PHYS 2D PROBLEM SESSION

2012/5/17

- Quiz 3 is graded
- Pick up quiz 3 today or next Tuesday
- Regrade request: 1 week

6.1

- Determine which wave functions are physical
- Conditions:
  - Continuous ~~(e)~~
  - Single-valued ~~(d)~~
  - Value is finite ~~(a)~~
- Valid ones: b & c

6.16

- Electron in a box, L=0.3 nm (x=0 to L), find the probability P_n the particle is found within [0, 0.1nm] for n=1 & n=100
- $\psi_n(x)=A\sin(n\pi x/L)$, $A=(2/L)^{1/2}$
- $$P_n = \int_0^{0.1} |\psi(x)|^2 dx = \frac{2}{L}\int_0^{0.1} \frac{1-\cos(2n\pi x/L)}{2} dx = \frac{1}{L}\left[x - \frac{L}{2n\pi}\sin(2n\pi x/L)\right]_0^{0.1}$$
  $$= 0.1/L - \left(\frac{\sin(0.2n\pi/L)}{2n\pi} - 0\right) = 1/3 - \frac{\sin(2n\pi/3)}{2n\pi}$$
- n=1, P_n=0.196; n=100, P_n=0.332; n=∞, P_n=1/3
- Correspondence principle: we get classical result with large n
- Classical: same probability everywhere, P=1/3

6.23

- Square well with V(x<0)=∞, V(0<x<L)=0, V(x>L)=U, particle has energy E, find ψ(x)
- Regions I, II, III for x<0, 0<x<L, x>L
- 1. Write out Schrodinger eq & ψ for each region
- Region I: ψ(x)=0
- Region II: Free space $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = E\psi(x)$

$$\psi(x) = A\cos(kx) + B\sin(kx), \quad E = \frac{\hbar^2 k^2}{2m}$$

- Region III: $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + U\psi(x) = E\psi(x), \quad E - U < 0$

$$\psi(x) = Ce^{k'x} + De^{-k'x}, \quad E - U = -\frac{\hbar^2 k'^2}{2m}$$

6.23

- 2. Apply boundary conditions
  - Wave function is continuous at x=0 & x=L
  - dψ/dx is continuous at x=L
  - Wave function is finite at x=∞
- At x=0: $\psi(0) = A\cos(0) + B\sin(0) = 0$, so A=0
- At x=∞: $\psi(\infty) = Ce^{k'\infty} + De^{-k'\infty} = Ce^{k'\infty}$ is finite, so C=0
- At x=L: $\psi(L) = B\sin(kL) = De^{-k'L}$

  $\psi'(L) = kB\cos(kL) = -k'De^{-k'L}$

- Dividing, we get tan(kL)=-k/k'

6.23

- $\tan(kL) = -k/k'$
- $E = \frac{\hbar^2 k^2}{2m}$, $E - U = -\frac{\hbar^2 k'^2}{2m}$
- $\cot^2(kL) = (U-E)/E$
- $\cot^2(kL) + 1 = 1/\sin^2(kL)$
- $(kL)^2/\sin^2(kL) > 1$, so $(kL)^2[(U-E)/E+1] > 1$
- $(kL)^2[(U-E)/E+1] = (kL)^2 U/E = 2mL^2U/\hbar^2 > 1$

- $2mL^2U/\hbar^2 > 1$ to have a solution
- No solution when $2mL^2U/\hbar^2 < 1$, or $U < \hbar^2/2mL^2$

6.24

- Show that $\psi(x) = Cxe^{-\alpha x^2}$ is a solution to Schrodinger's equation for quantum oscillator
- $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + \frac{1}{2}m\omega^2 x^2 \psi(x) = E\psi(x)$
- $\frac{d\psi}{dx} = -2\alpha x\psi(x) + Ce^{-\alpha x^2}$, $\frac{d^2\psi}{dx^2} = -2\alpha x\frac{d\psi}{dx} - 2\alpha\psi(x) - (2\alpha x)Ce^{-\alpha x^2} = (2\alpha x)^2\psi(x) - 6\alpha\psi(x)$
- Equate $x^2\psi(x)$ terms and $\psi(x)$ terms
- $\alpha = \frac{m\omega}{2\hbar}$, $E = \frac{3\alpha\hbar^2}{m} = \frac{3}{2}\hbar\omega$
- ψ(x) is the n=1 state
- Normalization: $1 = \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 2C^2\int_0^{\infty} x^2 e^{-2\alpha x^2} dx$
- We can find C in terms of $\alpha = \frac{m\omega}{2\hbar}$ ($2C^2\int_0^{\infty} x^2 e^{-2\alpha x^2} dx = 2C^2 \frac{1}{4\alpha}\sqrt{\frac{\pi}{2\alpha}}$)

6.32

- Find $<x>$ & $<x^2>$ & Δx for ground state $\psi(x) = Ce^{-\alpha x^2}$ of quantum oscillator
- $\alpha = \frac{m\omega}{2\hbar}$, $C = \left(\frac{1}{\pi}\frac{m\omega}{\hbar}\right)^{1/4}$
- $< x >= \int_{-\infty}^{\infty} x \,|\psi(x)|^2 \, dx = \int_{-\infty}^{\infty} x \, C^2 \, e^{-2\alpha x^2} \, dx = 0$

$$< x^2 >= \int_{-\infty}^{\infty} x^2 \,|\psi(x)|^2 \, dx = \int_{-\infty}^{\infty} x^2 \, C^2 \, e^{-2\alpha x^2} \, dx = 2\, C^2 \int_{0}^{\infty} x^2 \, e^{-2\alpha x^2} \, dx = 2\, C^2 \, \frac{1}{4\alpha} \sqrt{\frac{\pi}{2\alpha}}$$

$(\Delta x)^2 = < x^2 > - < x >^2$, $\Delta x = < x^2 >^{1/2}$