1240 eV \bullet nm}{-1.701 eV - (-6.804 eV)} = 243 nm$$

Lyman β is the $n = 3 \rightarrow n = 1$ transition.

$$\lambda_\beta = \frac{hc}{E_3 - E_1} = \frac{1240 eV \bullet nm}{-0.756 eV - (-6.804 eV)} = 205 nm$$

4-25. (a) The radii of the Bohr orbits are given by (see Equation 4-18)

$r = n^2 a_0 / Z$ where $a_0 = 0.0529 nm$ and $Z = 1$ for hydrogen.

For $n = 600$, $r = (600)^2 (0.0529 nm) = 1.90 \times 10^4 nm = 19.0 \mu m$

This is about the size of a tiny grain of sand.

(b) The electron's speed in a Bohr orbit is given by

$v^2 = ke^2 / mr$ with $Z = 1$

Substituting r for the $n = 600$ orbit from (a), then taking the square root,

$$v^2 = (8.99 \times 10^9 N \bullet m^2)(1.609 \times 10^{-19} C)^2 / (9.11 \times 10^{-31} kg)(19.0 \times 10^{-6} m)$$

$$v^2 = 1.33 \times 10^7 m^2 / s^2 \quad \rightarrow \quad v = 3.65 \times 10^3 m / s$$

For comparison, in the $n = 1$ orbit, v is about $2 \times 10^6 m / s$

4-26. (a) $\frac{1}{\lambda} = R(Z-1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$$\lambda_3 = \left[(1.097 \times 10^7 m^{-1})(42-1)^2 \left(\frac{1}{1^2} - \frac{1}{3^2} \right) \right]^{-1} = 6.10 \times 10^{-11} m = 0.0610 nm$$

$$\lambda_4 = \left[(1.097 \times 10^7 m^{-1})(42-1)^2 \left(\frac{1}{1^2} - \frac{1}{4^2} \right) \right]^{-1} = 5.78 \times 10^{-11} m = 0.0578 nm$$

(b) $\lambda_{limit} = \left[(1.097 \times 10^7 m^{-1})(42-1)^2 \left(\frac{1}{1^2} - 0 \right) \right]^{-1} = 5.42 \times 10^{-11} m = 0.0542 nm$

4-27. $\frac{1}{\lambda}=R(Z-1)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)=R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$ for K_α

$$Z-1=\left[\frac{1}{\lambda R\left(1-\frac{1}{4}\right)}\right]^{1/2}=\left[\frac{1}{(0.0794nm)(1.097\times10^{-2}/nm)(3/4)}\right]^{1/2}$$

$Z=1+39.1\approx 40$ Zirconium

4-29. $r_n=\frac{n^2a_0}{Z}$ (Equation 4-18)

The $n=1$ electrons "see" a nuclear charge of approximately $Z-1$, or 78 for Au.

$r_1=0.0529nm/78=6.8\times10^{-4}nm\left(10^{-9}m/nm\right)\left(10^{15}fm/m\right)=680fm$, or about 100 times the radius of the Au nucleus.

4-36. $\Delta E=\frac{hc}{\lambda}=\frac{1240eV\bullet nm}{790nm}=1.610eV$. The first decrease in current will occur when the voltage reaches $1.61V$.

4-42. Those scattered at $\theta=180°$ obeyed the Rutherford formula. This is a head-on collision where the α comes instantaneously to rest before reversing direction. At that point its kinetic energy has been converted entirely to electrostatic potential energy, so

$\frac{1}{2}m_\alpha v^2=7.7MeV=\frac{k(2e)(79e)}{r}$ where r = upper limit of the nuclear radius.

$$r=\frac{k(2)(79)e^2}{7.7MeV}=\frac{2(79)(1.440MeV\bullet fm)}{7.7MeV}=29.5fm$$

4-45. $\lambda=\left[R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)\right]^{-1}$ $\quad\Delta\lambda=\frac{d\lambda}{d\mu}\Delta\mu=\left(-R^{-2}\right)\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)^{-1}\frac{dR}{d\mu}\Delta\mu$

