

Lecture 22

Light!

In summary...

- Maxwell's equations:

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathbf{B}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_{\mathbf{E}} / dt$$

Maxwell's equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$



$$\Phi_E = \frac{Q_{inside}}{\epsilon_0}$$

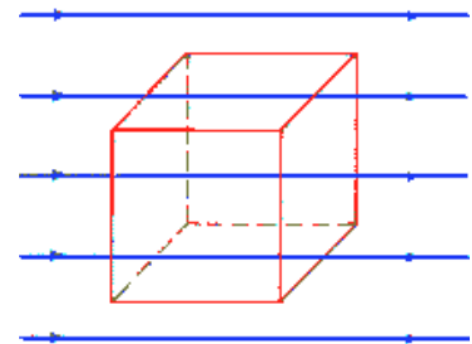
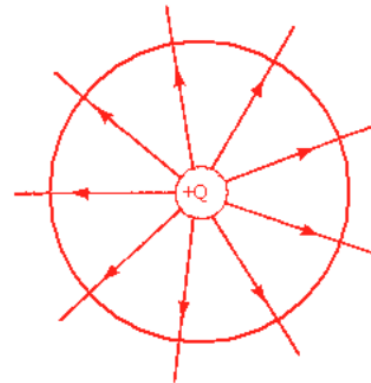
$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_B / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_E / dt$$

- Gauss's law:

- Where the permittivity of free space is $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2 / (\text{Nm}^2)$



Maxwell's equations

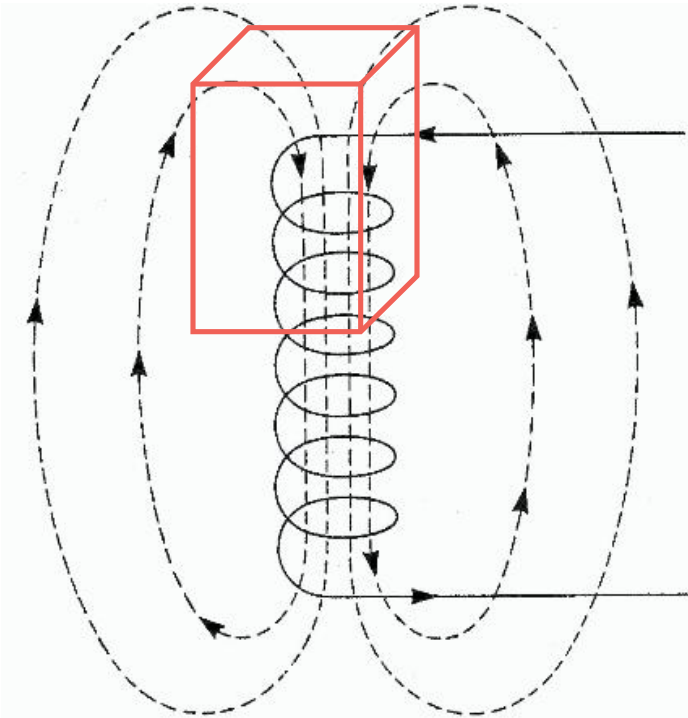
- Gauss's law for magnetic fields

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_B / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_E / dt$$



Just another way of saying that the field lines never begin or end. They are always endless loops.

Maxwell's equations

- Faraday's Law

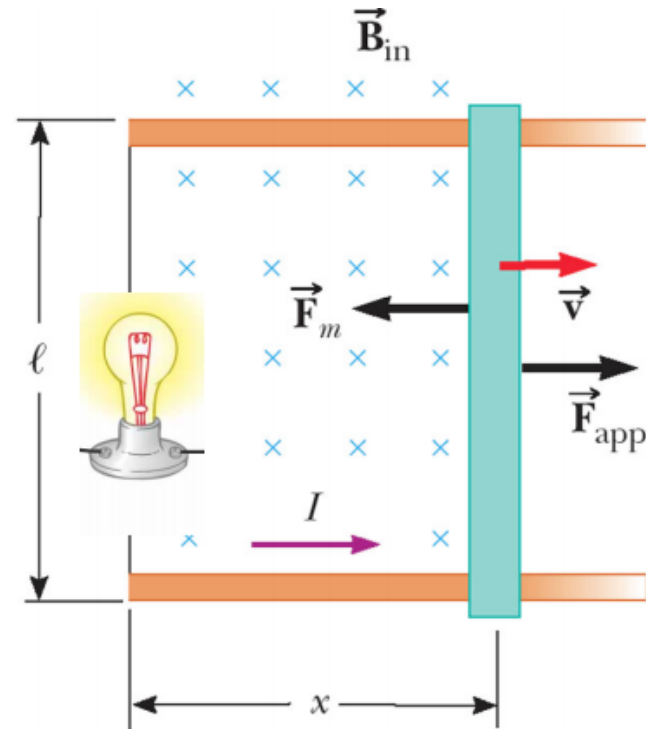
$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_B / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_E / dt$$



