The Nature of Scientific Progress, More Error Analysis, Exp #2

Lecture # 3

Physics 2BL

Spring Quarter 2012

Revised TA Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00 AM					
9:30 AM		A03		A07	
10:00 AM		**Boucheron**	A04 **Green**	**Boucheron**	A10 **Murphy**
11:00 AM		**Lin** 744463	**Conger**	**Lin** 744467	**Cabrera**
12:30 PM		A01	744464		744470
1:00PM		**Davis** **(Odd)**	A05 **Green**		
2:00 PM		**Green** **(Even)** 744461	**Conger**		
3:30 PM		A02 **Stergiou**	744465 A06 **Stergiou**	A08 **Green** **(even)**	
5:00 PM		**Ye** 744462	**Ye**	**Ye** **(odd)** 744468	
6:30 PM			744466	A09 **Murphy** **Cabrera** 744469	

Outline

- Last time introduced significant figures, standard deviations, standard deviation of the mean
- Today instigate clicker questions
- Reproducibility
- What you should know about error analysis (so far) and more
- Introduce limiting Gaussian distribution
- Exp. 1
- Reminder

Measurement and Observation

- Measurement: deciding the amount of a given property by observation
- Empirical
- Not logical deduction
- Not all measurements are created equal…

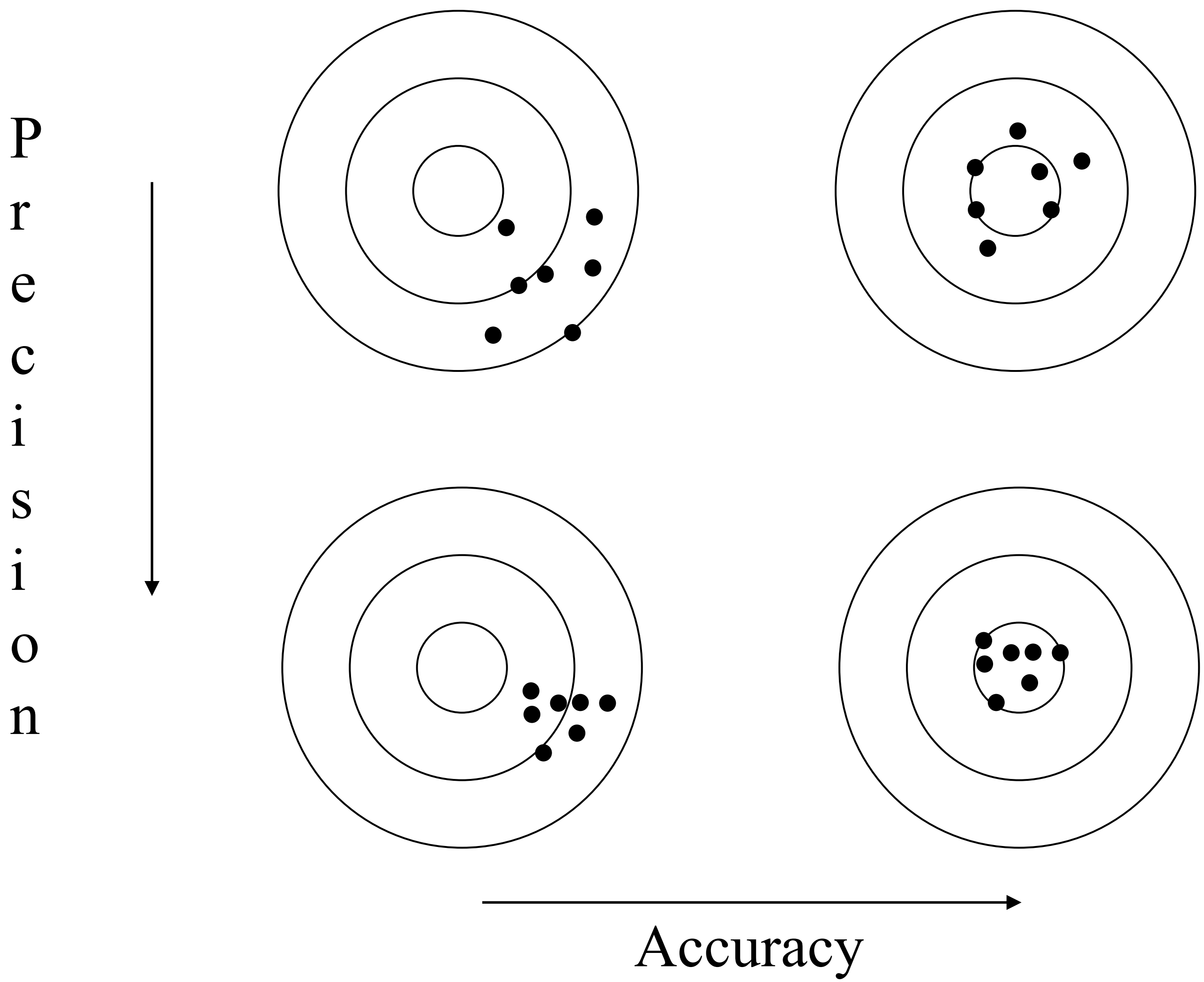
Reproducibility

- Same results under similar circumstances
  - Reliable/precise
- 'Similar' - a slippery thing
  - Measure resistance of metal
    - need same sample purity for repeatable measurement
    - need same people in room?
    - same potential difference?
  - Measure outcome of treatment on patients
    - Can't repeat on same patient
    - Patients not the same

Precision and Accuracy

- Precise - reproducible
- Accurate - close to true value
- Example - temperature measurement
  - thermometer with
    - fine divisions
    - or with coarse divisions
  - and that reads
    - 0 C in ice water
    - or 5 C in ice water

Accuracy vs. Precision



Random and Systematic Errors

- Accuracy and precision are related to types of errors
  - random (thermometer with coarse scale)
    - can be reduced with repeated measurements, careful design
  - systematic (calibration error)
