Quiz 4 answers

1A. Prob. for particle to be within $x = L/3$ and $x = 2L/3$ when $n = 3$

$$P_{2L/3 > x > L/3} = \int_{L/3}^{2L/3} \psi^*(x)\,\psi(x)\,dx$$

$$= \int_{L/3}^{2L/3} \left(\sqrt{\frac{2}{L}}\right)^2 \sin^2\left(\frac{3\pi}{L}x\right) dx$$

Using $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$$= \frac{2}{L}\int_{L/3}^{2L/3}\left[\frac{1}{2} - \frac{1}{2}\cos\left(\frac{6\pi}{L}x\right)\right]dx$$

$$= \frac{2}{L}\left[\frac{1}{2}x - \left(\frac{1}{2}\right)\frac{L}{6\pi}\sin\left(\frac{6\pi}{L}x\right)\right]\Bigg|_{L/3}^{2L/3}$$

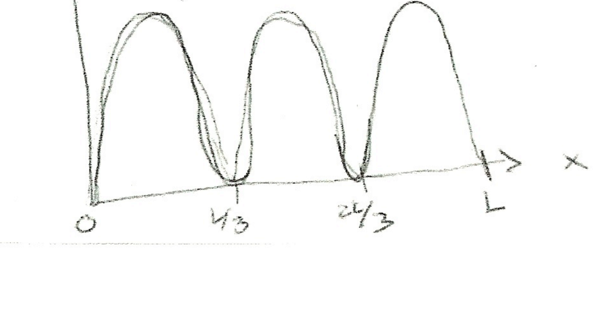
$$= \frac{2}{L}\left[\frac{1}{2}\left(\frac{2L}{3}\right) - \frac{L}{12\pi}\sin\left(\frac{6\pi}{L}\cdot\frac{2L}{3}\right) - \frac{1}{2}\left(\frac{L}{3}\right) + \frac{L}{12\pi}\sin\left(\frac{6\pi}{L}\cdot\frac{L}{3}\right)\right]$$

$$= \frac{2}{3} - \frac{1}{6\pi}\cancel{\sin(4\pi)}^{\,0} - \frac{1}{3} - \frac{1}{6\pi}\cancel{\sin(2\pi)}^{\,0}$$

$$= \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}$$

or $33.\overline{33}$ % prob of find the particle between $L/3$ and $2L/3$

If you Plot $P(x)$ for $n=3$



1.b. $$\langle KE \rangle = \int_0^L \psi^*[KE]\,\psi\,dx$$

$$= \frac{2}{L}\int_0^L \sin\left[\frac{3\pi}{L}x\right]\left(\frac{\hbar}{2me}\right)\sin\left[\frac{3\pi}{L}x\right]\left[\frac{3\pi}{L}\right]^2 dx$$

$$= \frac{2}{L}\,\frac{\hbar^2}{2m}\left(\frac{3\pi}{L}\right)^2 = \int_0^L \sin^2\left[\frac{3\pi}{L}x\right]dx$$

$$= \frac{2}{L}\,\frac{\hbar^2}{2m}\left(\frac{3\pi}{L}\right)^2 \int_0^L \left(\frac{1}{2} - \frac{1}{2}\cos\left(\frac{6\pi}{L}x\right)\right) dx$$

$$= \frac{2}{L}\,\frac{\hbar^2}{2m}\left(\frac{3\pi}{L}\right)^2 \left[\frac{1}{2}x - \frac{L}{6\pi}\frac{1}{2}\sin\left(\frac{6\pi}{L}x\right)\right]_0^L$$

$$= \frac{2}{L}\,\frac{\hbar^2}{2m}\left(\frac{3\pi}{L}\right)^2 \left[\frac{1}{2}L - \frac{L}{6\pi}\sin(6\pi)\right]$$

$$\langle KE \rangle = \frac{\hbar^2}{2m}\,\frac{3^2\pi^2}{L^2}$$

2\. a. $\psi_{II} = Ce^{\alpha x} + De^{-\alpha x}$

Shrödinger's Time indep. eq:

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(\cancel{x})\psi = E\psi$$

↳ and $U(x) = U$ inside barrier

so to find α, can use one of the general solutions

$$-\frac{\hbar^2}{2m}\frac{d^2(Ce^{\alpha x})}{dx^2} + U(Ce^{\alpha x}) = E(Ce^{\alpha x})$$

$$= \frac{-\hbar^2}{2m}\alpha^2(\cancel{Ce^{\alpha x}}) + U(\cancel{Ce^{\alpha x}}) = E(\cancel{Ce^{\alpha x}})$$

solving for α,

$$\alpha^2 = (E - U)\left(-\frac{2m}{\hbar^2}\right)$$

$$\boxed{\alpha = \sqrt{\frac{(U-E)2m}{\hbar^2}}}$$

or if you use the complete general sol.

$$\frac{-\hbar^2}{2m}\frac{d^2(Ce^{\alpha x}+De^{-\alpha x})}{dx^2} = (E-U)(Ce^{\alpha x}+De^{-\alpha x})$$

$$\Rightarrow \frac{-\hbar^2}{2m}\alpha^2(\cancel{Ce^{\alpha x}+De^{-\alpha x}}) = (E-U)(\cancel{Ce^{\alpha x}+De^{-\alpha x}})$$

$$\Rightarrow \alpha^2 = \sqrt{\frac{(U-E)2m}{\hbar^2}}$$

2b. at $x = 0$, $\psi_I(0) = \psi_{II}(0)$

$$\Rightarrow A e^{ik0} + B e^{-ik0} = C e^{\alpha \cdot 0} + D e^{-\alpha \cdot 0}$$

$$\boxed{A + B = C + D}$$

and at $x = 0$ $\quad \dfrac{d\psi_{II}(0)}{dx} = \dfrac{d\psi_{II}(0)}{dx}$

$$A ik \cancel{e^{ik0}} + B(-ik)\cancel{e^{-ik0}} = C\alpha \cancel{e^{\alpha \cdot 0}} - D\alpha \cancel{e^{\alpha \cdot 0}}$$

$$\boxed{A ik - B ik = C\alpha - D\alpha}$$