Weighted Averages
Simple Harmonic Motion and Damping

Lecture # 6

Physics 2BL

Spring 2012

Outline

- Weighted averages (Chapter 7, Taylor)
- Experiment 3 intro
- Physics of damping and SHM
- Experiment 3 objectives
- Register clickers
  iclicker.com/registration

Schedule

Meeting	Experiment
1 (Apr 2-6)	None (start Taylor)
2 (Apr 9-13)	1
3 (Apr 16-20)	1
4 (Apr 23-27)	2
5 (Apr 30-May4)	2
6 (May 7-11)	3
7 (May 14-18)	3
8 (May 21-25)	4
9 (May 28-June 1)	4
10 (June 4-8)	FINAL

Weighted averages (Chapter 7)

We can use maximum Likelihood (χ^2) to average measurements with different errors.

$$\chi^2 = \sum_{i=1}^{n}\left(\frac{x_i - X}{\sigma_i}\right)^2$$

We derived the result that: $$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Using error propagation, we can determine the error on the weighted mean:

$$\sigma_{\bar{x}} = \frac{1}{\sqrt{\sum_{i=1}^{n} w_i}}$$

What does this give in the limit where all errors are equal?

$$\frac{\partial \chi^2}{\partial X} = 0 = -2\sum_{i=1}^{n}\frac{x_i - X}{\sigma_i^2}$$

$$\sum_{i=1}^{n}\frac{x_i}{\sigma_i^2} - X\sum_{i=1}^{n}\frac{1}{\sigma_i^2} = 0$$

$$w_i \equiv \frac{1}{\sigma_i^2}$$

$$\sum_{i=1}^{n} w_i x_i = X\sum_{i=1}^{n} w_i$$

$$X = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Weighted averages

- $X = x = \dfrac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$ where $w_i = \dfrac{1}{\sigma_i^2}$

- $\sigma_{wav} = \dfrac{1}{\sqrt{\sum_{i=1}^{n} w_i}}$

Example: Weighted Average

Suppose 2 students measure the radius of Neptune.

- Student A gets r=80 Mm with an error of 10 Mm and
- Student B gets r=60 Mm with an error of 3 Mm

What is the best estimate of the true radius?

$$\bar{r} = \frac{w_A r_A + w_B r_B}{w_A + w_B} = \frac{\frac{1}{100}80 + \frac{1}{9}60}{\frac{1}{100} + \frac{1}{9}} = 61.65 \text{ Mm}$$

What does this tell us about the importance of error estimates?

Clicker Question 9

Two measurements of the speed of sound give the answers:

$u_A = (332 \pm 1)$ m/s and $u_B = (339 \pm 3)$ m/s.

What is the random chance of getting two results that show this difference?

(A) 2 %

(B) 3 %

(C) 4%

(D) 8 %

(E) 40%

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80

a) To check if the two measurements are consistent, we compute:

$$q = u_A - u_B = 339\text{-}332 = 7 \text{ m/s}$$

and: $$\sigma_q = \sqrt{\sigma_{uA}^2 + \sigma_{uB}^2} = 3.16 \text{ m/s}$$

so that: $$t = \frac{q}{\sigma_q} = \frac{339 - 332}{3.16} = 2.21$$

From Table A we get that 2.21 sigma corresponds to: 97.21%

Therefore the probability to get a worse result is 1-97% ~3%.

Clicker Question 10

Two measurements of the speed of sound give the answers:
$u_A = (332 \pm 1)$ m/s and $u_B = (339 \pm 3)$ m/s.
What is the best estimate (weighted mean)?

(A) 336.5 ± 2 m/s
(B) 336 ± 2 m/s
(C) 336.5 ± 0.9 m/s
(D) 332.7 ± 0.9 m/s
(E) 333 ± 2 m/s

b) Best estimate is the weighted mean:

$$\bar{u} = \frac{w_A u_A + w_B u_B}{w_A + w_B} = \frac{\frac{1}{1}332 + \frac{1}{9}339}{\frac{1}{1} + \frac{1}{9}} = 332.7 \text{ m/s}$$

$$\sigma_{\bar{u}} = \frac{1}{\sqrt{1/w_A + 1/w_B}} = \frac{1}{\sqrt{1/1 + 1/9}} = 0.9 m/s$$