    - difficult to detect with error analysis
    - compare to independent measurement

Observations in Practice

- Does a measurement measure what you think it does? Validity
- Are scope of observations appropriate?
  - Incidental circumstances
  - Sample selection bias
- Depends on model

Random and independent?

Yes

- Estimating between marks on ruler or meter
- Releasing object from ‘rest’
- Mechanical vibration

o **Judgment**

o **Problems of definition**

No

- End of ruler screwy
- Reading meter from the side (speedometer effect)
- Scale not zeroed
  Reaction time delay

o **Calibration**

o **Zero**

Clicker Question 1

Which of these is an example of a systematic error?

A) differences between four measurements using a stop watch.

B) a reading of time if the stop watch reads 2 s before started.

C) the recorded value if you mistakenly record 4 s rather than 3 s.

Clicker Question 2

Which of these is an example of a random error?

A) differences between four measurements using a stop watch.

B) a reading of time if the stop watch reads 2 s before started.

C) the recorded value if you mistakenly record 4 s rather than 3 s.

Random & independent errors:

$$q = x + y - z$$

$$\delta q = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$$

$$q = Bx$$

$$\delta q = |B| \delta x$$

$$\frac{\delta q}{|q|} = \frac{\delta x}{|x|}$$

$$q = x \times y \div z$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2}$$

$$q = q(x, y, z)$$

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x\right)^2 + \left(\frac{\partial q}{\partial y} \delta y\right)^2 + \left(\frac{\partial q}{\partial z} \delta z\right)^2}$$

Propagation in formulas

Independent

Propagate error in steps

For example:

$$q = \frac{x}{y - z}$$

- First find

$$p = y - z$$

$$\delta p = \sqrt{(\delta y)^2 + (\delta z)^2}$$

- Then

$$q = \frac{x}{p}$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta p}{p}\right)^2}$$

Clicker Question 3

What is the error in the calculated value for F if
$F = M/R^2$ and $\varepsilon_M = 0.1$ and $\varepsilon_t = 0.2$?

