Lecture 22

Light!

In summary…

- Maxwell's equations:

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \varepsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathbf{B}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \varepsilon_0 d\Phi_{\mathbf{E}} / dt$$

Maxwell's equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \varepsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathrm{B}} / dt$$

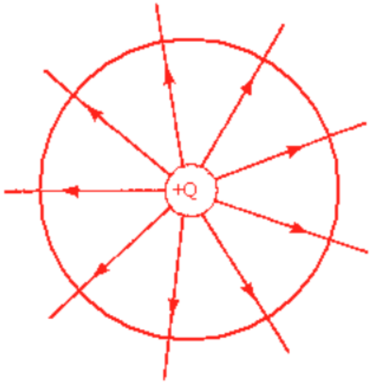
$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \varepsilon_0 d\Phi_{\mathrm{E}} / dt$$

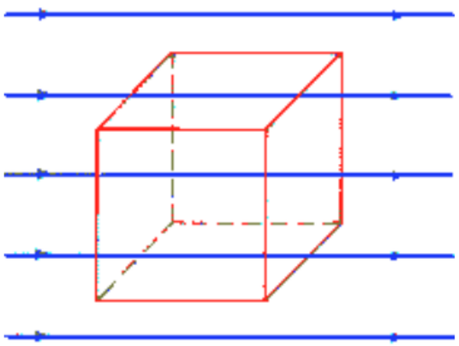
- Gauss's law:

$$\Phi_E = \frac{Q_{inside}}{\varepsilon_o}$$

- Where the permittivity of free space is

$\varepsilon_o = 8.85 \times 10^{-12} C^2/(Nm^2)$





Maxwell's equations

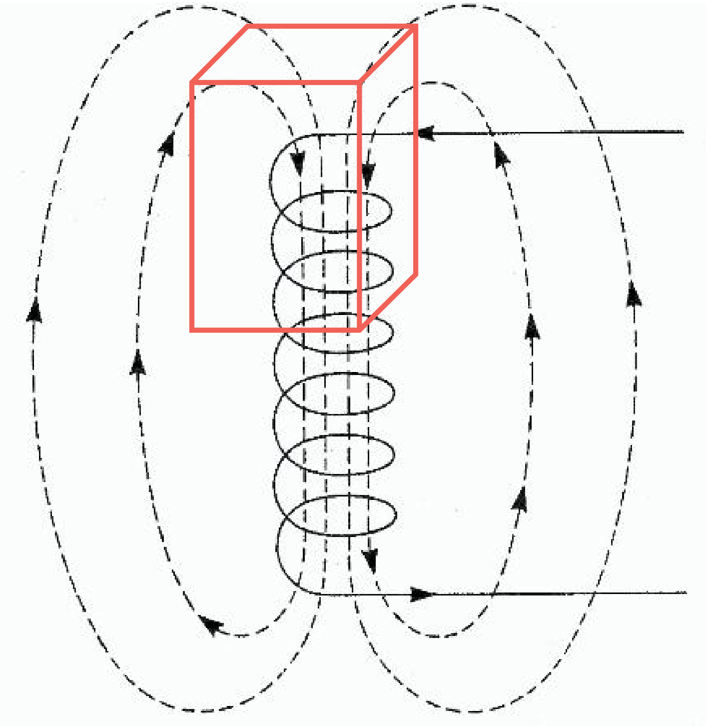
- Gauss's law for magnetic fields

$$\oint \mathbf{E}\cdot d\mathbf{A} = q/\varepsilon_0$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{S} = -d\Phi_{\mathbf{B}}/dt$$

$$\oint \mathbf{B}\cdot d\mathbf{S} = \mu_0 i + \mu_0\varepsilon_0 d\Phi_{\mathbf{E}}/dt$$



Just another way of saying that the field lines never begin or end. They are always endless loops.