Maxwell's equations

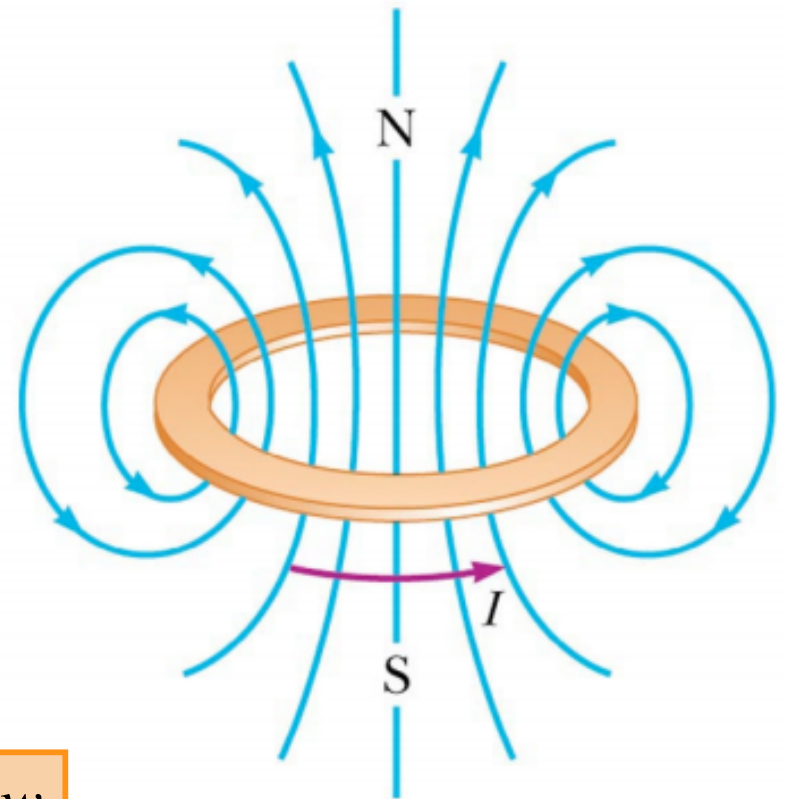
$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}} = 12.6 \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathbf{B}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_{\mathbf{E}} / dt$$



Ampere's law

Maxwell's equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathbf{B}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_{\mathbf{E}} / dt$$

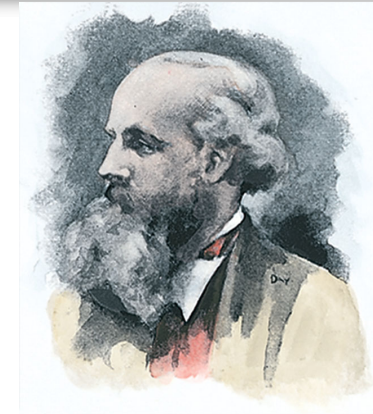
When you put Maxwell's equations together with the Lorentz force,

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

You have a complete description of all classical electromagnetic interactions.

James Clerk Maxwell

- First to put all of these equations together in a seminal paper that predicted the electromagnetic theory of light.

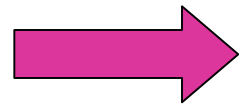


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He showed that if you had time dependent electric and magnetic fields in free space, they would satisfy a linear wave equation.

Wave Equation

- In free space, when current, I , is zero and charge, q , is zero, you can manipulate Maxwell's equations to see that a time varying electric field propagating obeys a wave equation.



