Electricity – Designing a Voltmeter
χ^2 testing
Review

Lecture # 8

Physics 2BL

Spring 2012

Announcements

- Next Monday Holiday
- Office hours
  - Tera – Monday 12 – 2 pm
  - Me – Tues, Thur 2 -3 pm
- Pick up 4th lab in lab room by noon on Monday before final
- CAPE evaluations:
  - Important for fine tuning of the course
  - Making changes
  - Giving feedback

Schedule

Meeting	Experiment
1 (Apr 2-6)	None (start Taylor)
2 (Apr 9-13)	1
3 (Apr 16-20)	1
4 (Apr 23-27)	2
5 (Apr 30-May4)	2
6 (May 7-11)	3
7 (May 14-18)	3
8 (May 21-25)	4
9 (May 28-June 1)	4
10 (June 4-8)	FINAL

The Four Experiments

- **Determine the average density of the earth**
  – Measure simple things like lengths and times
  – Learn to estimate and propagate errors
- **Non-Destructive measurements of densities, structure**
  – Measure moments of inertia
  – Use repeated measurements to reduce random errors
- **Test model for damping; Construct and tune a shock absorber**
  – Damping model based on simple assumption
  – Adjust performance of a mechanical system
- **Measure coulomb force and calibrate a voltmeter.**
  – Examine electrical forces, parallel plate capacitor, torsional pendulum.
  – Balancing forces.
  – Reduce systematic errors in a precise measurement.

Purpose

- Design a means to measure electrical voltage through force exerted on charged object

Method

- Use Torsional pendulum
- Balance forces, balance torques

Basic Equations

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

Force between point charges
Coulomb's low

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{\text{F}}{\text{m}}$$

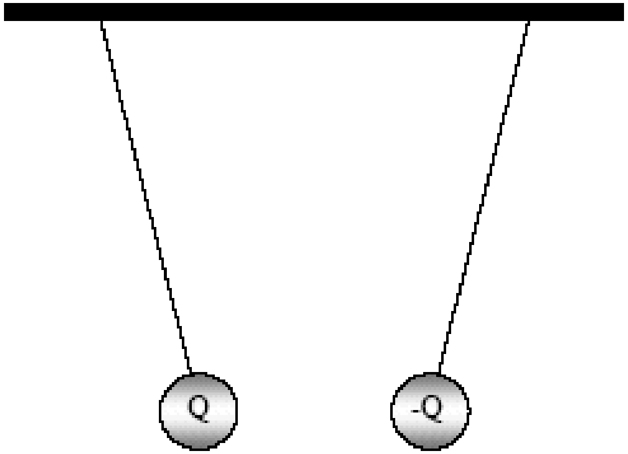
Permittivity constant

$$E = \frac{F}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

Electric field from a point charge Q_1
Coulomb force acting on a unit charge

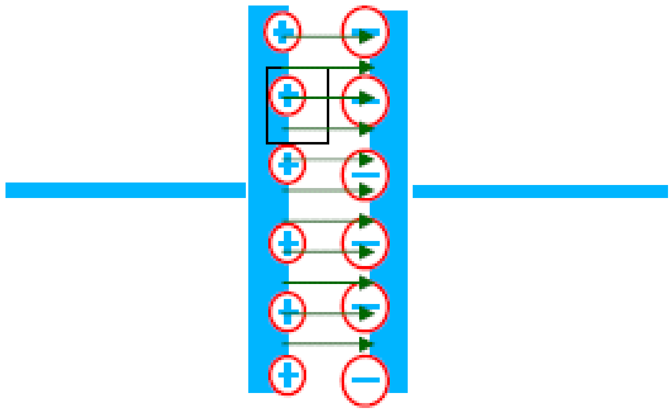
$$V = \frac{1}{Q_2}\int_r^\infty F dr = \frac{Q_1}{4\pi\varepsilon_0 r}$$

