2-2 (a) Scalar equations can be considered in this case because relativistic and classical velocities are in the same direction.

$$p = \gamma mv = 1.90mv = \frac{mv}{\left[1-(v/c)^2\right]^{1/2}} \Rightarrow \frac{1}{\left[1-(v/c)^2\right]^{1/2}} = 1.90 \Rightarrow v = \left[1-\left(\frac{1}{1.90}\right)^2\right]^{1/2} c$$

$$= 0.85c$$

(b) No change, because the masses cancel each other.

2-7 $E = \gamma mc^2$, $p = \gamma mu$; $E^2 = \left(\gamma mc^2\right)^2$; $p^2 = (\gamma mu)^2$;

$$E^2 - p^2c^2 = \left(\gamma mc^2\right)^2 - (\gamma mu)^2 c^2 = \gamma^2 \left\{\left(mc^2\right)^2 - (mc)^2 u^2\right\}$$

$$= \left(mc^2\right)^2 \left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-1} = \left(mc^2\right)^2 \text{ Q.E.D.}$$

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

2-8 (a) $E_R = mc^2 = \left(1.67\times10^{-27}\ \text{kg}\right)\left(3\times10^8\ \text{m/s}\right)^2 = 1.503\times10^{-10}\ \text{J} = 939.4\ \text{MeV}$ (Numerical round off gives a slightly larger value for the proton mass)

(b) $$E = \gamma mc^2 = \frac{1.503\times10^{-10}\ \text{J}}{\left(1-(0.95c/c)^2\right)^{1/2}} = 4.813\times10^{-10}\ \text{J} \approx 3.01\times10^3\ \text{MeV}$$

(c) $K = E - mc^2 = 4.813\times10^{-10}\ \text{J} - 1.503\times10^{-10}\ \text{J} = 3.31\times10^{-10}\ \text{J} = 2.07\times10^3\ \text{MeV}$

2-9 (a) When $K = (\gamma-1)mc^2 = 5mc^2$, $\gamma = 6$ and $E = \gamma mc^2 = 6(0.511\,0\ \text{MeV}) = 3.07\ \text{MeV}$.

(b) $$\frac{1}{\gamma} = \left[1-\left(\frac{v}{c}\right)^2\right]^{1/2} \text{ and } v = c\left[1-\left(\frac{1}{\gamma}\right)^2\right]^{1/2} = c\left[1-\left(\frac{1}{6}\right)^2\right]^{1/2} = 0.986c$$

2-12 (a) When $K_e = K_p$, $m_e c^2(\gamma_e - 1) = m_p c^2(\gamma_p - 1)$. In this case $m_e c^2 = 0.511\,0$ MeV and $m_p c^2 = 938$ MeV, $\gamma_e = \left[1-(0.75)^2\right]^{1/2} = 1.511\,9$. Substituting these values into the first equation, we find $\gamma_p = 1 + \dfrac{m_e c^2(\gamma - 1)}{m_p c^2} = 1 + \dfrac{(0.511\,0)(1.511\,9 - 1)}{939} = 1.000\,279$. But $\gamma_p = \dfrac{1}{\left[1-\left(u_p/c\right)^2\right]^{1/2}}$; therefore $u_p = c\left(1-\gamma_p^{-2}\right)^{1/2} = 0.023\,6c$.

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $u_p = \left(\dfrac{\gamma_e}{\gamma_p}\right)\left(\dfrac{m_e}{m_p}\right)u_e$,

$$u_p = \left(\frac{1.511\,9}{1.000\,279}\right)\left[\frac{0.511\,0/c^2}{939/c^2}\right](0.75c) = 6.17\times10^{-4}\,c.$$

2-13 (a)

$$E = 400mc^2 = \gamma mc^2$$

$$\gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2} = 400$$

$$\left(1-\frac{v^2}{c^2}\right) = \left(\frac{1}{400}\right)^2$$

$$\frac{v}{c} = \left[1-\frac{1}{400^2}\right]^{1/2}$$

$$v = 0.999\,997c$$

(b) $K = E - mc^2 = (400-1)mc^2 = 399mc^2 = (399)(938.3\text{ MeV}) = 3.744\times10^5\text{ MeV}$

2-15 (a) $K = \gamma mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1+\dfrac{Vq}{mc^2}\right)^2$ and $\dfrac{v}{c} = \left\{1-\left(1+\dfrac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{1-\frac{1}{1+\left(5.0\times10^4\text{ eV}/0.511\text{ MeV}\right)^2}\right\}^{1/2} = 0.412\,7$$

or $v = 0.413c$.