A) 0.1

B) 0.2

C) 0.3

D) 0.4

E) ??

An Important Simplifying Point

$$h = \frac{1}{2} g t^2$$

$$g = 2h/t^2 \; , \delta h/h = 5\%, \delta t / t = 0.1\%$$

Requires random & ind. errors!

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta h}{h}\right)^2 + \left(2\frac{\delta t}{t}\right)^2}$$

$$\delta g / g = \sqrt{5\%^2 + (2 \times 0.1\%)^2}$$

$$\delta g / g = 0.050039984 = 5\%$$

- Often the error is dominated by error in least accurate measurement

⇨Simplifies calc.

⇨Suggests improvements in experiment

Analyzing Multiple Measurements

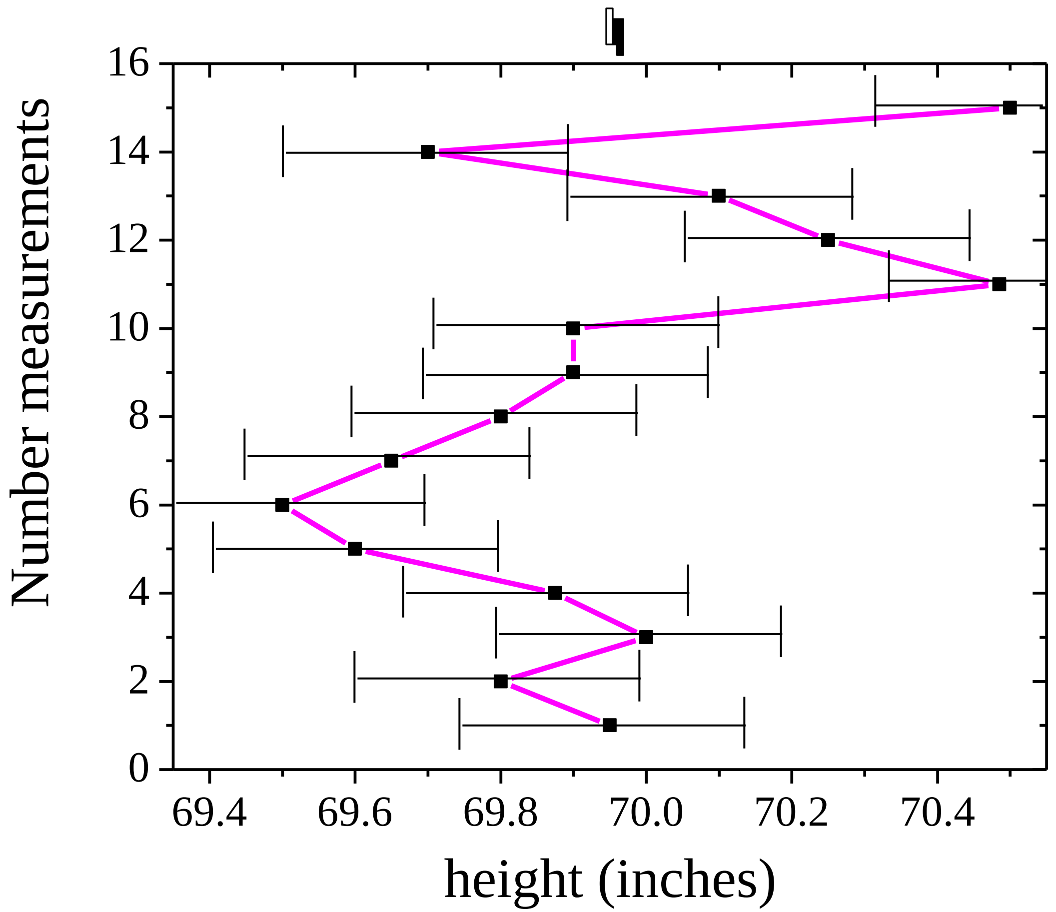
- Repeat measurement of *x* many times
- Best estimate of *x* is average (mean)