Voltage - potential energy per unit charge



Experiment #4: Parallel Plate Capacitor

We suggest the use of a <u>parallel plate capacitor</u> rather than charged spheres



$$E = \frac{Q}{A\varepsilon_0}$$ from Gauss's Law

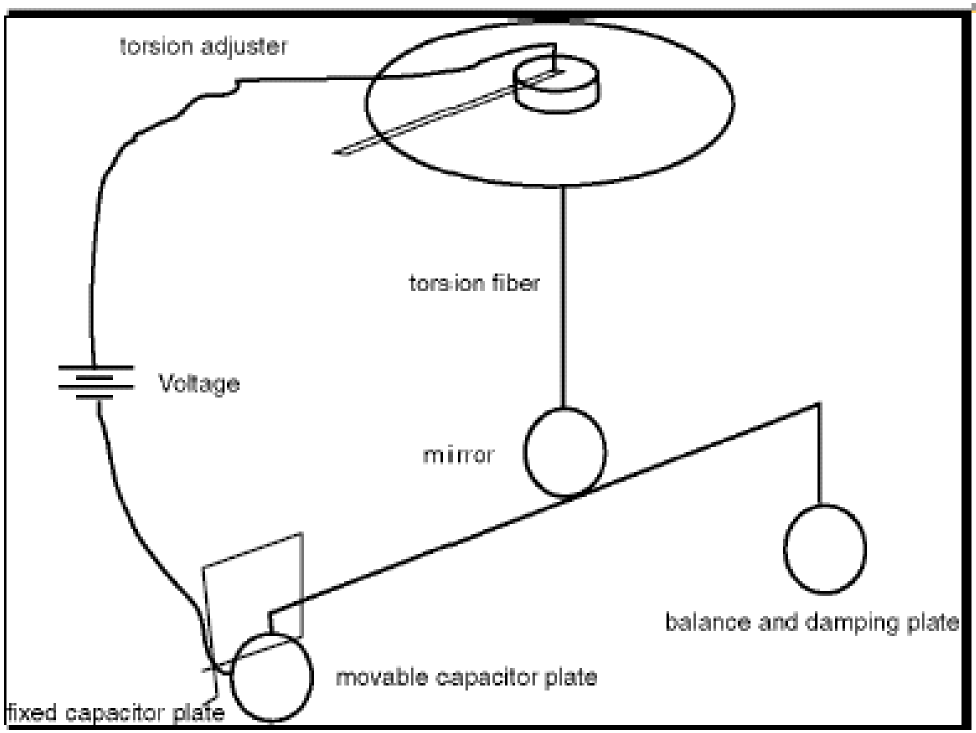
$$V = Ed = \frac{Qd}{A\varepsilon_0}$$ voltage difference

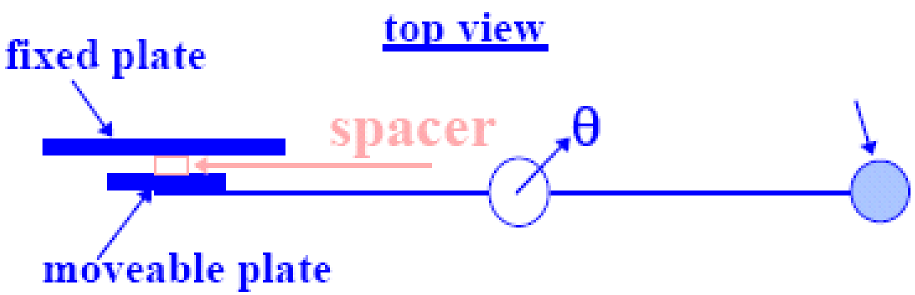
$$F = \frac{1}{2}EQ = \frac{1}{2}\frac{Q^2}{A\varepsilon_0} = \frac{1}{2}\frac{A\varepsilon_0}{d^2}V^2$$ the force

$$F = \frac{1}{2}\frac{\left(A = 3\ \text{cm}^2\right)\left(\varepsilon_0 = 8.8\times10^{-12}\ \frac{\text{F}}{\text{m}}\right)}{\left(d = 0.1\ \text{cm}\right)^2}\left(V = 1000\ \text{V}\right)^2 = 1.2\times10^{-3}\ \text{N}$$

The weight of 0.1 g.