Maxwell's equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \varepsilon_0$$

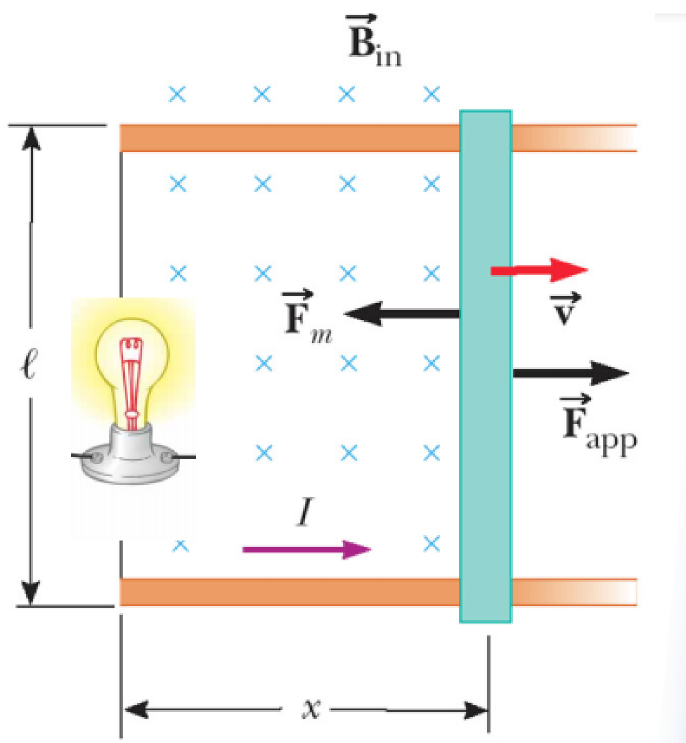
$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathbf{B}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \varepsilon_0 d\Phi_{\mathbf{E}} / dt$$

- Faraday's Law

$$\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t}$$



Maxwell's equations

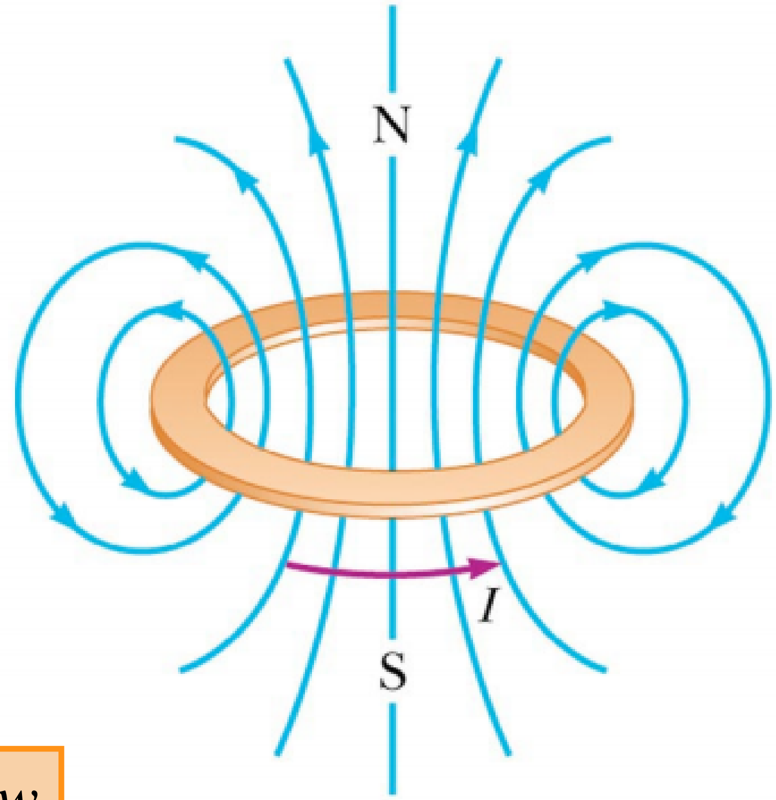
$$\mu_o = 4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}} = 12.6 \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \varepsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathbf{B}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \varepsilon_0 d\Phi_{\mathbf{E}} / dt$$



Ampere's law

Maxwell's equations

$$\oint \mathbf{E}\cdot d\mathbf{A} = q/\varepsilon_0$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{S} = -d\Phi_{\mathbf{B}}/dt$$

$$\oint \mathbf{B}\cdot d\mathbf{S} = \mu_0 i + \mu_0\varepsilon_0 d\Phi_{\mathbf{E}}/dt$$

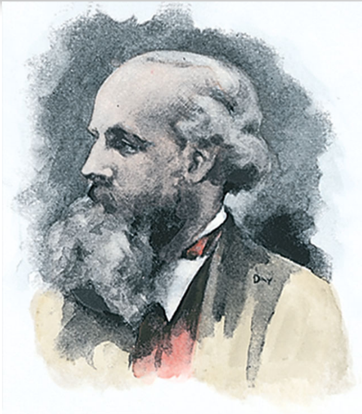
When you put Maxwell's equations together with the Lorentz force,

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$

You have a complete description of all classical electromagnetic interactions.

