PHYS 2D
PROBLEM SESSION

2012/5/31

8.1

- Particle in box with side $L_1 = L$, $L_2 = L_3 = 2L$. Find 6 lowest energy & degeneracy
- $E_1 = n_1^2 \pi^2 \hbar^2 / 2mL^2$, $E_2 = n_2^2 \pi^2 \hbar^2 / 2m(2L)^2$, $E_3 = n_3^2 \pi^2 \hbar^2 / 2m(2L)^2$
- $E = E_1 + E_2 + E_3$
- $E(1,1,1) = (1 + 1/4 + 1/4) = 1.5$ in unit of $\pi^2 \hbar^2 / 2mL^2$
- $E(1,1,2) = (1 + 1/4 + 1) = 2.25$, degeneracy $D = 2$
- $E(1,2,2) = (1 + 1 + 1) = 3$, $D = 1$
- $E(1,1,3) = (1 + 1/4 + 9/4) = 3.5$, $D = 2$
- $E(1,2,3) = (1 + 1 + 9/4) = 4.25$, $D = 2$
- $E(2,1,1) = (4 + 1/4 + 1/4) = 4.5$, $D = 1$

8.6

- Find stationary states for free particle in 3D

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x,y,z,t)+U(x,y,z)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,y,z,t)}{\partial t}$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

- U=0
- $\Psi(x,y,z,t)=\psi_1(x)\psi_2(y)\psi_3(z)\,\phi(t)$
- $\phi(t)=\exp(-i\omega t)$, $E=\hbar\omega$

$$-\frac{\hbar^2}{2\text{m}}\nabla^2\psi(x,y,z)=E\psi(x,y,z)$$

- $$\left(-\frac{\hbar^2}{2m}\frac{1}{\psi_1(x)}\frac{\partial^2\psi_1(x)}{\partial x^2}\right)+\left(-\frac{\hbar^2}{2m}\frac{1}{\psi_2(y)}\frac{\partial^2\psi_2(y)}{\partial y^2}\right)+\left(-\frac{\hbar^2}{2m}\frac{1}{\psi_3(z)}\frac{\partial^2\psi_3(z)}{\partial z^2}\right)=E=Const$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_1(x)}{\partial x^2}=E_1\psi_1(x) \qquad -\frac{\hbar^2}{2m}\frac{\partial^2\psi_2(y)}{\partial y^2}=E_2\psi_2(y) \qquad -\frac{\hbar^2}{2m}\frac{\partial^2\psi_3(z)}{\partial z^2}=E_3\psi_3(z)$$

8.6

- $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_1(x)}{\partial x^2} = E_1\psi_1(x)$$

- No boundary condition
  - $\psi_1(x)=A_1\sin(k_1x)+B_1\cos(k_1x)$, $E_1=\hbar^2k_1^2/2m$
  - $\psi_2(y)=A_2\sin(k_2y)+B_2\cos(k_2y)$, $E_2=\hbar^2k_2^2/2m$
  - $\psi_3(z)=A_3\sin(k_3z)+B_3\cos(k_3z)$, $E_3=\hbar^2k_3^2/2m$
- Sharp observables: E & k
  - Infinite uncertainty in t: stationary state
  - Infinite uncertainty in x: infinite plane wave

8.12

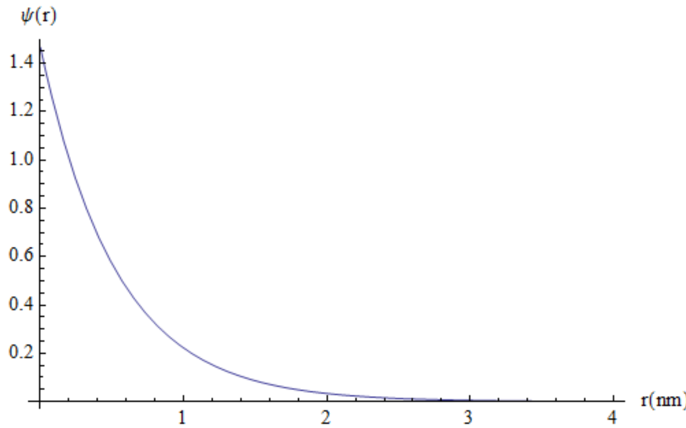
- Ground state of hydrogen
- Given $\psi(r) = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-\frac{r}{a_0}}$
- $1 = \int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi\, r^2 \sin\theta\, |\psi(r)|^2$

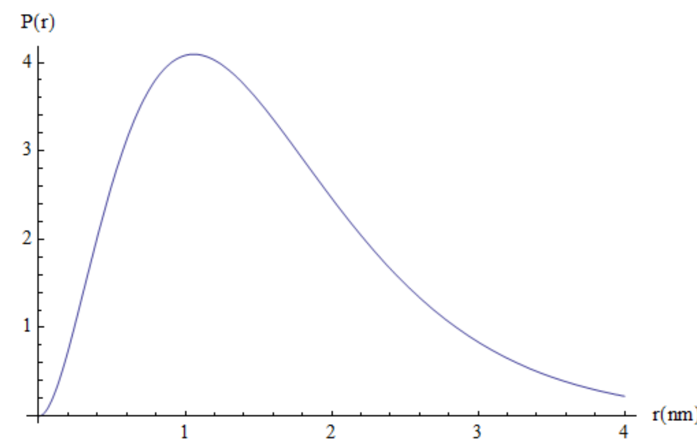
  $= \int_0^\infty dr\, r^2\, |\psi(r)|^2 \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$

  $= \int_0^\infty dr\, r^2\, |\psi(r)|^2 * 2 * 2\pi$

  $= \int_0^\infty dr\, 4\pi r^2\, |\psi(r)|^2$

  $= \int_0^\infty dr\, P(r)$
- $P(r) = 4\pi r^2 |\psi(r)|^2$
- P(r) has max when dP/dr=0
- $r_{Pmax} = a_0$





8.12

- Show that $\psi(r) = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-\frac{r}{a_0}}$ is normalized
  - $1 = \int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \, r^2 \sin\theta \, |\psi(r)|^2 = \int_0^\infty dr \, P(r)$

    $= \int_0^\infty dr \, 4\pi r^2 \left(\frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-\frac{r}{a_0}}\right)^2$

- Probability of finding electron in [a_0/2, 3a_0/2]
  - $\int_{a_0/2}^{3a_0/2} dr \, 4\pi r^2 \left(\frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-\frac{r}{a_0}}\right)^2 = 0.4965$

8.13

- Find l & m_l for He^+ ion in $n=3$ state, and its energy
  - l goes from 0 to $n-1=2$
  - m_l goes from $-l$ to l
    - $(n,l,m_l)=(3,0,0)$ or $(3,1,-1/0/1)$ or
      $(3,2,-2/-1/0/1/2)$
    - A total of 9 states
  - $E=-ke^2Z^2/2a_0n^2$
    - Depend only on n, not on l & m_l
    - 9 degenerate states
    - $Z=2$, $n=3$