

Experiment 4, Physics 2BL

Build and tune a voltmeter.

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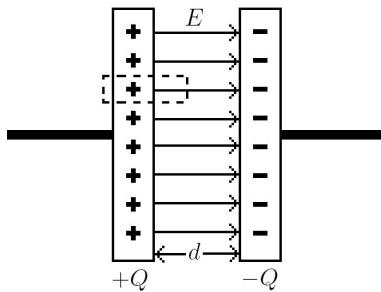
Preparation

Before this experiment, we recommend you review or familiarize yourself with the following:

- Chapters 8 and 12 in Taylor
- Capacitors
- Torque

1. PHYSICS

1.1. Capacitors: Derivation of Force Between Plates in Terms of Dimensions and Voltage



The figure above shows a capacitor that has two plates of area A and charged with $+Q$ on the left and $-Q$ on the right. We can find the electric field E , represented by the arrows, by using Gauss's law. Suppose we make a pill box that is the size of the dotted line. The cross sectional area of the pill box is a . Gauss's law is as follows.

$$\mathbf{E} \cdot \mathbf{a} = \frac{Q_{enc}}{\epsilon_0}$$

Where Q_{enc} is the total charge enclosed by the pill box and $\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$ is the permittivity of a vacuum. We will actually be conducting our experiment in air, but these permittivities are very close. Since the charge is evenly distributed over the plate we know that $Q_{enc} = Q \frac{a}{A}$ and

$$E = \frac{Q}{A\epsilon_0} \quad (1)$$

Next we want to calculate the voltage between the plates. To do that all we need is to integrate the electric field on a path from one plate to the other. $V = \int \mathbf{E} \cdot d\mathbf{r}$ gives us:

$$V = Ed \quad (2)$$

To calculate the force between the two plates we use Coulomb's force law which states that $F = QE$. However, since half the electric field creates a force on one plate and half the electric field creates a force on the other plate, the total force between them is only

$$F = \frac{1}{2}QE \quad (3)$$

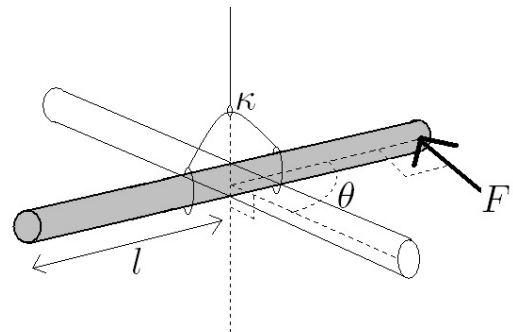
If we combine the last three equations we can solve for and eliminate E and Q . What we end up with is an equation for force in terms of the dimensions of the capacitor and the voltage.

$$F = \frac{1}{2} \frac{A\epsilon_0}{d^2} V^2 \quad (4)$$

Questions

1. Use equations (1), (2), and (3) to eliminate E and Q and derive equation (4).
2. Use equations (1) and (2) and $V = \frac{Q}{C}$ to derive an expression for the capacitance, C , of a parallel plate capacitor. What are two ways of improving the capacitance by changing the dimensions of the capacitor, A and d ?

1.2. Using Torque to Measure a Force



In this experiment we will be constructing a torsional pendulum, which has a period

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad (5)$$

where I is the moment of inertia of whatever is being rotated and κ is a torsional constant similar to a spring constant. See the Appendix 1 for more information on torsional pendulums. There is an equivalent form of Hooke's Law for torsional pendulums. Recall that for a regular spring the force is $F = -kx$, where x is the displacement of the spring from equilibrium. For a torsional pendulum we have that the torque is $\tau = -\kappa\theta$, where θ is the angular displacement from equilibrium.

Now, if we construct a parallel plate capacitor on one end of a torsional pendulum and apply a voltage across it, then there will be a force on the end of the moment arm given by equation (4) (This electrostatic force is perpendicular to the moment arm). You now have a torque on this torsional pendulum given by $\tau = \mathbf{F} \times \mathbf{l} = Fl$, where F is given by equation (4). Now, to separate the plates from each other, we must rotate the torsional pendulum in a direction opposite than the direction the capacitor forces the pendulum. In doing this, you are applying a torque (now in the opposite direction as the torque due to the capacitor) given by $\tau = -\kappa\theta$. When the system is in equilibrium, the sum of the torques should be zero. The result is

$$Fl = \kappa\theta \quad (6)$$

Note that the system is in equilibrium the moment just before the capacitor plates separate. This is when the torques are equal.