$$x_1, x_2, \cdots, x_N$$

$$x_{best} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

$$\bar{x} = \frac{\sum x_i}{N}$$

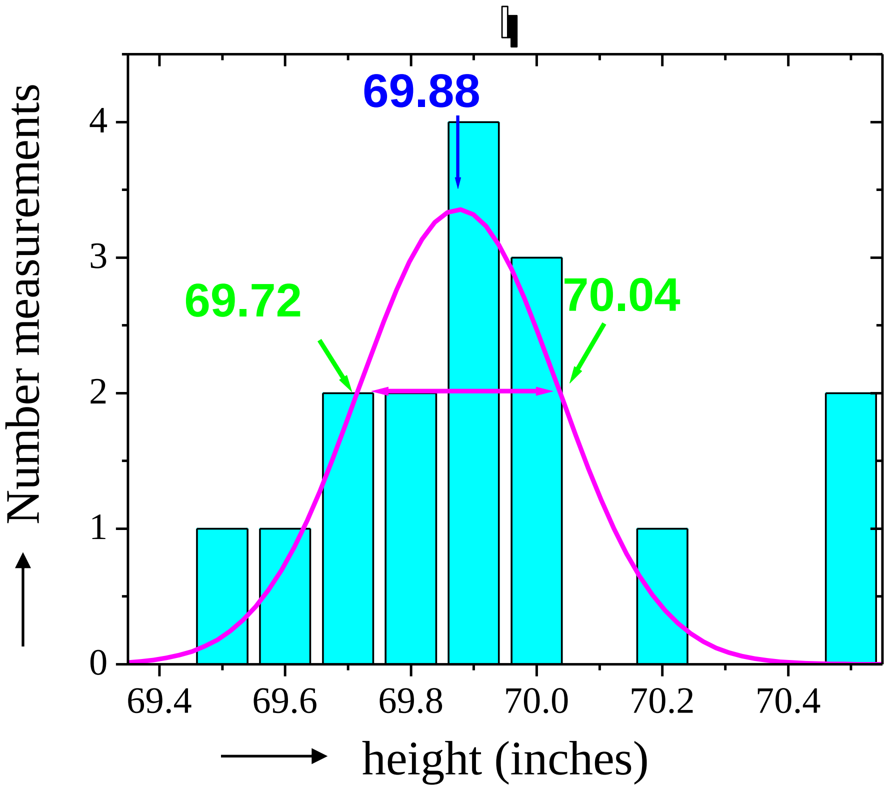
Repeated Measurements



How are Measured Values Distributed?

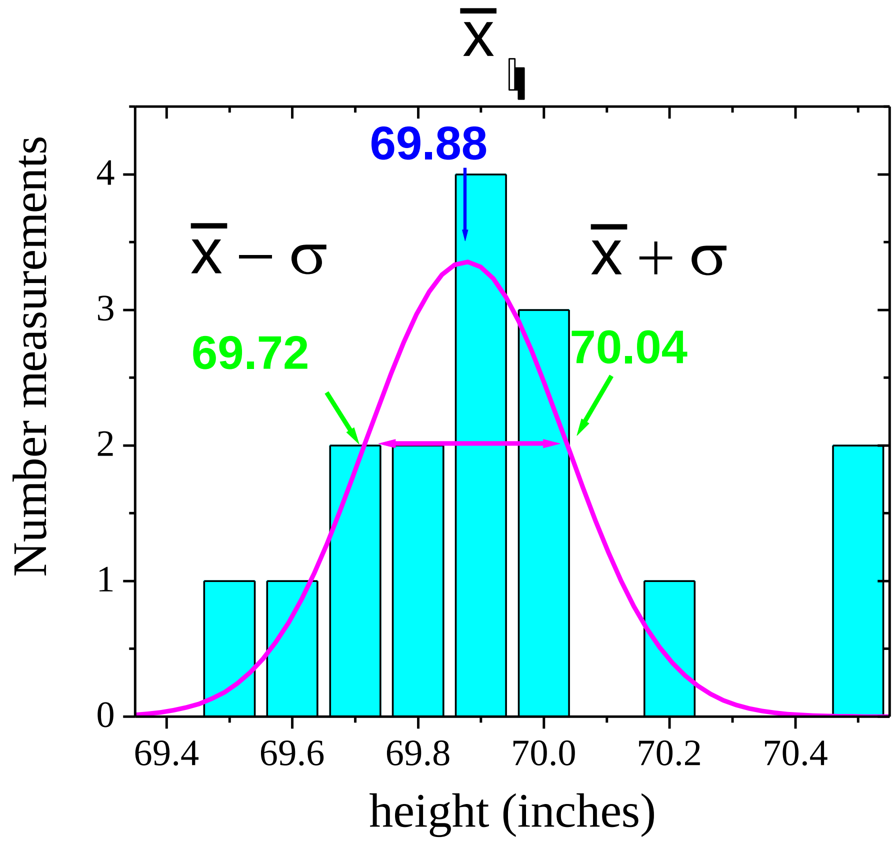
- If errors are random and independent:
  - Expect most values near true value
  - Expect few values far from true value