Because $R\propto\mu$, $dR/d\mu=R/\mu$. $\Delta\lambda\approx\left(-R^{-2}\right)\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)^{-1}(R/\mu)\Delta\mu=-\lambda(\Delta\mu/\mu)$

$$\mu_H = \frac{m_e m_p}{m_e + m_p} \qquad \mu_D = \frac{m_e m_d}{m_e + m_d}$$

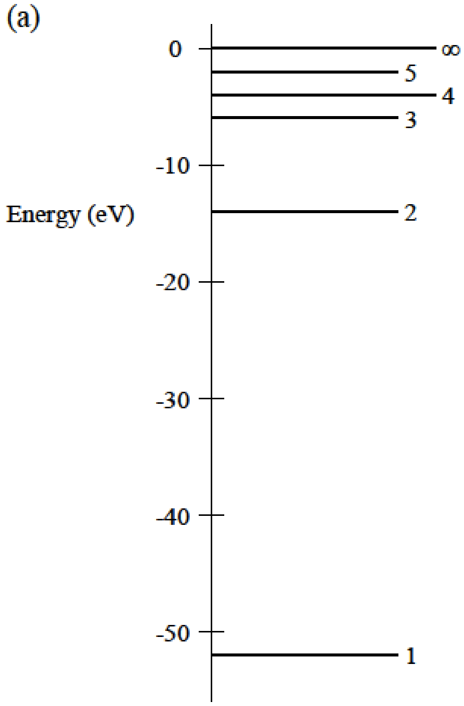
$$\frac{\Delta\mu}{\mu} = \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{m_e m_d / (m_e + m_d)}{m_e m_p / (m_e + m_p)} - 1 = \frac{m_d / (m_e + m_d)}{m_p / (m_e + m_p)} - 1 = \frac{m_e (m_d - m_p)}{m_p (m_e + m_d)}$$

If we approximate $m_d = 2m_p$ and $m_e \ll m_d$, then $\frac{\Delta\mu}{\mu} \approx \frac{m_e}{2m_p}$ and

$$\Delta\lambda = -\lambda(\Delta\mu/\mu) = -(656.3nm)\frac{0.511MeV}{2(938.28MeV)} = -0.179nm$$

4-52. For He: $E_n = -13.6eV\,Z^2/n^2 = -54.4eV/n^2$ (Equation 4-20)

(a)



$E_1 = -54.4\ eV$

$E_2 = -13.6\ eV$

$E_3 = -6.04\ eV$

$E_4 = -3.04\ eV$

$E_5 = -2.18\ eV$

$E_\infty = 0\ eV$

(b) Ionization energy is 54.5eV.

(c) H Lyman α: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 3.4eV) = 121.6nm$

H Lyman β: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 1.41eV) = 102.6nm$

He^+ Balmer α: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 6.04eV) = 164.0nm$

He^+ Balmer β: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 3.40eV) = 121.6nm$

$\Delta\alpha = 42.4nm \quad \Delta\beta = 19.0nm$

(The reduced mass correction factor does not change the energies calculated above to three significant figures.)

(d) $E_n = -13.6eV\, Z^2/n^2$ because for He^+, $Z = 2$, then $Z^2 = 2^2$. Every time n is an even number a 2^2 can be factored out of n^2 and cancelled with the $Z^2 = 2^2$ in the numerator; e.g., for He^+,

$E_2 = -13.6eV{\bullet}2^2/2^2 = -13.6eV$ (H ground state)

$E_4 = -13.6eV{\bullet}2^2/4^2 = -13.6eV/2^2$ (H -1^{st} excited state)

$E_6 = -13.6eV{\bullet}2^2/6^2 = -13.6eV/3^2$ (H -2^{nd} excited state)

⋮

etc.

Thus, all of the H energy level values are to be found within the He^+ energy levels, so He^+ will have within its spectrum lines that match (nearly) a line in the H spectrum.