Questions

3. If the moment arm is increased do we need more or less of an angle to torque the plates apart from each other? Is the torsion constant, κ , altered if you change the moment of inertia of the hanger?

We can now determine the voltage V applied across this capacitor (of area A , thickness d , and a distance l from the pendulum axis) by combining equations (4) and (6).

$$V = d\sqrt{\frac{2\kappa\theta}{lA\epsilon_0}} \quad (7)$$

Where θ here is the rotation angle that puts the torsional pendulum in a state of equilibrium (i.e. when the torques are equal).

Questions

4. In order to determine the voltage in terms of Volts, what must be the appropriate units of d , κ , θ , l , and A in equation (7)?
5. Use equations (4) and (6) to derive equation (7).
6. Given that $I = 2ml^2$, use equation (5) to derive an expression for σ_κ if the fractional error in the measurement of T is much bigger than the fractional errors for m and l

2. METHODS FOR STATISTICAL ANALYSIS

2.1. Curve Fitting

Suppose you have a series of N points that have coordinates (x_i, y_i) . In order to find the equation of a line that makes a least-squares fit to $y = A + Bx$ use the following formulas.

$$A = \frac{\sum x^2 \sum y - \sum x \sum xy}{\Delta}$$

$$B = \frac{N \sum xy - \sum x \sum y}{\Delta}$$

$$\Delta = N \sum x^2 - (\sum x)^2$$

Note that $\sum x^2 = x_1^2 + x_2^2 + \dots$ whereas $(\sum x)^2 = (x_1 + x_2 + \dots)^2$.

For extra practice see Taylor #8.6 and #8.10

2.2. Chi Squared

In order to get a measure of how good your fit is, first use the following formula:

$$\chi^2 = \sum \frac{(y_i - f(x_i))^2}{\sigma_{y_i}^2}$$

where $f(x_i)$ is a function that represents your fitted curve evaluated at x_i and σ_{y_i} is the uncertainty on the data point y_i .

It is then necessary to find the reduced Chi Squared, $\tilde{\chi}^2$ given by:

$$\tilde{\chi}^2 = \frac{\chi^2}{d}$$

where d is the number of degrees of freedom of the system and is given by $d = N - c$ where N is the number of data points/bins and c is the number of parameters calculated from the data. The probability of obtaining a certain $\tilde{\chi}^2$ is given by the following chart.

d	$\tilde{\chi}_o^2$													
	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2	3	4	5	6	7
1	100	62	48	39	32	26	22	19	16	8	5	3	1	—
2	100	78	61	47	37	29	22	17	14	5	2	0.7	0.2	—
3	100	86	68	52	39	29	21	15	11	3	0.7	0.2	—	—
5	100	94	78	59	42	28	19	12	8	1	0.1	—	—	—
10	100	99	89	68	44	25	13	6	3	0.1	—	—	—	—
15	100	100	94	73	45	23	10	4	1	—	—	—	—	—

For extra practice see Taylor #12.3 and #12.14

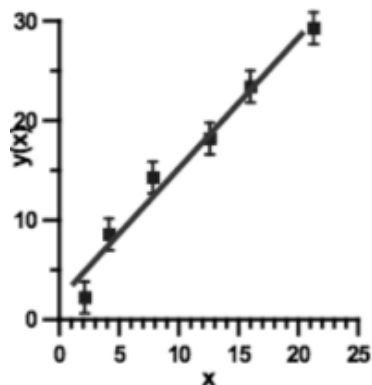
Question

7. χ^2 is formulaically similar to the t -score.

How is t -score different from χ^2 ?

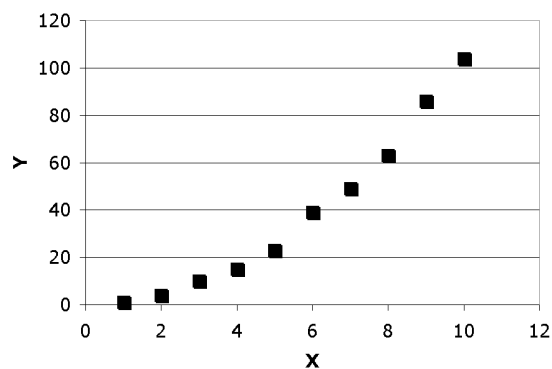
8. See the plot below. A set of data has been plotted and fitted by a straight line. If you were calculating $\tilde{\chi}^2$, what would be the number of degrees of freedom d ?

Don't just state the number, explain how you determined the number.