$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

A general solution to this differential equation is a sinusoidal wave with speed v , and wavelength, λ .



$$E = E_0 \sin\left(2\pi \frac{x - vt}{\lambda}\right)$$

Wave Equation

$$E = E_0 \sin\left(2\pi \frac{x - vt}{\lambda}\right)$$




$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

- Plug your general solution into your differential equation that was derived from Maxwell's equation.

$$\frac{\partial^2 E}{\partial x^2} = -E_0 \left(\frac{2\pi}{\lambda}\right)^2 \sin\left(2\pi \frac{x - vt}{\lambda}\right) \quad \text{and} \quad \frac{\partial^2 E}{\partial t^2} = -E_0 \left(\frac{2\pi v}{\lambda}\right)^2 \sin\left(2\pi \frac{x - vt}{\lambda}\right)$$

- Solve for the velocity, see board...


$$v^2 = \frac{1}{\mu_0 \epsilon_0}$$

Wave Equation

$$v^2 = \frac{1}{\mu_0 \epsilon_0} \quad \longrightarrow \quad = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

- Plug in numbers:

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}} = 12.6 \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2 / (\text{Nm}^2)$$

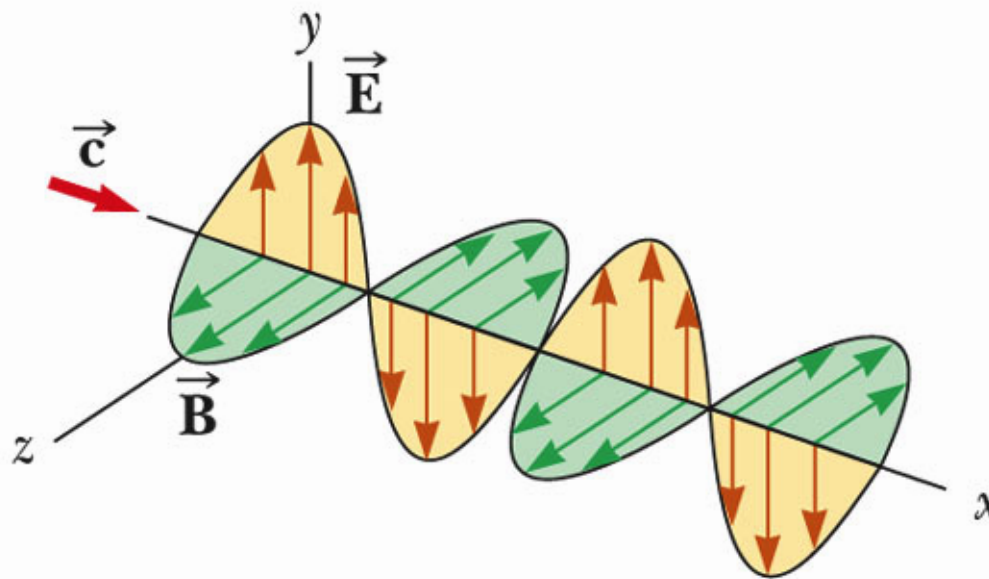


$$v = 2.99 \times 10^8 \text{ m/s}$$

Equals the speed of light!!!!

Electromagnetic Wave

- The same exercise can be done for a B, a magnetic wave in free space.