The Four Experiments

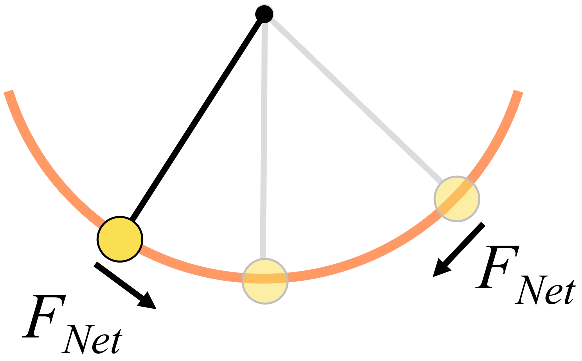
- **Determine the average density of the earth**
  – Measure simple things like lengths and times
  – Learn to estimate and propagate errors
- **Non-Destructive measurements of densities, structure**
  – Measure moments of inertia
  – Use repeated measurements to reduce random errors
- **Test model for damping; Construct and tune a shock absorber**
  – Damping model based on simple assumption
  – Adjust performance of a mechanical system
  – Demonstrate critical damping of your shock absorber
  – Does model work? Under what conditions? If needed, what more needs to be considered?
- **Measure coulomb force and calibrate a voltmeter.**
  – Reduce systematic errors in a precise measurement.

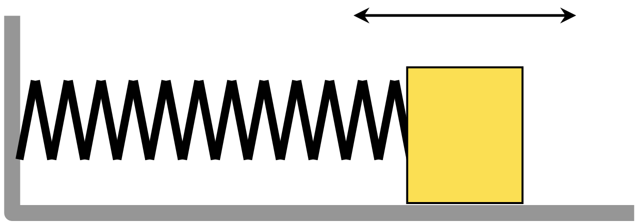
Experiment 3

- Goals: Test model for damping
- Model of a shock absorber in car
- Procedure: develop and demonstrate critically damped system
- check out setup, take data, do data make sense?
- Write up results - Does model work under all conditions, some conditions? Need modification?

Simple Harmonic Motion

- Position oscillates if force is always directed towards equilibrium position (restoring force).
- If restoring force is ~ position, motion is easy to analyze.





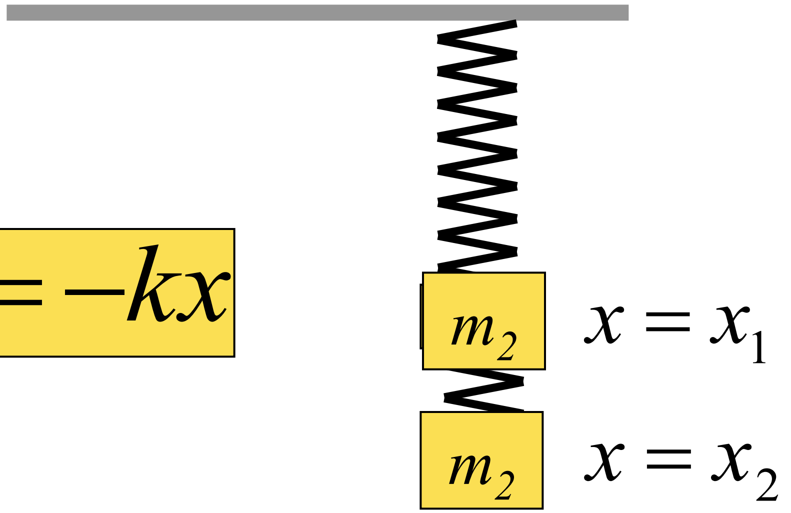
Springs

- Mag. of force from spring ~ extension (compression) of spring
- Mass hanging on spring: forces due to gravity, spring
- Stationary when forces balance

