7-2 (a) To the left of the step the particle is free with kinetic energy $E$ and corresponding wavenumber $k_1 = \left(\frac{2mE}{\hbar^2}\right)^{1/2}$:

$$\psi(x) = Ae^{ik_1 x} + Be^{-ik_1 x} \qquad x \le 0$$

To the right of the step the kinetic energy is reduced to $E - U$ and the wavenumber is now $k_2 = \left[\frac{2m(E-U)}{\hbar^2}\right]^{1/2}$

$$\psi(x) = Ce^{ik_2 x} + De^{-ik_2 x} \qquad x \ge 0$$

with $D = 0$ for waves incident on the step from the left. At $x = 0$ both $\psi$ and $\frac{d\psi}{dx}$ must be continuous: $\psi(0) = A + B = C$

$$\left.\frac{d\psi}{dx}\right|_0 = ik_1(A - B) = ik_2 C\ .$$

(b) Eliminating $C$ gives $A + B = \frac{k_1}{k_2}(A - B)$ or $A\left(\frac{k_1}{k_2} - 1\right) = B\left(\frac{k_1}{k_2} + 1\right)$. Thus,

$$R = \left|\frac{B}{A}\right|^2 = \frac{(k_1/k_2 - 1)^2}{(k_1/k_2 + 1)^2} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$$

$$T = 1 - R = \frac{4k_1 k_2}{(k_1 + k_2)^2}$$

(c) As $E \to U$, $k_2 \to 0$, and $R \to 1$, $T \to 0$ (no transmission), in agreement with the result for any energy $E < U$. For $E \to \infty$, $k_1 \to k_2$ and $R \to 0$, $T \to 1$ (perfect transmission) suggesting correctly that very energetic particles do not *see* the step and so are unaffected by it.

7-3 With $E = 25\ \text{MeV}$ and $U = 20\ \text{MeV}$, the ratio of wavenumber is $\frac{k_1}{k_2} = \left(\frac{E}{E-U}\right)^{1/2} = \left(\frac{25}{25-20}\right)^{1/2} = \sqrt{5} = 2.236$. Then from Problem 7-2 $R = \frac{(\sqrt{5}-1)^2}{(\sqrt{5}+1)^2} = 0.146$ and $T = 1 - R = 0.854$. Thus, 14.6% of the incoming particles would be reflected and 85.4% would be transmitted. For electrons with the same energy, the transparency and reflectivity of the step are unchanged.

7-11 (a) The matter wave reflected from the trailing edge of the well $(x = L)$ must travel the extra distance $2L$ before combining with the wave reflected from the leading edge $(x = 0)$. For $\lambda_2 = 2L$, these two waves interfere destructively since the latter suffers a phase shift of 180° upon reflection, as discussed in Example 7.3.

(b) The wave functions in all three regions are free particle plane waves. In regions 1 and 3 where $U(x) = U$ we have

$$\Psi(x,\ t) = Ae^{i(k'x-\omega t)} + Be^{i(-k'x-\omega t)} \qquad x < 0$$
$$\Psi(x,\ t) = Fe^{i(k'x-\omega t)} + Ge^{i(-k'x-\omega t)} \qquad x < 0$$

with $k' = \dfrac{[2m(E-U)]^{1/2}}{\hbar}$. In this case $G = 0$ since the particle is incident from the left. In region 2 where $U(x) = 0$ we have

$$\Psi(x,\ t) = Ce^{i(-kx-\omega t)} + De^{i(kx-\omega t)} \qquad 0 < x < L$$

with $k = \dfrac{(2mE)^{1/2}}{\hbar} = \dfrac{2\pi}{\lambda_2} = \dfrac{\pi}{L}$ for the case of interest. The wave function and its slope are continuous everywhere, and in particular at the well edges $x = 0$ and $x = L$. Thus, we must require

$$A + B = C + D \qquad [\text{continuity of } \Psi \text{ at } x = 0]$$
$$k'A - k'B = kD - kC \qquad \left[\text{continuity of } \frac{\partial \Psi}{\partial x} \text{ at } x = 0\right]$$
$$Ce^{-ikL} + De^{ikL} = Fe^{ik'L} \qquad [\text{continuity of } \Psi \text{ at } x = L]$$
$$kDe^{ikL} - kCe^{-ikL} = k'Fe^{ik'L} \qquad \left[\text{continuity of } \frac{\partial \Psi}{\partial x} \text{ at } x = L\right]$$

For $kL = \pi$, $e^{\pm ikL} = -1$ and the last two requirements can be combined to give $kD - kC = k'C + k'D$. Substituting this into the second requirement implies $A - B = C + D$, which is consistent with the first requirement only if $B = 0$, i.e., no reflected wave in region 1.