PHYS 2D
PROBLEM SESSION

2012/4/12

1.14

Problem: Rod with length L_0 tilted by θ_0 in S', L and θ in S? S' is moving at v relative to S.

- Only x coordinates are transformed. y is unchanged

S' frame first:

- $x_1'=0$, $x_2'=L_0\cos\theta_0$, $\Delta x'=L_0\cos\theta_0$

S frame:

- Length measurement: $\Delta t=0$
- $\Delta x'=\gamma(\Delta x-v\Delta t/c^2)=\gamma\Delta x$
- $\Delta x=L_0\cos\theta_0/\gamma$

1.14

- Length of rod in S:

$L^2=(\Delta x)^2+(\Delta y)^2=(L_0\sin\theta_0)^2+(L_0\cos\theta_0/\gamma)^2$

$=(L_0\sin\theta_0)^2+(1-v^2/c^2)\,(L_0\cos\theta_0)^2$

- Angle of rod in S:

$\tan\theta=\Delta y/\Delta x=L_0\sin\theta_0/(L_0\cos\theta_0/\gamma)=\gamma\tan\theta_0$

1.16

- Red light (650nm) appear green (550nm). How fast are you going toward light source?
- Doppler shift: $f' = [(c+v)/(c-v)]^{1/2} f$
- $c = f\lambda = f'\lambda'$
- $\lambda' = [(c-v)/(c+v)]^{1/2} \lambda$

- If moving away from light source, v becomes -v

$$f' = [(c-v)/(c+v)]^{1/2} f$$

1.19

Problem: 2 spaceships A & B moving at same speed v_0 toward Earth with relative speed 0.7c, find v_0

- A ⟹ E ⟸ B
- Transformation of velocity $u'_X = \dfrac{u_X - v}{1 - u_X v/c^2}$
- Transform velocity of B from Earth frame into rest frame of A
- u_x: velocity of B in Earth frame=$-v_0$
- v: velocity of new frame (rest frame of A) relative to old frame (Earth frame)=v_0
- u_x':velocity of B in A frame=-0.7c

1.35

Problem: Ages of twins S & G traveling to a planet 20 ly away at 0.95c and 0.75c

- Intervals $(\Delta s)^2=(\Delta s')^2$

Twin S first

- Earth frame: $(\Delta s)^2=(c\Delta t)^2-(\Delta x)^2$, $\Delta t=20ly/0.95c$, $\Delta x=20ly$, $(\Delta s)^2=43.2\ ly^2$
- S frame: $\Delta x'=0$, $(c\Delta t')^2-0=(\Delta s')^2=(\Delta s)^2$
- $\Delta t'=6.57ly/c=6.57y$
- $\Delta t=20ly/0.95c=21.05y$

1.35

Twin G

- Earth frame:$(\Delta s)^2=(c\Delta t)^2-(\Delta x)^2$, $\Delta t=20ly/0.75c$, $\Delta x=20ly$, $(\Delta s)^2=311.11\ ly^2$
- G frame: $\Delta x''=0$, $(c\Delta t'')^2-0=(\Delta s'')^2=(\Delta s)^2$
- $\Delta t''=17.64ly/c=17.64y$
- $\Delta t=20ly/0.75c=26.67y$

1.35

- In Earth frame: both leave at $t=0$
- S arrives at $t=21.05$y, having aged $\Delta t'=6.57$y
- G arrives at $t=26.67$y, having aged $\Delta t''=17.64$y
- S spends an additional 26.67-21.05=5.62y in Earth frame
- When G arrives, S has aged a total of 6.57+5.62=12.19y

- Difference in age=17.64-12.19=5.45y
- G is older

1.37

Lorentz transform time coordinates

- Event A: (x,t)=(50,0)
- Event B: (x,t)=(150,0)

What's Δt' in S' frame moving at 0.8c?

- $\Delta t = t_B - t_A = 0$, $\Delta x = x_B - x_A = 100$
- $\Delta t' = t_B' - t_A' = \gamma(\Delta t - \Delta x\ v/c^2) = 1.67(0 - 100m * 0.8/c)$

$= -4.44 * 10^{-7} s$

- $t_A' > t_B'$, which means A happened later in S'

1.23

- Lightning strikes simultaneously in S, t’ & s’ in S’?
- Lorentz transform time & coordinate