$$F_S = -kx$$

$$F_G = -mg$$

$$F_G = F_S$$
$$mg = kx$$

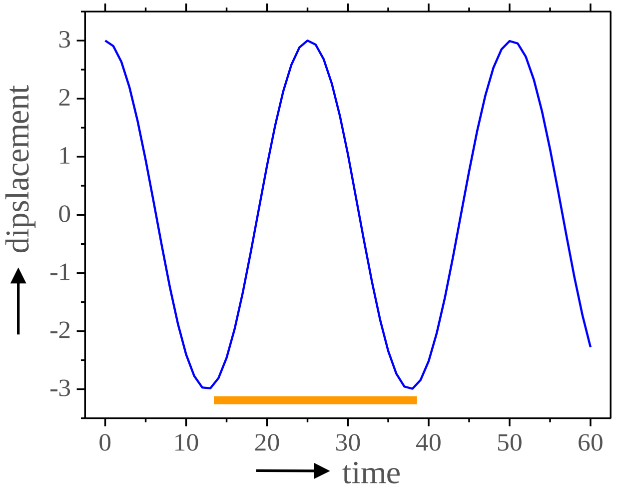


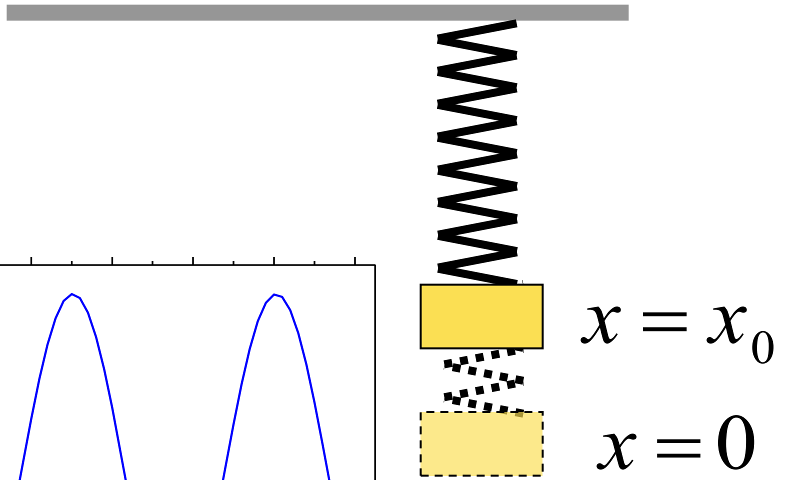
Action Figure

Simple Harmonic Motion

- Spring provides linear restoring force

$\Rightarrow$ Mass on a spring is a harmonic oscillator





$$F = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$x(t) = x_0 \cos \omega t$$

$$T = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Damping

- Damping force opposes motion, magnitude depends on speed
- For falling object, constant gravitational force
- Damping force increases as velocity increases until damping force equals gravitational force
- Then no net force so no acceleration (constant velocity)

$$\vec{F}_{damping} = -b\vec{v}$$

$$F_{gravity} = -mg$$

$$bv = mg$$

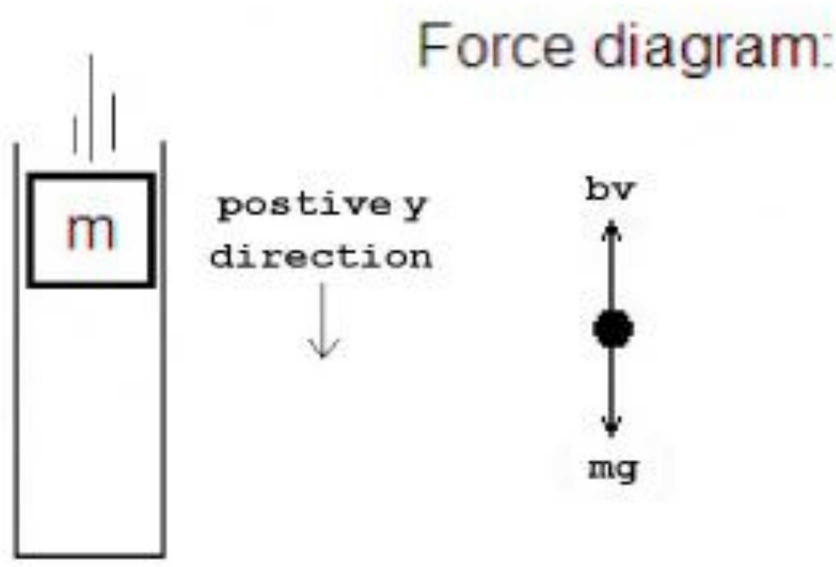
$$v_{terminal} = (mg)/b$$

Terminal velocity

- What is terminal velocity?
- How can it be calculated?