⇨ Assume values are distributed *normally*



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-(x-\overline{x})^2 / 2\sigma^2\right)$$

Normal Distribution



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-(x-\bar{x})^2 / 2\sigma^2)$$

Error of an Individual Measurement

- How precise are measurements of *x*?
- Start with each value's deviations from mean
- Deviations average to zero, so square, then average, then take square root
- ~68% of time, x_i will be w/in σ_x of true value

$$d_i \equiv x_i - \bar{x}$$

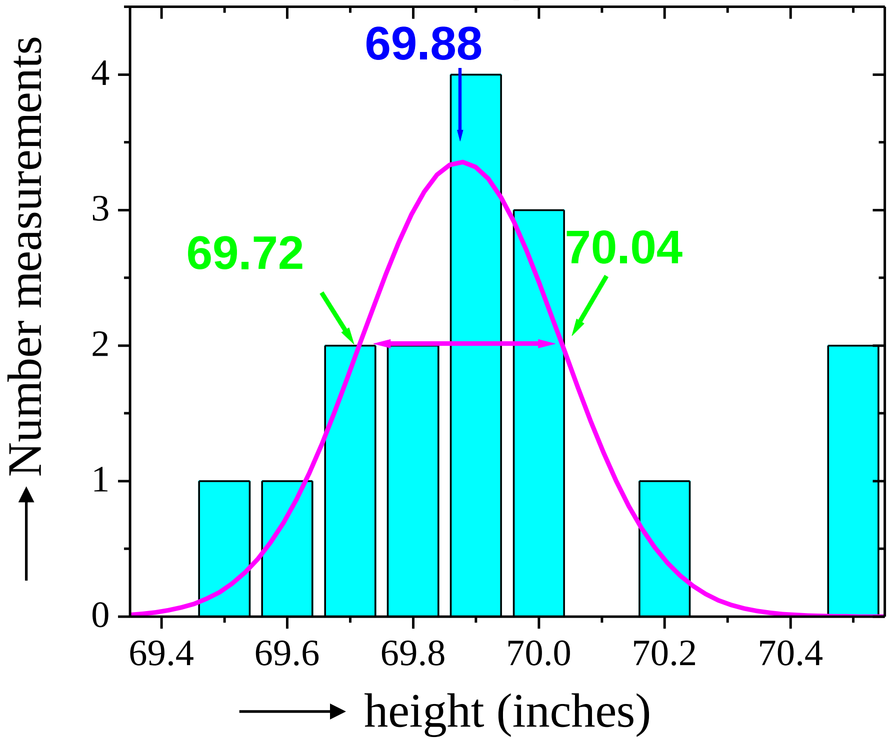
$$\bar{d} = 0$$

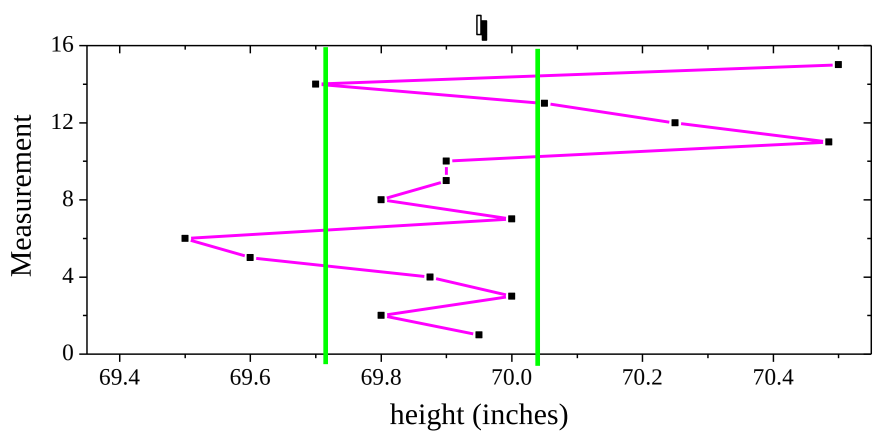
$$\sigma_x \equiv \sqrt{\overline{(d_i)^2}}$$

$$= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Take σ_x as error in individual measurement - called standard deviation

Standard Deviation





Drawing a Histogram

1. Determine the range of your data by subtracting the smallest number from the largest one.

2. The number of bins should be approximately $\sqrt{N}$ and the width of a bin should be the range divided by $\sqrt{N}$.

3. Make a list of the boundaries of each bin and determine which bin each of your data points should fall into.

4. Draw the histogram. The y axis should be the number of values that fall into each bin.

5. Sometimes this procedure will not produce a good histogram. If you make too many bins the histogram will be flat and too few bins will not show the curve on either side of the maximum. You might need to play around with the number of bins to produce a better histogram.