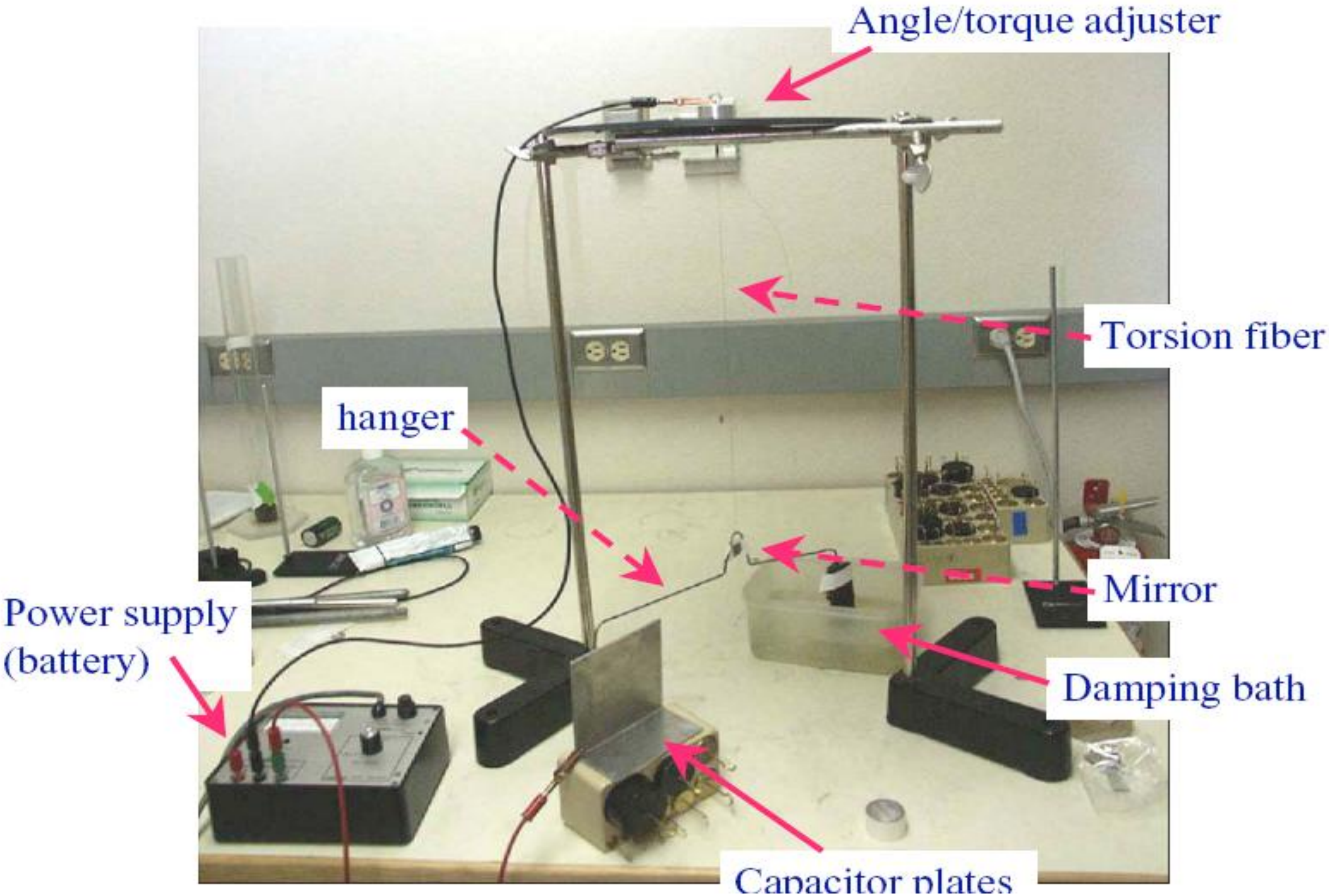
Calibrate Voltmeter



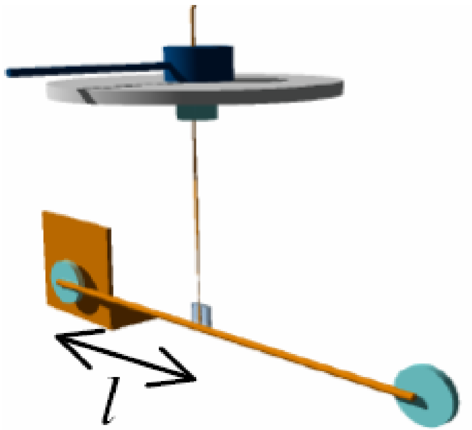


- Set up the apparatus.
- Keep table dry.
- Make the plates parallel for spacer in contact.
- Measure the spacer.
- Measure κ.
- Find Voltage that just causes plates to move apart.
- Try calibration at about 1000 Volts.
- Now get several measurements at lower voltage.
- Water must be stable.
- Move slowly.
- Protect your apparatus from air currents.
- Estimate errors

Voltmeter Appartus



Measure κ using Torsional Pendulum



$$F = \kappa\theta / l$$

l - Distance from the suspension to the disk is measured with a ruler

θ - Deflection angle is measured with a protractor

How do we measure the torsion constant κ?