Action
Figure

Falling Mass and Drag

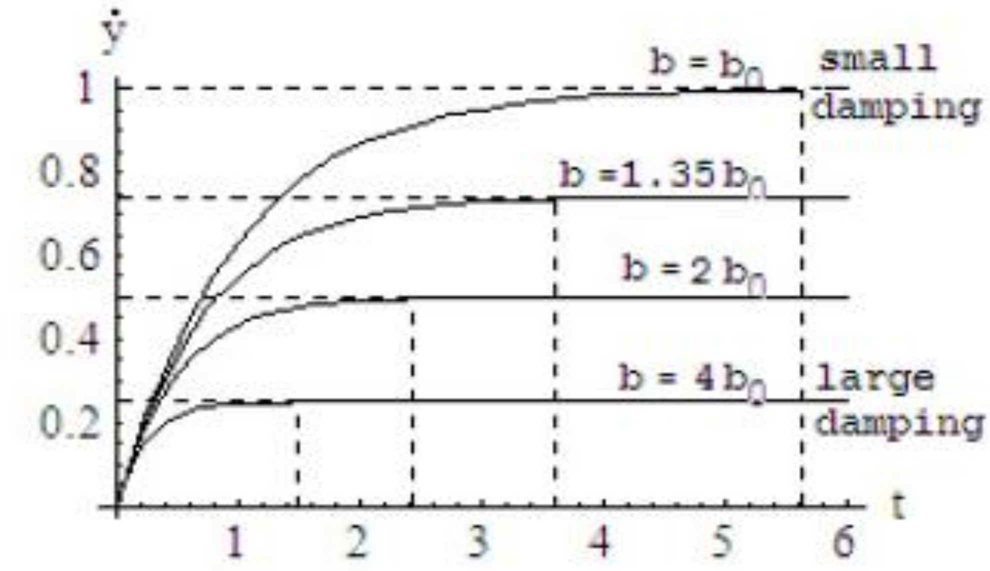


At steady state: $F_{drag} = F_{gravity}$

$$bv_t = mg$$

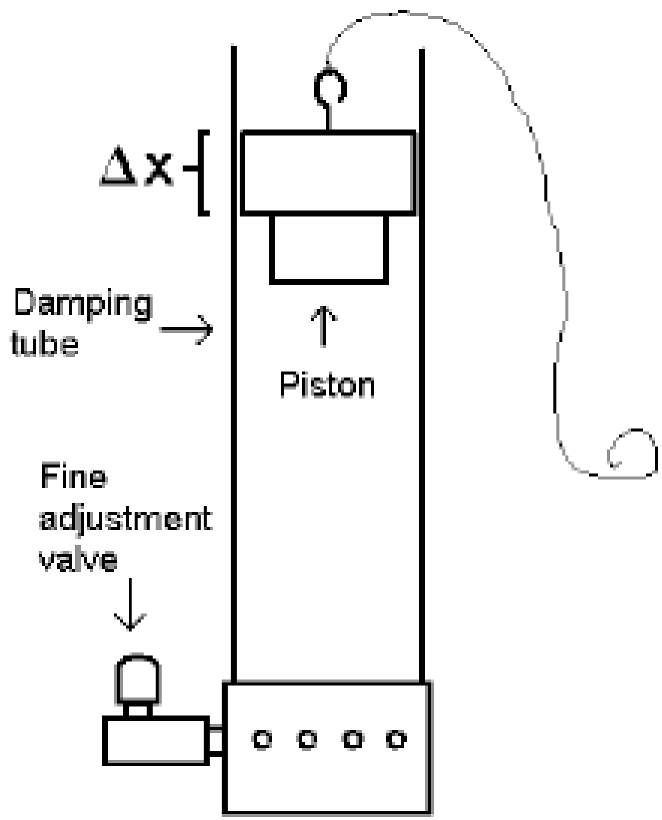
From rest: $y(t) = v_t[(m/b)(e^{-(b/m)t} - 1) + t]$

Terminal Velocity



For velocity: $\dot{y}(t) = v_t[1 - e^{-(b/m)t}]$

Experimental Setup for Falling Mass and Drag



How do you measure velocity?