James Clerk Maxwell

- First to put all of these equations together in a seminal paper that predicted the electromagnetic theory of light.



He showed that if you had time dependent electric and magnetic fields in free space, they would satisfy a linear wave equation.

Wave Equation

- In free space, when current, I, is zero and charge, q, is zero, you can manipulate Maxwell's equations to see that a time varying electric field propagating obeys a wave equation.

$$\frac{\partial^2 E}{\partial^2 x} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

A general solution to this differential equation is a sinusoidal wave with speed v, and wavelength, λ.

$$E = E_0 \sin\left(2\pi \frac{x - vt}{\lambda}\right)$$

Wave Equation

$$E = E_0 \sin\left(2\pi \frac{x - vt}{\lambda}\right) \Rightarrow \frac{\partial^2 E}{\partial^2 x} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

- Plug your general solution into your differential equation that was derived from Maxwell's equation.

$$\frac{\partial^2 E}{\partial^2 x} = -E_0 \left(\frac{2\pi}{\lambda}\right)^2 \sin\left(2\pi \frac{x - vt}{\lambda}\right) \quad \text{and} \quad \frac{\partial^2 E}{\partial^2 t} = -E_0 \left(\frac{2\pi v}{\lambda}\right)^2 \sin\left(2\pi \frac{x - vt}{\lambda}\right)$$

- Solve for the velocity, see board…

$$\Rightarrow \quad v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Wave Equation

$$v^2 = \frac{1}{\mu_0 \varepsilon_0} \quad \Rightarrow \quad = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

- Plug in numbers:

$$\mu_o = 4\pi \times 10^{-7}\ \frac{\mathrm{T \cdot m}}{\mathrm{A}} = 12.6 \times 10^{-7}\ \frac{\mathrm{T \cdot m}}{\mathrm{A}}$$

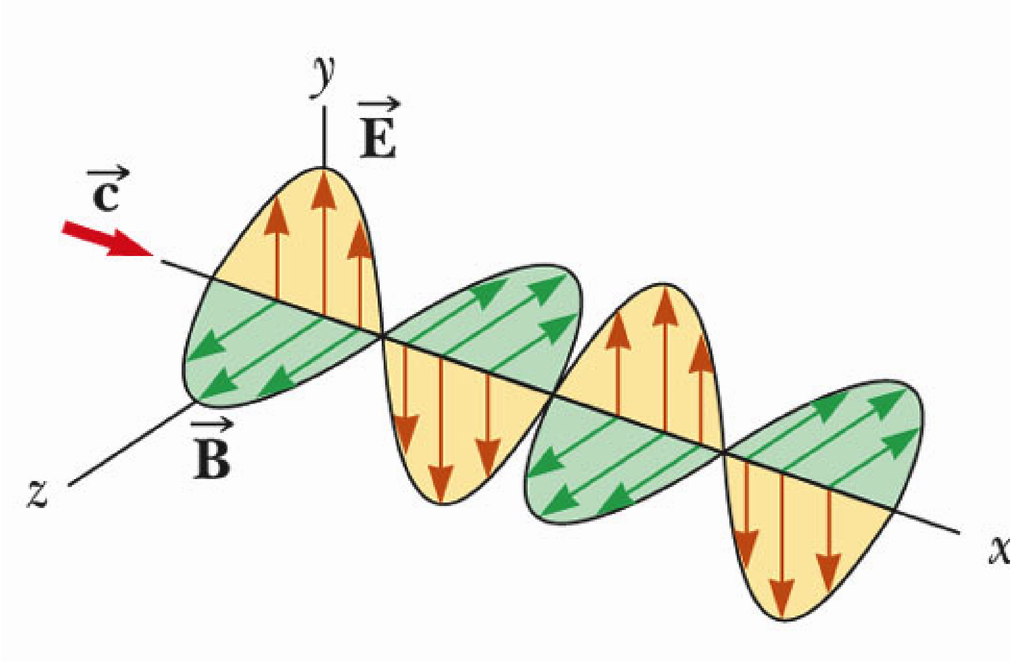
$$\varepsilon_o = 8.85 \times 10^{-12} C^2/(Nm^2)$$

$\Rightarrow$ v=2.99 x 10 ^{8} m/s

Equals the speed of light!!!!!