Question

9. See the plot in the next column. A set of data has been plotted. Suppose you decided to fit a second order polynomial to the data (i.e. $y = Ax^2 + Bx + C$). If you were calculating $\tilde{\chi}^2$, what would be the number of degrees of freedom d ? Don't just state the number, explain how you determined the number.



3. PROCEDURE

This experiment is very sensitive and usually very time consuming. Come well prepared and work as a team in order to complete the experiment in a timely fashion.

3.1. Setup Description

This section is meant to familiarize you with the setup. There are further details about the setup in the next few sections.

A large grey disk is first raised a couple feet from the table and clamped in a horizontal position. A circular protractor should be taped to the top and one of the metal pistons (with eye hooks screwed into both its ends) used in experiment 3 should be placed in the hole of the large grey disk. This piston should fit snugly. Also, obtain a small rod and slide it through the small hole on the side of the piston. Next, a torsional pendulum should hang from the eye hook that points down. This torsional pendulum is composed of a copper wire with a hanger tied to its end. The hanger should have a copper disk taped to the ends of each of its arms. One of the copper disks should have a rubber stopper attached to its center. This disk should be taped such that the rubber stopper points away from the hanger wire. Also, a small mirror should be taped to the center of the hanger.

3.2. Moment of Inertia of Hanger+Disks

Measure the mass and dimensions of the hanger+disks: This includes the mass of the hanger, m , the masses of the copper disks, m_1 and m_2 (One of these masses should have a rubber stopper attached to it.), the length of the moment arm, l (see diagram on page 1), and the radius of one of the disks, R . Also, using calipers, measure the thickness d of the rubber stopper. (This last measurement is not needed to calculate the moment of inertia. It is needed in the voltage calculation.)

Calculate the moment of inertia of the hanger+disks and its error σ_I .

$$I = \frac{1}{3}ml^2 + (m_1 + m_2)l^2 + (m_1 + m_2)\frac{R^2}{4}$$

Note that this is the moment of inertia of the hanger with a copper disk attached to either end of the hanger.

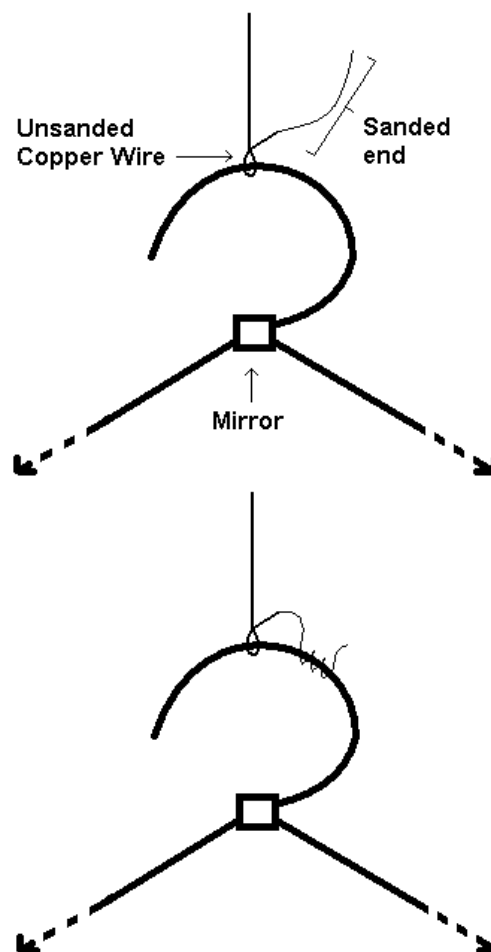
3.3. Measuring the Torsion Constant κ

In order to calculate the voltage using equation (7), you need to know κ of the copper wire. You can determine κ of the wire if you know the moment of inertia and rotational period for an object that hangs from the wire. You've already determined the moment of inertia of your object (hanger+disks) in the previous section. Next you need to construct this torsional pendulum so you can measure the rotational period of the object.

Before constructing the pendulum, using sand paper, sand the hook of the hanger and the end of one of its arms. Obtain a piece of copper wire (the length is up to you but around 15-20 cm should be fine). Then sand the last few centimeters of each end of the copper wire and sand the two eye hooks attached to small piston. Next, tape the copper disks on either end of the hangers arms. *Be sure that the disk with the rubber stopper is taped to the end of the hanger that has been sanded and that the rubber stopper points away from the hanger wire.* Because you won't be able to deconstruct your setup later to resand things you may have sanded poorly, you now need to check that everything is conducting well. *Before moving on, ask one of your TAs to check that everything conducts well (This will be done using a multimeter).*

Next, construct the torsional pendulum according to the setup description in section 3.1 BUT also consider the following detail. The ends of the copper wire have been sanded and are now weak, so it would be a good idea to tie the copper wire to the hanger hook using the unsanded portion of the copper wire that comes just before the sanded end. Then wrap the small piece of *sanded* copper wire to the hanger hook. See the diagram below for clarification. Do the same when tying the copper wire to the eye hook. Note: If you tie the sanded portion of the copper wire to the hanger, the copper wire may break during your experiment and then you will have to start over from the beginning. Ask your TA to check that your current setup conducts well.