Error of the Mean

- Expect error of mean to be lower than error of the measurements it's calculated from
- Divide SD by square root of number of measurements
- Decreases slowly with more measurements

Standard Deviation of the Mean (SDOM)

or

Standard Error

or

Standard Error of the Mean

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$$

Summary

- Average

$$\bar{x} = \frac{\sum x_i}{N}$$

- Standard deviation

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- Standard deviation of the mean

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$$

The Four Experiments

- **Determine the average density of the earth**

Weigh the Earth, Measure its volume

– Measure simple things like lengths and times

– Learn to estimate and propagate errors

- **Non-Destructive measurements of densities, inner structure of objects**

– Absolute measurements *vs.* Measurements of variability

– Measure moments of inertia

– Use repeated measurements to reduce random errors

- **Construct and tune a shock absorber**

– Adjust performance of a mechanical system

– Demonstrate critical damping of your shock absorber

- **Measure coulomb force and calibrate a voltmeter.**

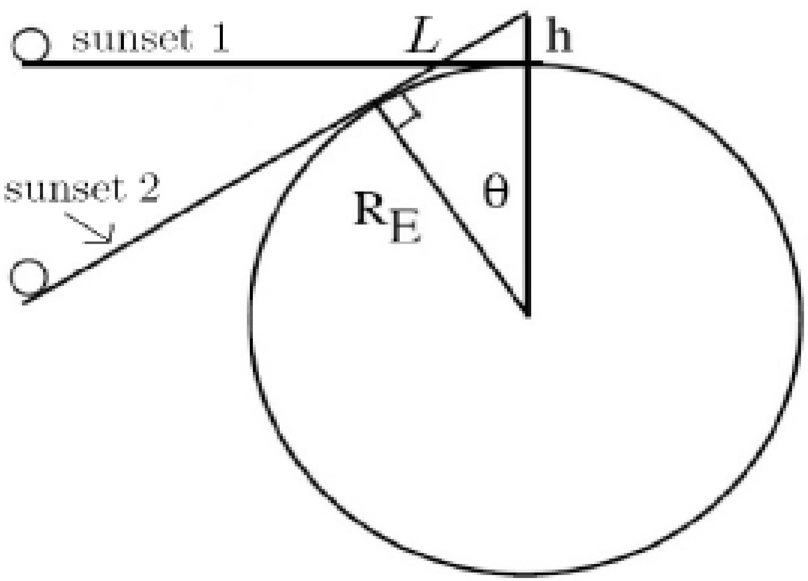
– Reduce systematic errors in a precise measurement.