Electromagnetic Wave

- The same exercise can be done for a B, a magnetic wave in free space.



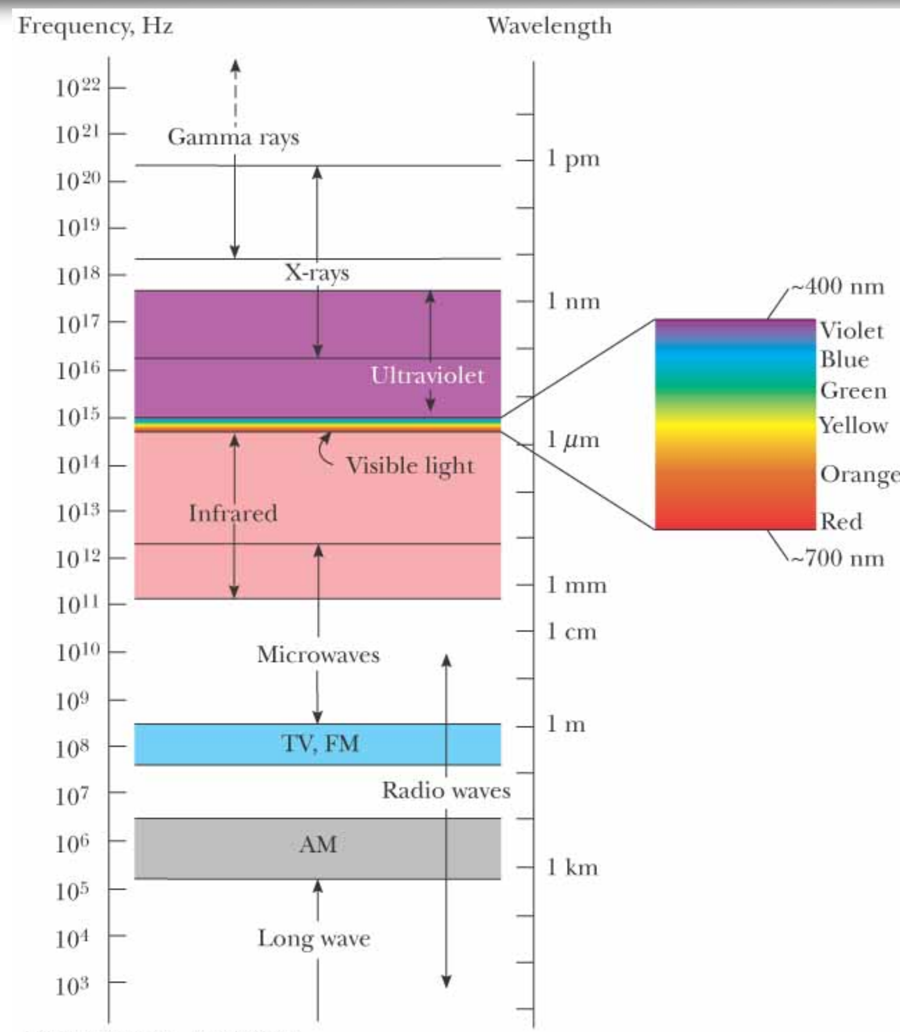
Where c = the speed of light. This was the first indication that light was an electromagnetic wave.