Torsional oscillations $T = 2\pi\sqrt{\frac{I}{\kappa}}$

$$\kappa = \left(\frac{2\pi}{T}\right)^2 I$$

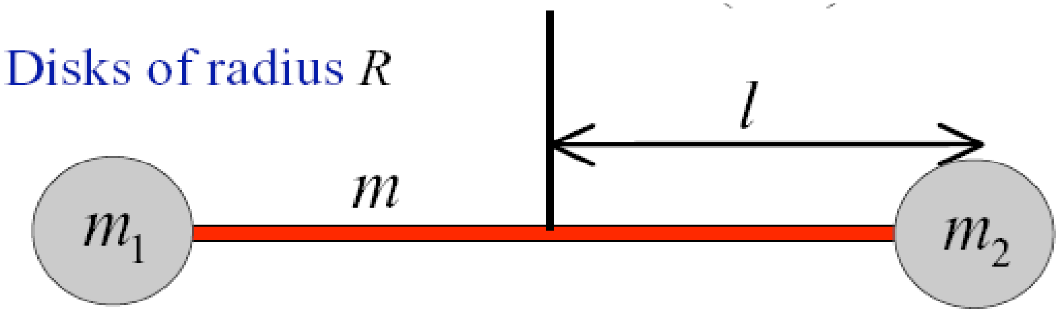
I - Moment of inertia

.

$$\varepsilon_\kappa = \sqrt{(\varepsilon_I)^2 + (2\varepsilon_T)^2}$$

$$\sigma_\kappa = \kappa * (\varepsilon_\kappa)$$

Moment of Inertia



You want to weigh the support beam and disks separately

$$I = \frac{1}{3} m l^2 + (m_1 + m_2) l^2 + (m_1 + m_2) \frac{R^2}{4}$$

rod | parallel axis theorem | disks - CM

Error Propagation…

$$\sigma_I = \sqrt{[(\delta I/\delta m)\sigma_m]^2 + [(\delta I/\delta m_1)\sigma_{m1}]^2 + [(\delta I/\delta m_2)\sigma_{m2}]^2 + [(\delta I/\delta l)\sigma_l]^2 + [(\delta I/\delta R)\sigma_r]^2}$$

