Problem 2

In the "rest" frame the velocity of the first car ($m_1 = 2000kg$) is $u_{1x} = 20m/s$, that of the second car ($m_2 = 1500kg$) is $u_{2x} = 0$. So the total momentum before the collision is

$$p_x = m_1 u_{1x} + m_2 u_{2x} = 4 \times 10^4 kg\, m/s$$

which must be conserved and is the same after the collision. The total mass of the wreck (the two cars stuck together) after the collision is $M = m_1 + m_2 = 3500kg$. Thus, the velocity of the wreck after the collision is

$$u_x = \frac{p_x}{M} = \frac{4 \times 10^4 kg\, m/s}{3500kg} \approx 11.43m/s$$

Now let us transform to the "moving" frame with velocity $v = 10m/s$. Applying the Galilean transformation law (1.2) for velocities we get that the velocity of the first car before the collision in this frame is

$$u'_{1x} = u_{1x} - v = 10m/s$$

the velocity of the second car is

$$u'_{2x} = u_{2x} - v = -10m/s$$

and that of the wreck is

$$u'_x = u_x - v \approx 1.43m/s$$

The total momentum before the collision in the "moving" frame is thus

$$p'_{x,initial} = m_1 u'_{1x} + m_2 u'_{2x} = 5000kg\, m/s$$

while after the collision we have

$$p'_{x,final} = M u'_x = 5000kg\, m/s$$

Hence $p'_{x,final} = p'_{x,initial}$ and the momentum is conserved in the "moving" frame. Note that the final result may be slightly off because of the rounding of the numerical results above.

1-5 This is a case of dilation. $T=\gamma T'$ in this problem with the proper time $T'=T_0$

$$T=\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}T_0 \Rightarrow \frac{v}{c}=\left[1-\left(\frac{T_0}{T}\right)^2\right]^{1/2};$$

in this case $T=2T_0$, $v=\left\{1-\left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2}=\left[1-\left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v=0.866c$.

1-8 $L=\frac{L'}{\gamma}$

$$\frac{1}{\gamma}=\frac{L}{L'}=\left[1-\frac{v^2}{c^2}\right]^{1/2}$$

$$v=c\left[1-\left(\frac{L}{L'}\right)^2\right]^{1/2}=c\left[1-\left(\frac{75}{100}\right)^2\right]^{1/2}=0.661c$$

1-12 (a) 70 beats/min or $\Delta t'=\frac{1}{70}$ min

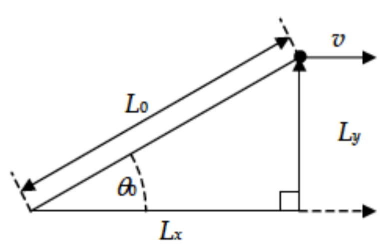
(b) $\Delta t=\gamma\,\Delta t'=\left[1-(0.9)^2\right]^{-1/2}\left(\frac{1}{70}\right)\text{ min}=0.032\,8\text{ min/beat}$ or the number of beats per minute $\approx 30.5\approx 31$.

1-13 (a) $\tau=\gamma\tau'=\left[1-(0.95)^2\right]^{-1/2}(2.2\ \mu\text{s})=7.05\ \mu\text{s}$

(b) $\Delta t'=\frac{d}{0.95c}=\frac{3\times10^3\text{ m}}{0.95c}=1.05\times10^{-5}\text{ s}$, therefore,

$$N=N_0\exp\left(-\frac{\Delta t}{\tau}\right)=\left(5\times10^4\text{ muons}\right)\exp(-1.487)\approx 1.128\times10^4\text{ muons}.$$

1-14 (a) Only the x-component of L_0 contacts.



$$L_{x'} = L_0 \cos\theta_0 \Rightarrow \frac{L_x [L_0 \cos\theta_0]}{\gamma}$$
$$L_{y'} = L_0 \sin\theta_0 \Rightarrow L_y = L_0 \sin\theta_0$$
$$L = \left[(L_x)^2 + (L_y)^2\right]^{1/2} = \left[\left(\frac{L_0 \cos\theta_0}{\gamma}\right)^2 + (L_0 \sin\theta_0)^2\right]^{1/2}$$
$$= L_0\left[\cos^2\theta_0\left(1 - \frac{v^2}{c^2}\right) + \sin^2\theta_0\right]^{1/2} = L_0\left[1 - \frac{v^2}{c^2}\cos^2\theta_0\right]^{1/2}$$

(b) As seen by the stationary observer, $\tan\theta = \frac{L_y}{L_x} = \frac{L_0 \sin\theta_0}{L_0 \cos\theta_0/\gamma} = \gamma \tan\theta_0$.