Electromagnetic wave

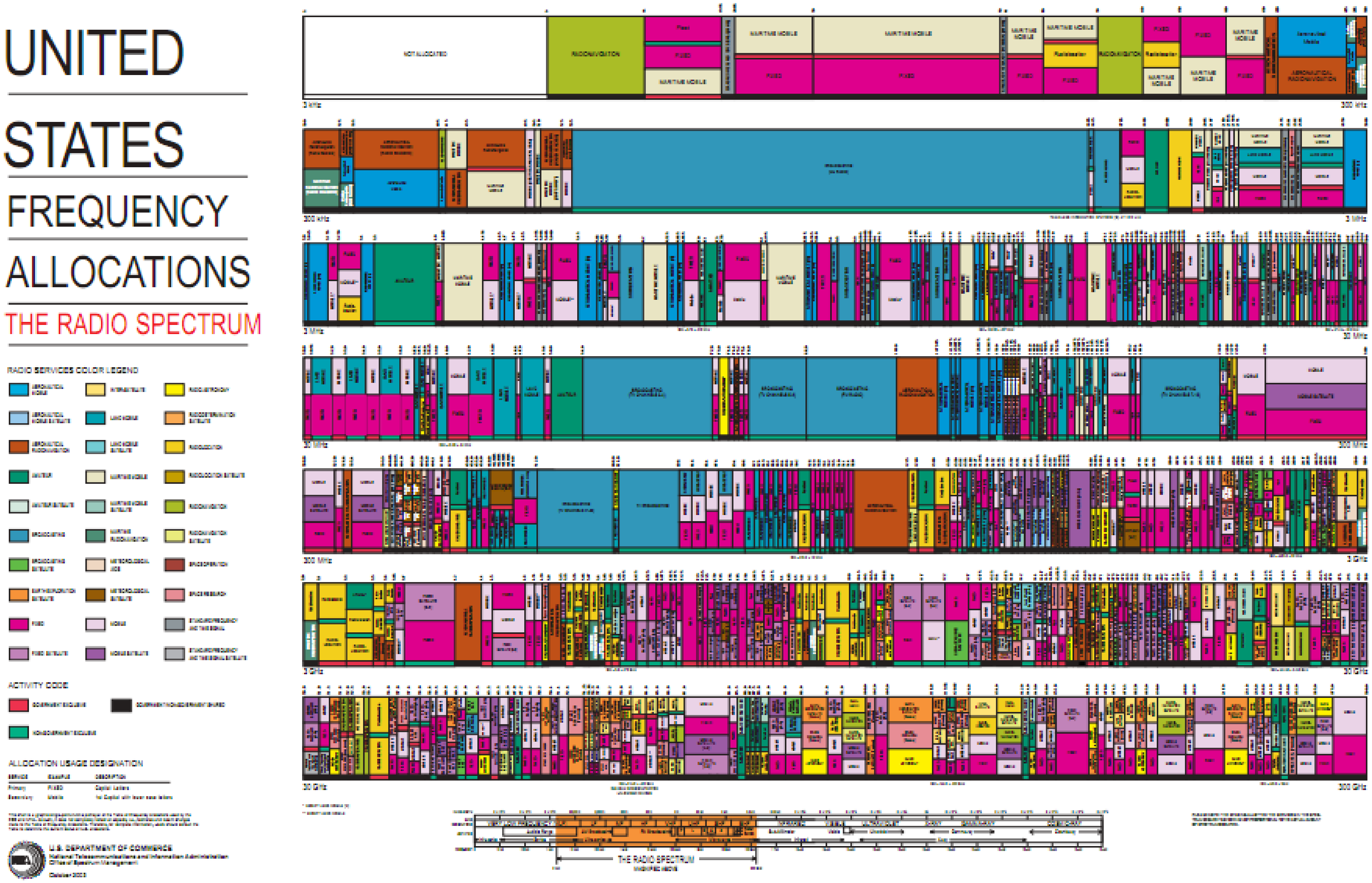
- Speed of light is constant for all EM waves.