4-54. (a) $E_n = -\dfrac{ke^2}{2r_n} = -\dfrac{ke^2}{2n^2 r_o} \qquad E_{n-1} = -\dfrac{ke^2}{2(n-1)^2 r_o}$

$$hf = E_n - E_{n-1} = -\frac{ke^2}{2n^2 r_o} - \left(-\frac{ke^2}{2(n-1)^2 r_o}\right)$$

$$f = \frac{ke^2}{2hr_o}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{ke^2}{2hr_o}\frac{n^2 - (n^2 - 2n + 1)}{n^2(n-1)^2}$$

$$= \frac{ke^2}{2hr_o}\frac{2n-1}{n^2(n-1)^2} \approx \frac{ke^2}{r_o h n^3} \quad \text{for } n \gg 1$$

(b) $f_{rev} = \dfrac{v}{2\pi r} \quad \rightarrow \quad f_{rev}^2 = \dfrac{v^2}{4\pi^2 r^2} = \dfrac{1}{4\pi^2 mr}\dfrac{mv^2}{r} = \dfrac{1}{4\pi^2 mr}\dfrac{ke^2}{r^2} = \dfrac{ke^2}{4\pi^2 m r_o^3 n^6}$

(c) The correspondence principle implies that the frequencies of radiation and revolution are equal.

$$f^2 = \left(\frac{ke^2}{r_o h n^3}\right)^2 = \frac{ke^2}{4\pi^2 m r_o^3 n^6} = f_{rev}^2 \qquad r_o = \frac{ke^2}{4\pi^2 m n^6}\left(\frac{hn^3}{ke^2}\right)^2 = \frac{h^2}{4\pi^2 mke^2} = \frac{\hbar^2}{mke^2}$$

which is the same as a_0 in Equation 4-19.

4-55. $\frac{kZe^2}{r} = \frac{mv^2}{r} \rightarrow \frac{kZe^2}{r^2} = \frac{(\gamma mv)^2}{mr}$ (from Equation 4-12)

$$\gamma v = \left(\frac{kZe^2}{mr}\right)^{1/2} = \frac{v}{\sqrt{1-\beta^2}}$$

$\frac{c^2\beta^2}{1-\beta^2} = \left(\frac{kZe^2}{mr}\right)$ Therefore, $\beta^2\left[c^2 + \left(\frac{kZe^2}{mr}\right)\right] = \left(\frac{kZe^2}{mr}\right)$

$$\beta^2 \approx \frac{1}{c^2}\left(\frac{kZe^2}{ma_o}\right) \rightarrow \beta = 0.0075Z^{1/2} \rightarrow v = 0.0075cZ^{1/2} = 2.25\times10^6 m/s \times Z^{1/2}$$

$$E = KE - kZe^2/r = mc^2(\gamma - 1) - \frac{kZe^2}{r} = mc^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right] - \frac{kZe^2}{r}$$

And substituting $\beta = 0.0075$ and $r = a_o$

$$E = 511\times10^3 eV\left[\frac{1}{\sqrt{1-(0.0075)^2}} - 1\right] - 28.8Z\ eV$$

$$= 14.4eV - 28.8Z\ eV = -14.4Z\ eV$$

4-59. Refer to Figure 4-16. All possible transitions starting at $n = 5$ occur.

$n = 5$ to $n = 4, 3, 2, 1$

$n = 4$ to $n = 3, 2, 1$

$n = 3$ to $n = 2, 1$

$n = 2$ to $n = 1$

Thus, there are 10 different photon energies emitted.

n_i	n_f	fraction	no. of photons
5	4	1/4	125
5	3	1/4	125
5	2	1/4	125
5	1	1/4	125
4	3	$1/4\times1/3$	42
4	2	$1/4\times1/3$	42
4	1	$1/4\times1/3$	42
3	2	$1/2\left[1/4+1/4(1/3)\right]$	83
3	1	$1/2\left[1/4+1/4(1/3)\right]$	83
2	1	$\left[\left(1/2(1/4+1/4)(1/3)\right)+1/4(1/3)+1/4\right]$	250

Total = 1,042

Note that the number of electrons arriving at the $n = 1$ level (125+42+83+250) is 500, as it should be.