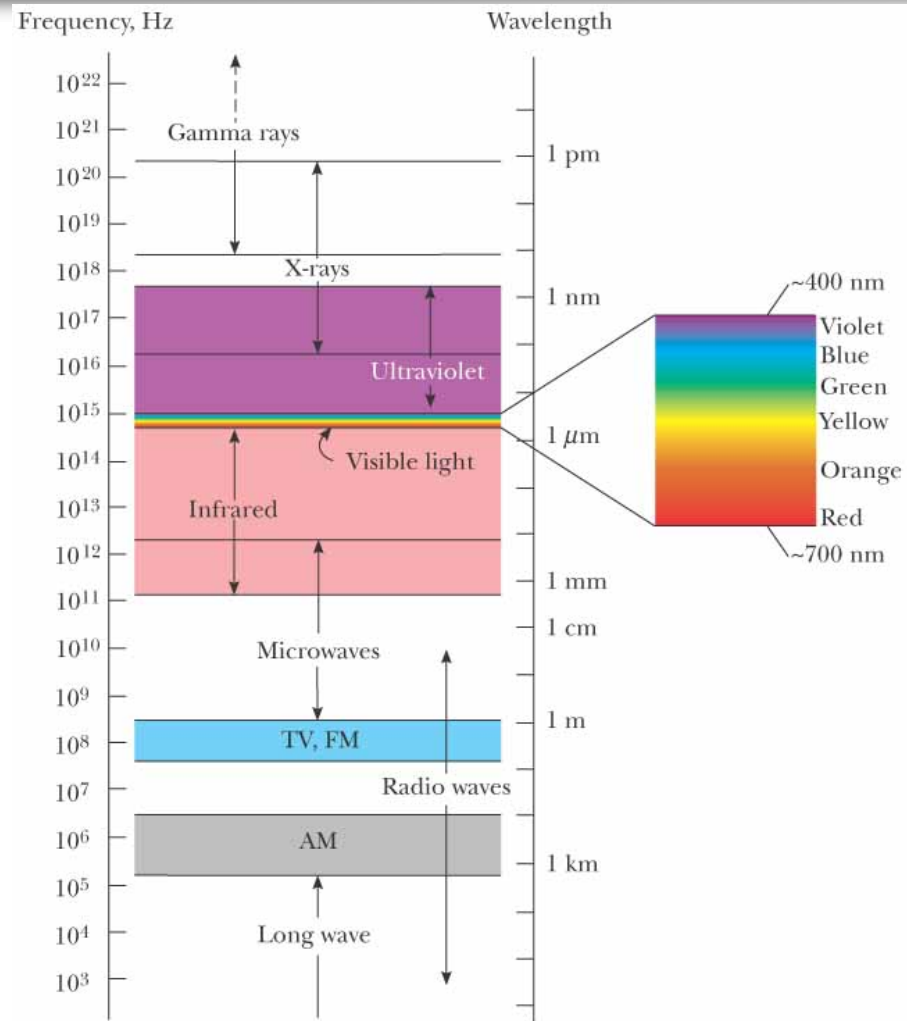
Where c = the speed of light. This was the first indication that light was an electromagnetic wave.

Electromagnetic wave

- Speed of light is constant for all EM waves.

$$\lambda f = c$$

- Can parameterize all light by it's frequency, f , or wavelength.



Example Radio Waves

UNITED STATES FREQUENCY ALLOCATIONS THE RADIO SPECTRUM

RADIO SERVICES COLOR LEGEND

Aeronautical Mobile	Maritime Mobile	Amateur Radio
Aeronautical Mobile Earth Station	Land Mobile	Aeronautical Earth Station
Aeronautical Mobile Earth Station	Land Mobile Earth Station	Navigation
Maritime	Maritime Mobile	Aeronautical Earth Station
Maritime Earth Station	Maritime Mobile Earth Station	Aeronautical Earth Station
Broadcasting	Maritime Radio	Aeronautical Earth Station
Broadcasting Earth Station	Maritime Radio Earth Station	Fixed
Broadcasting Earth Station	Maritime Radio Earth Station	Fixed Earth Station
Fixed	Fixed Earth Station	Fixed Radio Earth Station
Fixed Earth Station	Fixed Earth Station	Fixed Radio Earth Station