$$\lambda f = c$$

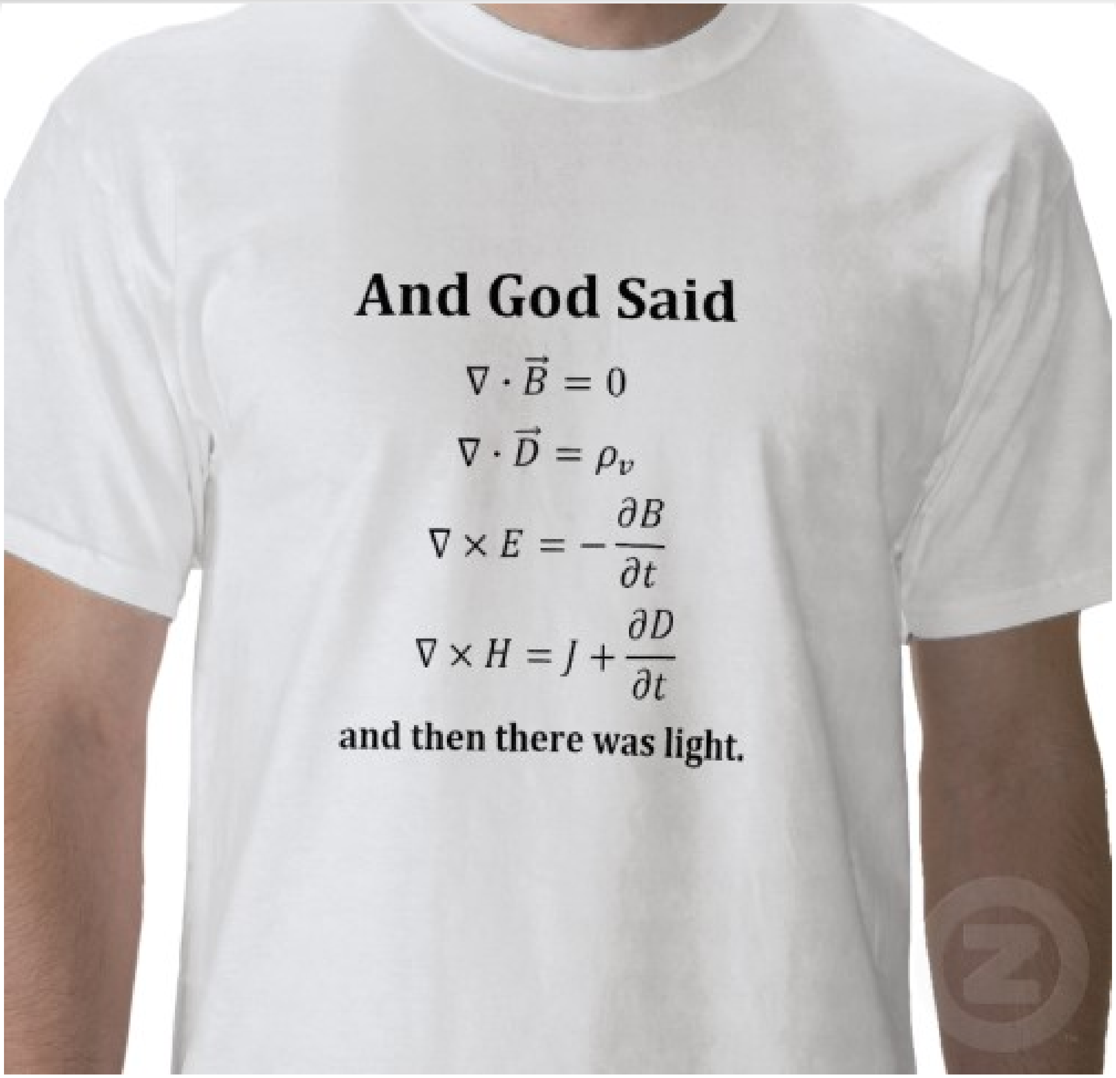
- Can parameterize all light by it's frequency, f, or wavelength.



Example Radio Waves



Nerd Alert!


And God Said
$\nabla \cdot \vec{B} = 0$
$\nabla \cdot \vec{D} = \rho_v$
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times H = J + \frac{\partial D}{\partial t}$
and then there was light.

Interesting applications

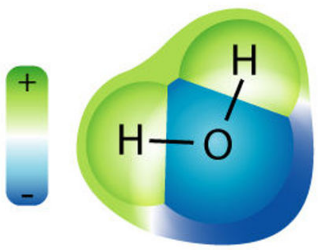
- Light as an electromagnetic wave.
- Microwaves!

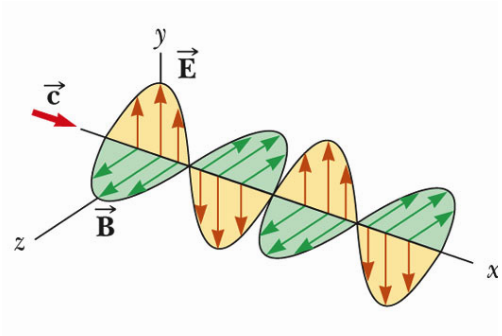
Use radio waves to heat the molecules in your food.

How?

Polarized molecules like water, and less polarized materials such as fats and sugars have an electric dipole moment.

f=2.45 GHz





Faraday Cage

- No signal in an elevator?

For Next Time

- Work on Chapter 23 homework. Solutions are now posted.