The Earth

Volume – radius
Mass
Density

Experiment 1 Overview: Density of Earth

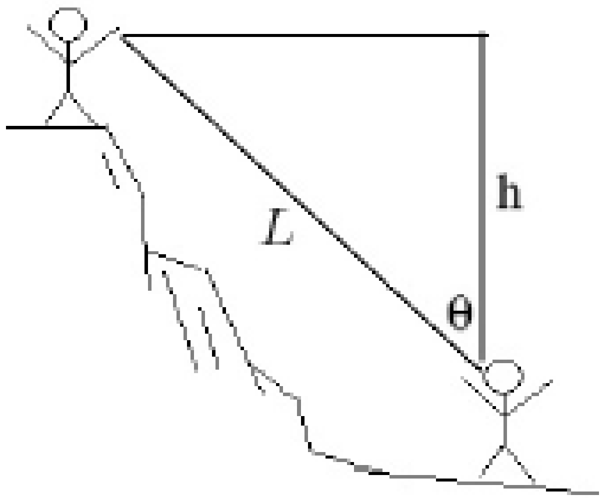
density $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{3g}{4\pi G R_E}$ $= \frac{G M_E m}{R_E^2} = mg$

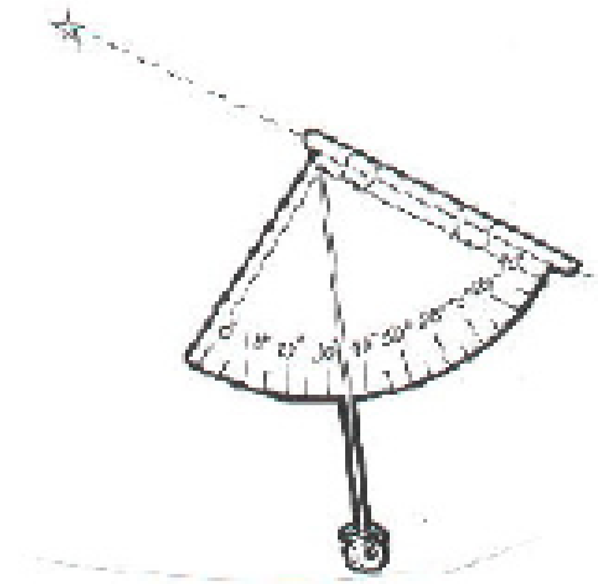


$$R_E = \frac{2h}{\omega^2(\Delta t)^2}$$

measure Δt between sunset on cliff and at sea level

Experiment 1: Height of Cliff





rangefinder to get *L*

Sextant to get θ

Wear comfortable shoes

Make sure you use θ and not (90 – θ)

"The Equation" for Experiment 1a

$$t = \frac{T}{2\pi}\sqrt{\frac{2Ch}{R_e}} = \frac{1}{\omega}\sqrt{\frac{2Ch}{R_e}}$$

from previous page.

$$\omega = \frac{2\pi}{24\text{ hr}}$$

Which are the variables that contribute to the error significantly?

$$\Delta t = t_1 - t_2 = \frac{1}{\omega}\sqrt{\frac{2C}{R_e}}\left(\sqrt{h_1} - \sqrt{h_2}\right)$$

Time difference between the two sunset observers.

$$C \equiv \frac{1}{\cos^2(\lambda)\cos^2(\lambda_s) - \sin^2(\lambda)\sin^2(\lambda_s)}$$

Season dependant factor slightly greater than 1.