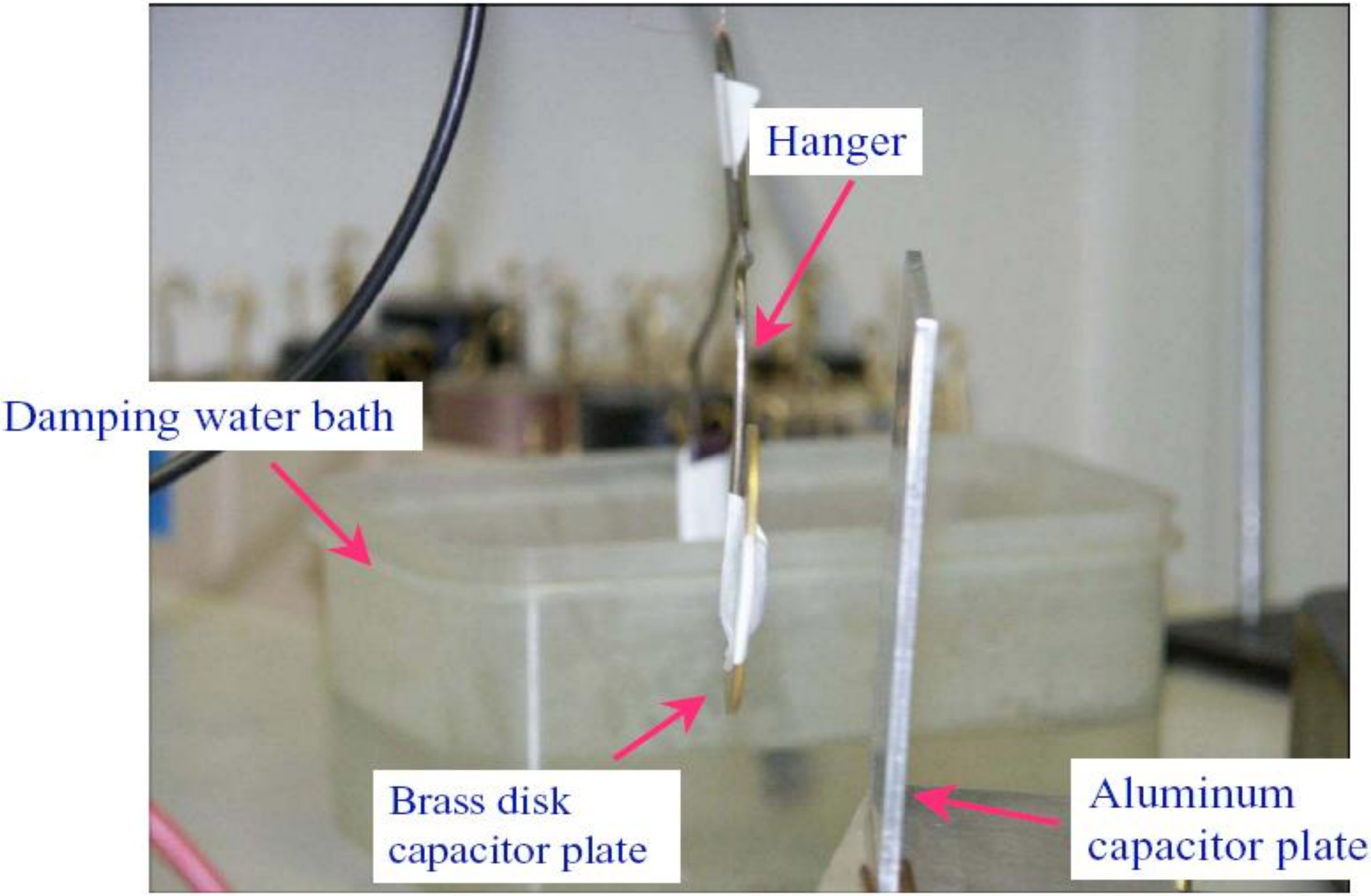
Error in Moment of Inertia

$$I = m(l_1 + l_2)^2/12 + m(l_1 - l_2)^2/4$$
$$m_1 l_1^2 + m_2 l_2^2 + (m_1+m_2)R^2/4$$

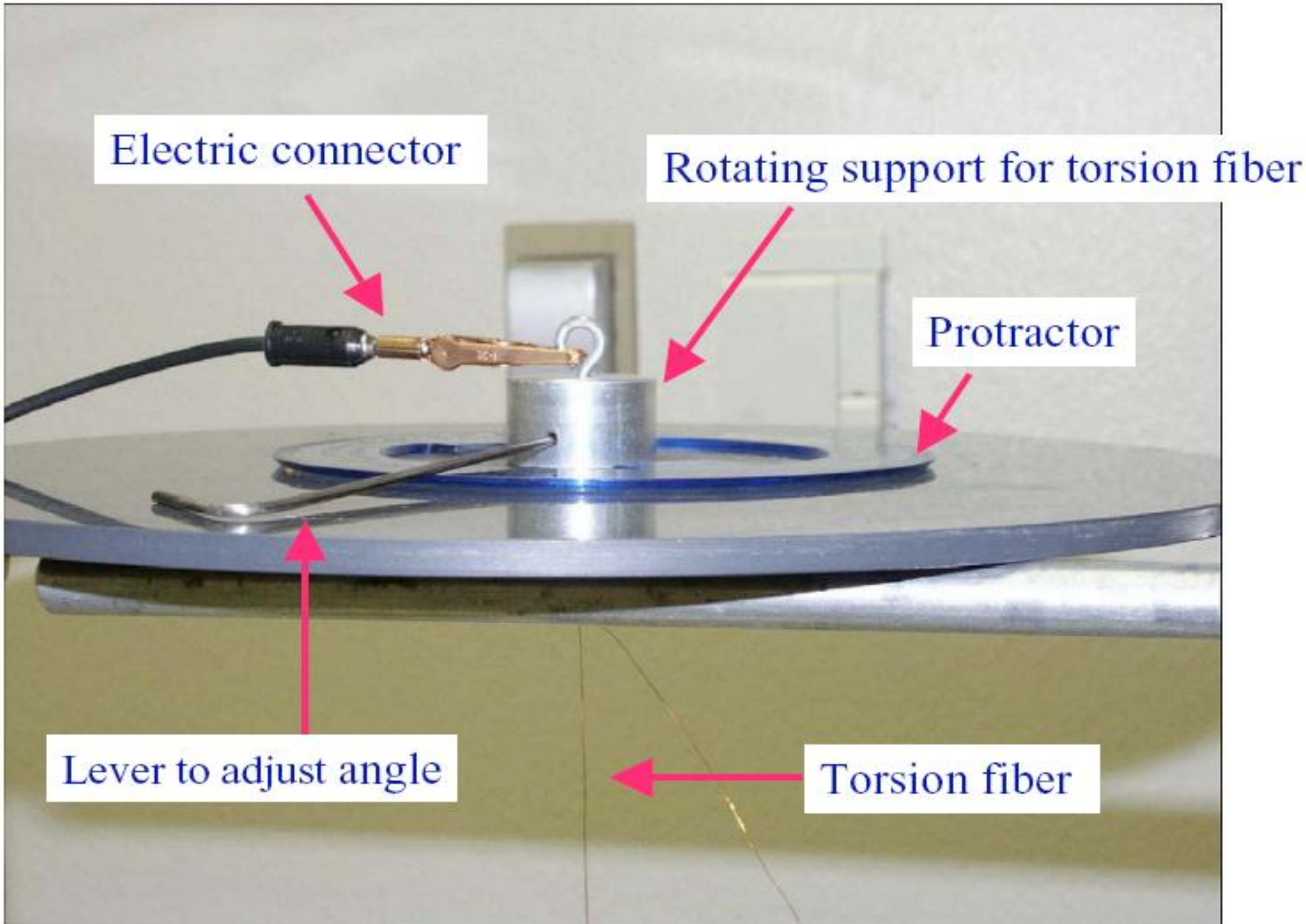
$$\sigma_I = \sqrt{[(\delta I/\delta m)\sigma_m]^2 + [(\delta I/\delta m_1)\sigma_{m1}]^2 + [(\delta I/\delta m_2)\sigma_{m2}]^2 [(\delta I/\delta l_1)\sigma_{l1}]^2 + [(\delta I/\delta l_2)\sigma_{l2}]^2 + [(\delta I/\delta R)\sigma_r]^2}$$