1-16 For an observer approaching a light source, $\lambda_{\text{ob}} = \left[\frac{(1 - v/c)^{1/2}}{(1 + v/c)^{1/2}}\right]\lambda_{\text{source}}$. Setting $\beta = \frac{v}{c}$ and after some algebra we find,

$$\beta = \frac{\lambda_{\text{source}}^2 - \lambda_{\text{obs}}^2}{\lambda_{\text{source}}^2 + \lambda_{\text{obs}}^2} = \frac{(650\ \text{nm})^2 - (550\ \text{nm})^2}{(650\ \text{nm})^2 + (550\ \text{nm})^2} = 0.166$$
$$v = 0.166c = \left(4.98 \times 10^7\ \text{m/s}\right)(2.237\ \text{mi/h})(\text{m/s})^{-1} = 1.11 \times 10^8\ \text{mi/h}.$$

1-19 $u_{XA} = -u_{XB}$; $u'_{XA} = 0.7c = \frac{u_{XA} - u_{XB}}{1 - u_{XA}u_{XB}/c^2}$; $0.70c = \frac{2u_{XA}}{1 + (u_{XA}/c)^2}$ or $0.70u_{XA}^2 - 2cu_{XA} + 0.7c^2 = 0$.

Solving this quadratic equation one finds $u_{XA} = 0.41c$ therefore $u_{XB} = -u_{XA} = -0.41c$.

1-23 (a) Let event 1 have coordinates $x_1 = y_1 = z_1 = t_1 = 0$ and event 2 have coordinates $x_2 = 100\text{ mm}$, $y_2 = z_2 = t_2 = 0$. In S', $x_1' = \gamma(x_1 - vt_1) = 0$, $y_1' = y_1 = 0$, $z_1' = z_1 = 0$, and $t_1' = \gamma\left[t_1 - \left(\frac{v}{c^2}\right)x_1\right] = 0$, with $\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-1/2}$ and so $\gamma = \left[1 - (0.70)^2\right]^{-1/2} = 1.40$. In system S', $x_2' = \gamma(x_2 - vt_2) = 140\text{ m}$, $y_2' = z_2' = 0$, and

$$t_2' = \gamma\left[t_2 - \left(\frac{v}{c^2}\right)x_2\right] = \frac{(1.4)(-0.70)(100\text{ m})}{3.00\times 10^8\text{ m/s}} = -0.33\ \mu\text{s}.$$

(b) $\Delta x' = x_2' - x_1' = 140\text{ m}$

(c) Events are not simultaneous in S', event 2 occurs $0.33\ \mu\text{s}$ earlier than event 1.

1-35 In the Earth frame, Speedo's trip lasts for a time $\Delta t = \frac{\Delta x}{v} = \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 21.05$ Speedo's age advances only by the proper time interval: $\Delta t_p = \frac{\Delta t}{\gamma} = 21.05\text{ yr}\sqrt{1 - 0.95^2} = 6.574\text{ yr}$ during his trip. Similarly for Goslo, $\Delta t_p = \frac{\Delta x}{v}\sqrt{1 - \frac{v^2}{c^2}} = \frac{20.0\text{ ly}}{0.750\text{ ly/yr}}\sqrt{1 - 0.75^2} = 17.64\text{ yr}$. While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\text{ ly}}{0.750\text{ ly/yr}} - \frac{0.20\text{ ly}}{0.950\text{ ly/yr}}\sqrt{1 - 0.75^2} = 17.64\text{ yr}.$$

Then Goslo ends up older by $17.64\text{ yr} - (6.574\text{ yr} + 5.614\text{ yr}) = 5.45\text{ yr}$.

1-37 Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, event B, as occurring first. We may take the *S*-frame coordinates of the events as ($x = 0$, $y = 0$, $z = 0$, $t = 0$) and ($x = 100\text{ m}$, $y = 0$, $z = 0$, $t = 0$). Then the coordinates in S' are given by Equations 1.23 to 1.27. Event A is at ($x' = 0$, $y' = 0$, $z' = 0$, $t' = 0$). The time of event B is:

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1 - 0.8^2}}\left(0 - \frac{0.8c}{c^2}(100\text{ m})\right) = 1.667\left(\frac{80\text{ m}}{3\times 10^8\text{ m/s}}\right) = -4.44\times 10^{-7}\text{ s}.$$

The time elapsing before A occurs is 444 ns.