Plotting Graphs

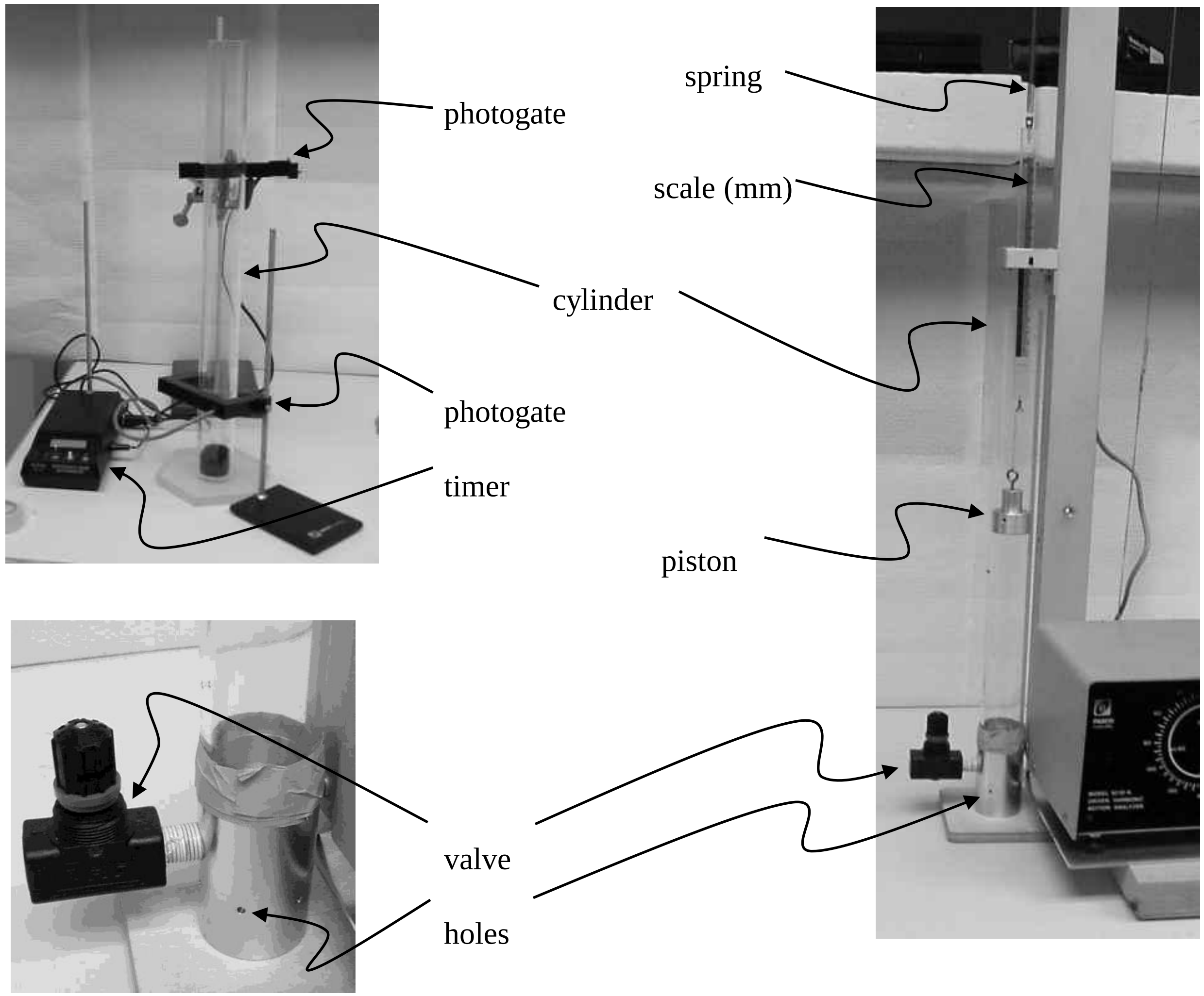
Give each graph a title

Determine independent and dependent variables

Determine boundaries

Include error bars

Experimental setup



Experiment 3: achieve critical damping

- Show/test method
  - Determine spring constant, predict critical damping coefficient
  - Determine how damping coefficient depends on air flow (valve position)
    - easy at terminal velocity
    - how do you know it's $v_{terminal}$?
  - Set damping to critical level

Demonstrate critical damping: show convincing evidence that critical damping was achieved

- Demonstrate that damping is critical
  - No oscillations (overshoot)
  - Shortest time to return to equilibrium position

Clicker Question 11

What is the uncertainty formula for *P* if

$P = q/t^{1/2}$

(a) $\delta P = [(\delta q)^2 + (\delta t)^2]^{1/2}$
(b) $\delta P = [(\delta q)^2 + (2\delta t)^2]^{1/2}$
(c) $\varepsilon P = [(\varepsilon q)^2 + (\varepsilon t)^2]^{1/2}$
(d) $\varepsilon P = [(\varepsilon q)^2 + (2\varepsilon t)^2]^{1/2}$
(e) $\varepsilon P = [(\varepsilon q)^2 + (0.5\varepsilon t)^2]^{1/2}$

Error propagation

(1) $k_{spring} = 4\pi^2 m/T^2$

$\sigma_{kspring} = \varepsilon_{kspring} * k_{spring}$

$\varepsilon_{kspring} = \sqrt{\varepsilon_m^{\ 2} + (2\varepsilon_T)^2}$

(2) $k_{by\text{-}eye} = m(g\Delta t^*/2\Delta x)^2$

$\sigma_{by\text{-}eye} = \varepsilon_{by\text{-}eye} * k_{by\text{-}eye}$

$\varepsilon_{by\text{-}eye} = \sqrt{(2\varepsilon_{\Delta t^*)}^{\ 2} + (2\varepsilon_{\Delta x})^2 + \varepsilon_m^{\ 2}}$

Remember

- Prepare for Quiz 3 and Experiment 3
- Review ideas - Taylor through Chapter 9