$$\delta I/\delta m = (l_1 + l_2)^2/12 + (l_1 - l_2)^2/4$$
$$\delta I/\delta m_1 = l_1^2 + R^2/4 \sim \delta I/\delta m_2$$
$$\delta I/\delta l_1 = m(l_1 + l_2)/6 + m(l_1 - l_2)/2 + 2m_1 l_1 \sim \delta I/\delta l_2$$
$$\delta I/\delta R = 1/2(m_1+m_2)R$$

Capacitor – Electrical Force



Torsion



Electric connector
Rotating support for torsion fiber
Protractor
Lever to adjust angle
Torsion fiber

Equilibrium Positions

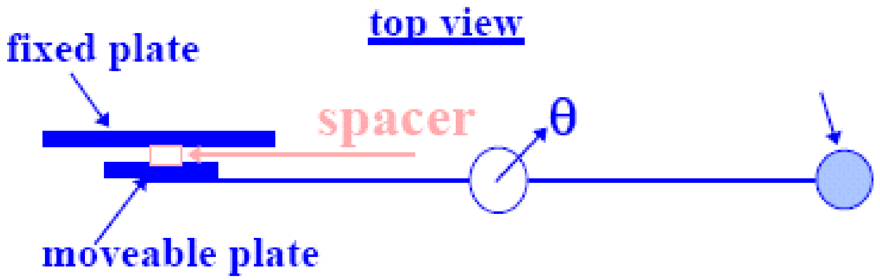
$$F = \frac{1}{2}\frac{A\varepsilon_0}{d^2}V^2$$ electrostatic attraction

Fl torque resulting from the electrostatic force

$k\theta$ torque resulting from the fiber

hold separation between the capacitor plates fixed as the voltage between them is increased by twisting the top end of the fiber

$$V = d\sqrt{\frac{2k\theta}{lA\varepsilon_0}}$$



Clicker Question # 13

Do you expect a plot of V versus θ to be linear?

$$V = d\sqrt{2\kappa\theta/lA\varepsilon_0}$$

(A)Yes
(B)No

Error Propagation

$$V = d\sqrt{2\kappa\theta / l A \varepsilon_0}$$

$$\varepsilon_V = \sqrt{(\varepsilon_d)^2 + (\varepsilon_\kappa/2)^2 + (\varepsilon_\theta/2)^2 + (\varepsilon_A/2)^2 + (\varepsilon_l/2)^2}$$

$$\sigma_V = V * (\varepsilon_V)$$

Analysis

Make a graph of your data where:

- x-axis is the voltage read from the power supply (600-1000V)
- y-axis is the calculated voltage from the torsional pendulum

Fit to straight line

Calculate χ^2

Discuss goodness of fit

Calculate probability of result.