(b)

$$K = \frac{1}{2}mv^2 = Vq$$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2\left(5.0\times10^4\text{ eV}\right)}{0.511\text{ MeV}/c^2}\right\}^{1/2} = 0.442c$$

(c) The error in using the classical expression is approximately $\dfrac{3}{40}\times100\%$ or about 7.5% in speed.

2-18 (a) The mass difference of the two nuclei is

$$\Delta m = 54.927\,9\text{ u} - 54.924\,4\text{ u} = 0.003\,5\text{ u}$$
$$\Delta E = (931\text{ MeV/u})(0.003\,5\text{ u}) = 3.26\text{ MeV}.$$

(b) The rest energy for an electron is 0.511 MeV. Therefore,

$$K = 3.26\text{ MeV} - 0.511\text{ MeV} = 2.75\text{ MeV}.$$

2-20 $\Delta m = m\ \ - m_p - m_e = 1.008\,665\text{ u} - 1.007\,276\text{ u} - 0.000\,548\,5\text{ u} = 8.404\times10^{-4}\text{ u}$

$E = c^2\left(8.404\times10^{-4}\text{ u}\right) = \left(8.404\times10^{-4}\text{ u}\right)(931.5\ \text{MeV/u}) = 0.783\text{ MeV}.$

2-23 In this problem, M is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2 = \frac{m_l c^2}{\sqrt{1-v_l^2/c^2}} + \frac{m_h c^2}{\sqrt{1-v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987c)(6.22) = (m_h)(0.868c)(2.01)$$
$$m_l = \frac{(m_h)(0.868c)(2.01)}{(0.987)(6.22)} = 0.284 m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains $3.34\times10^{-27}\text{ kg} = 6.22 m_l + 2.01 m_h$ which in turn gives

$3.34\times10^{-27}\text{ kg} = (6.22)(0.284)m_l + 2.01 m_h$ or $m_h = \dfrac{3.34\times10^{-27}\text{ kg}}{3.78} = 8.84\times10^{-28}\text{ kg}$ and

$m_l = (0.284)m_h = 2.51\times10^{-28}\text{ kg}.$

2-26 Energy conservation: $\dfrac{1}{\sqrt{1-0^2}}1\,400\text{ kg}c^2 + \dfrac{900\text{ kg}c^2}{\sqrt{1-0.85^2}} = \dfrac{Mc^2}{\sqrt{1-v^2/c^2}}$; $3\,108\text{ kg}\sqrt{1-\dfrac{v^2}{c^2}} = M.$

Momentum conservation: $0 + \dfrac{900\text{ kg}(0.85c)}{\sqrt{1-0.85^2}} = \dfrac{Mv}{\sqrt{1-v^2/c^2}}$; $1\,452\text{ kg}\sqrt{1-\dfrac{v^2}{c^2}} = \dfrac{Mv}{c}.$

(a) Dividing gives $\dfrac{v}{c} = \dfrac{1\,452}{3\,108} = 0.467$ $\qquad v = 0.467c.$

(b) Now by substitution $3\,108\text{ kg}\sqrt{1-0.467^2} = M = 2.75\times10^3\text{ kg}.$

2-33 The energy that arrives in one year is

$$E = \mathscr{P}\,\Delta t = \left(1.79\times10^{17}\ \text{J/s}\right)\left(3.16\times10^{7}\ \text{s}\right) = 5.66\times10^{24}\ \text{J}.$$

Thus, $m = \dfrac{E}{c^2} = \dfrac{5.66\times10^{24}\ \text{J}}{\left(3.00\times10^{8}\ \text{m/s}\right)^2} = 6.28\times10^{7}\ \text{kg}$.