The formula for your error analysis.

$$R_e = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1} - \sqrt{h_2}}{\Delta t}\right)^2$$

What other methods could we use to measure the radius of the earth?

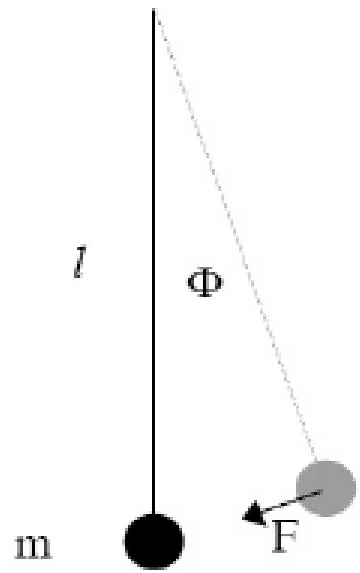
Eratosthenes

angular deviation = angle subtended

From Yagil

Experiment 1: Determine g

pendulum



$F = -mg\sin(\phi) = -mg\phi$

$F = m\alpha = ml\ddot{\phi}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

period

Experiment 1: Pendulum

- For release angle θ_i, you should have a set of time data $(t_1^p, t_2^p, t_3^p, ..., t_N^p)$.

- Calculate the average, $\bar{t^p}$, and the the standard deviation, σ_{t^p}, of this data.

- Divide $\bar{t^p}$ and σ_{t^p} by p to get average time of a *single* period, $\bar{T}$ and standard deviation of a single period σ_T.

- Calculate SDOM, $\sigma_{\bar{T}} = \frac{\sigma_T}{\sqrt{N}}$.

- Now you should have $\bar{T} \pm \sigma_{\bar{T}}$ for you data at θ_i.

- Repeat these calculations for data at each release angle.

Grading rubric uploaded on website

Error Propagation - example

Ws saw earlier how to determine the acceleration of gravity, g.

Using a simple pendulum, measuring its length and period:

-Length l : $l = l_{best} \pm \delta l$

-Period T : $T = T_{best} \pm \delta T$

Determine g by solving: $g = l \cdot (2\pi / T)^2$

The question is what is the resulting uncertainty on g, **δg** ??

From Yagil

Propagating Errors for Experiment 1

$$\rho = \frac{3}{4\pi}\frac{g}{GR_e}$$ **Formula for density.**

$$\sigma_\rho = \frac{3}{4\pi}\frac{1}{GR_e}\sigma_g \oplus \frac{-3}{4\pi}\frac{g}{GR_e^2}\sigma_{R_e}$$ **Take partial derivatives and add errors in quadrature**

Or, in terms of relative uncertainties: $\frac{\sigma_\rho}{\rho} = \frac{\sigma_g}{g} \oplus \frac{\sigma_{R_e}}{R_e}$

shorthand notation for quadratic sum: $\sqrt{a^2 + b^2} = a \oplus b$

From Yagil

Propagating Errors for R_e

$$R_e = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1}-\sqrt{h_2}}{\Delta t}\right)^2$$

basic formula

$$\sigma_{R_e} = \frac{\partial R_e}{\partial \Delta t}\sigma_{\Delta t} \oplus \frac{\partial R_e}{\partial h_1}\sigma_{h_1} \oplus \frac{\partial R_e}{\partial h_2}\sigma_{h_2}$$

Propagate errors (use shorthand for addition in quadrature)

$$\sigma_{R_e} = \frac{2R_e}{\Delta t}\sigma_{\Delta t} \oplus \frac{R_e}{\sqrt{h_1}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_1} \oplus \frac{R_e}{\sqrt{h_2}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_2}$$

Note that the error blows up at $h_1=h_2$ and at $h_2=0$.

From Yagil

Remember

- Finish Lab #1
- Read lab description, prepare for Lab #2 for next week.
- Read Taylor through Chapter 5
- Problems 5.2, 5.36