To measure the period of oscillations line up the laser so that it reflects off of the mirror on the hanger. Give the pendulum a very small kick so that it starts moving in a smooth horizontal path. Watch the motion of the laser across the room. Mark off a point near the center of the path with a piece of tape. Measure the time for the laser to go from the point you marked off to its extreme point, and back through the marked point and off to the other side, and finally back to the starting point. This full oscillation should take approximately 1 minute. Measure about 5 of these periods.



The advantage of using a point near the center of the path is that the laser moves relatively quickly through that point, as opposed to the extreme points where the laser can linger for several seconds without moving very much.

Calculate $\kappa = \frac{4\pi^2 I}{T^2}$ and its error σ_κ .

3.4. Setting up the Capacitor

Wait for the hanger to reach an equilibrium position. Place an aluminum plate up against the copper disk with the rubber stopper. Orient the aluminum plate such that it is PARALLEL to the copper disk. Obtain a small bucket of water and submerge the other disk in it to minimize vibrations from air currents. Obtain a voltage source and some wires. Attach one wire to the aluminum plate and plug it into the ground terminal on the voltage source. Then, plug a wire into the positive terminal and connect the other end to the eye hook pointing up from the piston (You might have to connect a couple wires together in order to have a cable long enough to reach the top of the setup.). There may be some charge build up on the plates, so you should discharge the plates by

touching them together. Finally, flip the black switch on the voltage source to 1000V.

Question

10. In this experiment, why must you first wait for the hanger to reach an equilibrium position before placing the aluminum plate next to the copper plate?

3.5. Measuring the Voltage

Set the voltage source to 1000 V, record an initial angle, then rotate the axel by pushing the small rod that sticks out of the piston. Rotate the axel slowly until the plates of the capacitor separate and record the angle of rotation. Do the same for the voltage settings 900 V, 800 V, 700 V, and 600 V. Voltages lower than 600V are often too hard to measure because the force is weak.

Repeat this procedure at least 4 more times so that you will have 5 angle measurements per voltage setting. Then you can calculate an average angle and its standard deviation at each voltage setting. If you have measured the rubber stopper thickness d with calipers, then, using equation (7) you can calculate the voltage across the capacitor for each supplied voltage.

4. CAUTIONS

4.1. Minimizing Vibrations

Make sure that no one is leaning on the table while the measurements are being taken. Try to block out air currents by surrounding your setup with boxes or boards.

4.2. Risk of Shock

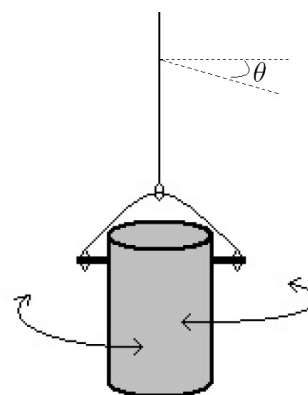
Do not touch the electrical connections when a voltage is being applied. Keep the desk and your hands dry.

5. ERROR ANALYSIS

Use error propagation to find the error of your measurements, σ_V .

Make a graph of your data where the x axis is the values of the voltage supplied to the capacitor by the voltage source (600-1000 V) and the y axis is the calculated values obtained from the torsional pendulum. Make sure to include vertical error bars on your points. Use the procedure outlined in section 2.1 of this lab guide to calculate a line of best fit and plot that line on your graph. Then calculate a χ^2 and find the probability of obtaining it using the chart in the back of Taylor.

Appendix 1: Torsional Pendulum



Instead of gravity, a torsional pendulum begins its motion when it is given an angular displacement. The equations for torque are as follows, where the dot signifies a derivative in time and κ is the torsional constant of the fiber.

$$\tau = -\kappa\theta = I\ddot{\theta}$$

where I is the moment of inertia of a cylinder about its longitudinal axis. Rearranging gives a differential equation for θ .

$$\ddot{\theta} + \frac{\kappa}{I}\theta = 0$$

The solution is periodic motion where the frequency of oscillations, $\omega = \sqrt{\frac{\kappa}{I}}$. The period of this pendulum is easily measured and is given by the formula:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$$