χ^2 Testing
(Taylor Chapter 12)

- You take N measurements of some parameter x which you believe should be distributed in a certain way (e.g., based on some hypothesis).
- You divide them into n bins (k=1,2,...,n) and count the number of observations that fall into each bin (O_k).
- You also calculate the expected number of measurements (E_k), in the same bins, based on some hypothesis.
- Calculate:

$$\chi^2 = \sum_{i=1}^{n} \frac{\left(O_k - E_k\right)^2}{E_k}$$

- If $\chi 2 < n$, then the agreement between the observed and expected distributions is acceptable.
- If $\chi 2 >> n$, there is significant disagreement.

Degrees of Freedom

- Number of degrees of freedom, d = number of observations, O_k, minus the number of parameters computed from the data and used in the calculation.
- d=n-c,
  - Where c is the number of parameters that were calculated in order to compute the expected numbers, E_k.
  - It can be shown that the expected average value of $\chi 2$ is d.
- Therefore, we define "reduced chi-squared":

$$\tilde{\chi}^2 = \frac{\chi^2}{d}$$

- If the reduced chi-squared is <1, there is no reason to doubt the expected distribution.

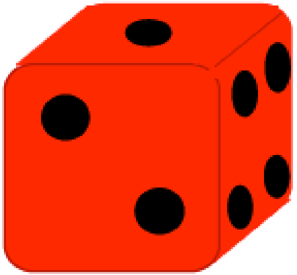
Fitting Summary

- You have a set of measurements and a hypothesis that relates them.
- The hypothesis has some unknown parameters that you want to determine.
- You "fit" for the parameters by maximizing the odds of all measurements being consistent with your hypothesis.

- Evaluate your fit based on the goodness of fit.

Example - Dice

Die is tossed 600 times



Expectation: each face has same likelihood of showing up

Verification of expectation by computing the χ^2

v	1	2	3	4	5	6
obs	91	137	111	87	80	94
exp	100	100	100	100	100	100
Δ^2	81	1369	121	169	400	36
χ^2_i	0.81	13.7	1.21	1.69	4.0	0.36

Total χ^2 21.76

n_{dof} 5

reduced χ^2 4.35

This term is the squared difference between observation and expectation.

In computation of χ^2 the Δ^2 term is divided by expectation. σ is square root of expectation ($E_y = \sigma_y^2$)

Application of χ^2 – Use of Table D

Just calculated:

Total χ^2	21.77
n_{dof}	5
Reduced $\tilde{\chi}^2_{\circ}$	4.35

	$\tilde{\chi}_o^2$												
d	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2	3	4	5	6
1	100	62	48	39	32	26	22	19	16	8	5	3	1
2	100	78	61	47	37	29	22	17	14	5	2	0.7	0.2
3	100	86	68	52	39	29	21	15	11	3	0.7	0.2	—
5	100	94	78	59	42	28	19	12	8	1	0.1	—	—
10	100	99	89	68	44	25	13	6	3	0.1	—	—	—
15	100	100	94	73	45	23	10	4	1	—	—	—	—

Prob that $\tilde{\chi}_0^2 > 4$ chance

Die is loaded at 99.9% Confidence Level

χ^2 Test for a Fit

- We have used χ^2 minimization to fit data.
- We can also use the value of χ^2 to determine if the data fit the hypothesis.

$$\chi^2 = \sum_{i=1}^{n} \frac{\left(y_i - f(x_i)\right)^2}{\sigma_y^2}$$

- On average, the χ^2 value is about one per degree of freedom.

$$\left\langle \chi^2 \right\rangle = n_{\text{d.o.f.}}$$

- The number of degrees of freedom is the number of measurements minus the number of fit parameters.

$$n_{\text{d.o.f.}} = n_{\text{data}} - n_{\text{parameters}}$$

- We will use the χ^2 per degree of freedom to compute a probability that the data are consistent with the hypothesis. (table D)

$$\tilde{\chi}^2 = \frac{\chi^2}{n_{\text{d.o.f.}}}$$

- This probability of χ^2 is like the confidence level.

If *Prob*($\tilde{\chi}^2 > \tilde{\chi}^2_o$) is less than 5% - disagreement “significant”

If *Prob*($\tilde{\chi}^2 > \tilde{\chi}^2_o$) is less than 1% - disagreement “highly significant”

Review

Determination of errors from measurements

Two types – random (statistical) and systematic

Random errors – intrinsic uncertainty (limitations)

Can be determined from multiple measurements

Mean and standard deviation, standard deviation of the mean

Propagation or uncertainties through formulas

Simple formula for adding two terms (a=b+c)

Simple formula for multiplying two terms (a=b*c)

General formula for g(x,y,z)

Overview

Will be given basic physics equations

Need to know how to use them (labs)