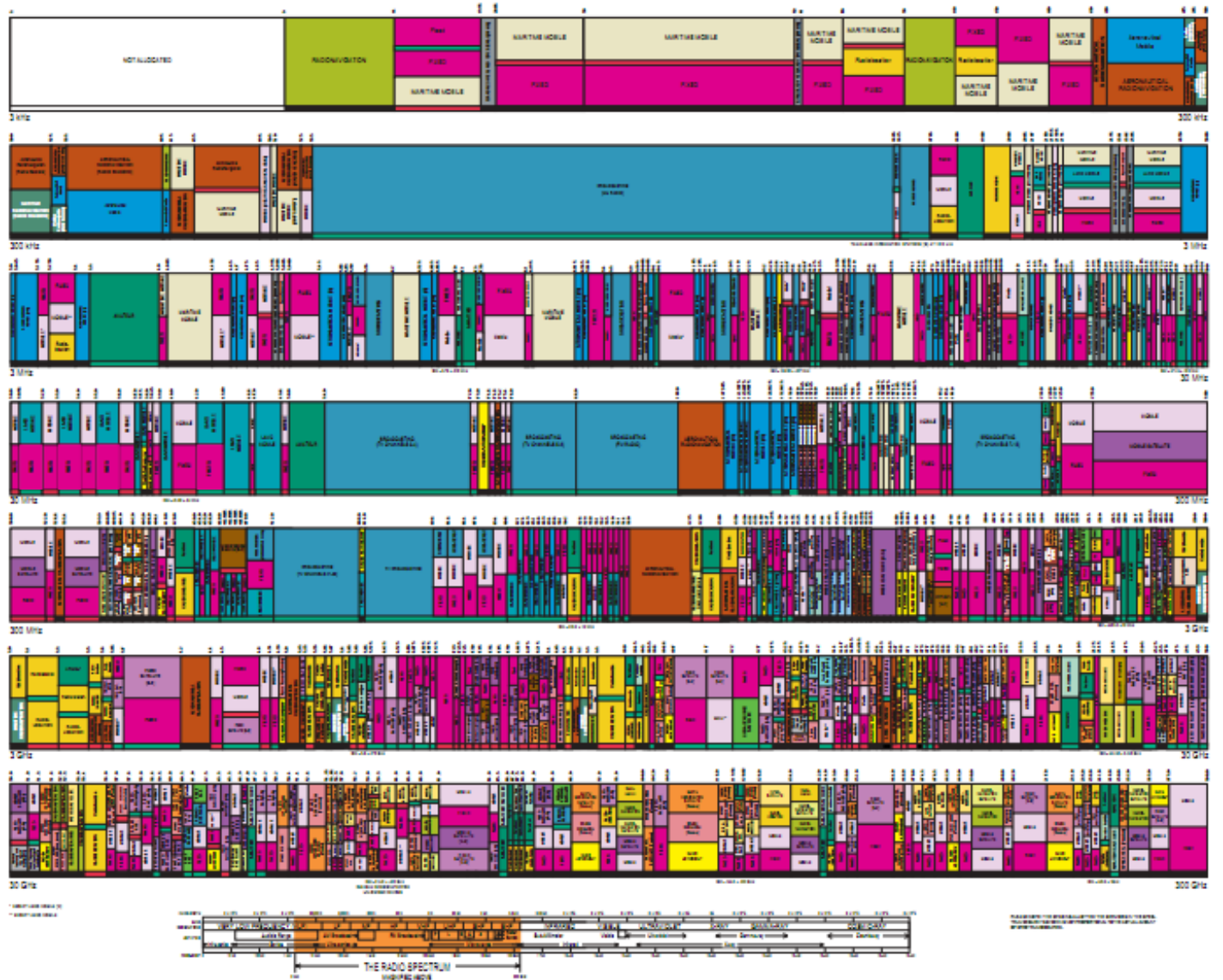
ACTIVITY CODE

Administrative	Communication
Non-Administrative	

ALLOCATION USAGE DESIGNATION

SERVICE	EXAMPLE	DESCRIPTION
Primary	FIXED	Capital Letters
Secondary	MOBILE	no Capital with lower case letters

This chart is a simplified representation of the radio frequency spectrum available to the United States. It is not intended to be used as a legal reference. For more information on the radio frequency spectrum, please refer to the Federal Communications Commission's website at www.fcc.gov.



Nerd Alert!

And God Said

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

and then there was light.

Interesting applications

- Light as an electromagnetic wave.
- Microwaves!

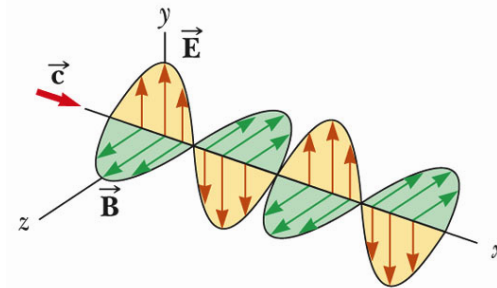
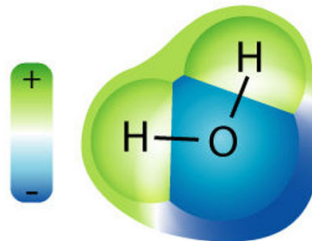
Use radio waves to heat the molecules in your food.

How?



Polarized molecules like water, and less polarized materials such as fats and sugars have an electric dipole moment.

$f=2.45 \text{ GHz}$



Faraday Cage



- No signal in an elevator?

For Next Time

- Work on Chapter 23 homework. Solutions are now posted.