Understand significant figures and how to quotes values properly

Need to know basic error propagation formulas

Need to know Gaussian distributions

mean, standard deviation, standard deviation of the mean

Overview

Know how to determine t-values
extract probability information from those values

Understand rejection of data – Chauvenet's principle

Know how to calculate weighted averages

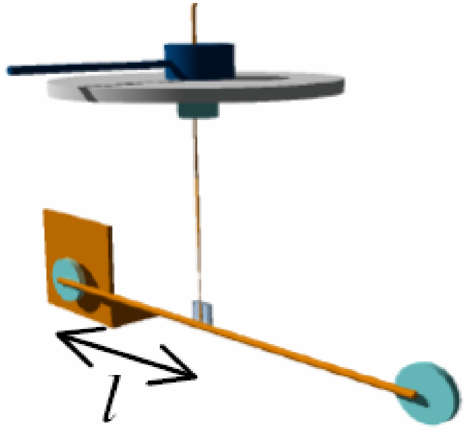
Let's do an example

Example Exam Question

You want to determine the torsional constant for the wire you used in the last experiment. You do this by measuring the period of oscillation. You make 5 measurements of 15.1 s, 13.2s, 14.4 s, 15.4 s and 14.6 s. What is the best value for the torsional constant κ with the proper number of significant figures and uncertainty. You also determined the moment of inertia to be (2420 ± 120) g cm^2.

Example Solution

(1) Draw diagram



(2) Identify given parameters

Given T values and I

(3) Write the equation(s) necessary to calculate κ

Torsional oscillations $T = 2\pi\sqrt{\frac{I}{\kappa}}$

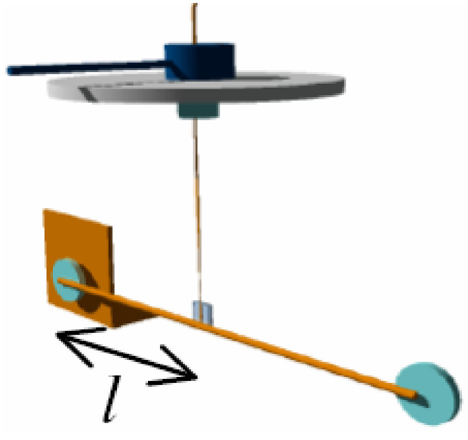
$$\kappa = \left(\frac{2\pi}{T}\right)^2 I$$

I - Moment of inertia

(4) Calculate best value for T

$$T_{best} = T_{ave} = 14.54 \text{ s}$$

Example Solution



(5) Calculate uncertainty in T

$$\sigma_T = 0.847 \text{ s}$$

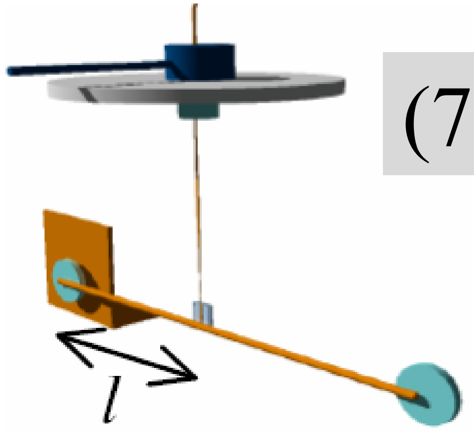
$$\sigma_{\overline{T}} = 0.424 \text{ s} = 0.4 \text{ s}$$

$$T_{best} = (14.5 \pm 0.4) \text{ s}$$

(6) Calculate κ from best values

$$\kappa = 4\pi^2 I/T^2 = 454.4 \text{ g cm}^2/\text{s}^2$$

Example Solution



(7) Calculate uncertainty for κ

$$\varepsilon_\kappa = \sqrt{(\varepsilon_I)^2 + (2\varepsilon_T)^2}$$

$$\varepsilon_\kappa = \sqrt{(120/2420)^2 + (2*0.4/14.5)^2}$$

$$\varepsilon_\kappa = \sqrt{(.0496)^2 + (.0552)^2}$$

Most significant source of uncertainty?

$$\varepsilon_\kappa = .07$$

$$\sigma_\kappa = \kappa * (\varepsilon_\kappa) = 30 \text{ g cm}^2/\text{s}^2$$

Thus, $\kappa = (450 \pm 30) \text{ g cm}^2/\text{s}^2$

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Reminder

- Prepare for Experiment 4
- Last lecture - No more lectures
- Remember to prepare for the final which will be like an extended quiz, designed for ~50 minutes and have a higher weighting then a single lab quiz