Physics 225A: Intro to General Relativity April 2012

Professor: Kim Griest

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1 Analogy of the Surveryors

1.1 Curved Spacetime

GR says that gravity is not really a "force"; but instead is curved spacetime. What does that mean?

Galileo and Newton view motion with respect to a rigid Euclidean reference frame that extends throughout all space and endures forever. Within this ideal frame, there exists the mysterious force of gravity – a foreign influence. Einstein says, "there is no such thing". Climb into a spaceship and see for yourself – no gravity there! Suppose you are floating in a space ship with no windows. Can you tell you whether you are out in the middle of free space or orbiting the Earth? Not really!

This is the starting point of GR, *Physics is locally gravity free.*All free particles move in straight lines and constant speed. In an inertial frame, physics looks simple. But such frames are inertial only in a limited region, i.e. *local*. Complications arise when motion is described in nearby local frames. Any difference between direction in one local frame and a nearby frame is described in terms of "curvature of spacetime". Curvature implies it is impossible to use a single Euclidean frame for all space. In a small region, curvature is small, that is it looks flat. Einstein adds together many local regions and has a theory with no gravity force. Newton has a single flat space and an extra force. These are radically different views. Einstein is right, but usually Newton's view is good enough for calculation.

1.2 Example of two surveyors

Fig 1: Surveyors on Earth going north.

Let me give an example that is extremely helpful in understanding what I just said. Consider two surveyors standing 100 meters apart on the equator. They both decide to start out perfectly parallel towards the north by rolling a big ball directly north. Some time later as they roll their balls, one notices that the distance between the two balls is less than the initial 100m. "Hey" one surveyor calls, you aren't going straight, you are coming towards me. The other says "I'm going perfectly straight, it's you that's moving." After a lot of checking they decide they both are rolling the balls straight, but that there must be some mysterious force that is pulling the balls toward each other. (What is happening of course, is that both balls are approaching the north pole, and would hit each other there.) They try the experiment with bigger balls and discover that the big balls come closer as the go north by the same amount. Since $F = ma$, the bigger balls require a bigger force and thus they decide this force is proportional to the mass of the object. In fact, it seems all objects moving north attract all other objects with a force proportional to their mass. “We have made a great discovery; let’s call this force gravity”, the surveyors decide.

The surveyors think they have a new force because they think they are moving on a flat surface, but in reality are on the large curved surface of the Earth. They don’t realize the reason for the balls coming together is the curvature of the Earth’s surface. In fact, you can do the math for the radius of the Earth and even find the value of the effective “Newton’s constant G” (not the same of course as our normal G, and this “gravity” does not fall-off as r^{-2}.)

From Einstein’s view, there is no force. The movement together of the balls is proof that the Earth’s surface is curved. Einstein says the same thing with regard to actual gravity that pulls the falling apple toward the Earth. No force, but curved spacetime. Note in the example of the surveyors only space (Earth surface) was curved; in GR both space and time are curved. This view in fact explains a major mystery of Newton’s law. Newton had two types of mass: $m = F/a$ is “inertial mass”, telling how hard it is to accelerate things, while the m in $F = GMm/r^2$ is the gravitational mass, telling how much gravity comes off the object. Why are these masses the same? In Coulomb’s law, the source of the force is the charge, and it is not the same as the mass. This is a mystery, but it has been tested carefully many times and the two masses are always equal. Einstein’s answer is that there is only the inertial mass, which curves spacetime. Gravity as a force, doesn’t exist.

1.3 Tidal force as curvature

The principle of relativity you learned in Special Relativity says physics is the same in all inertial frames. Consider traveling in a moving train or plane. Drop a ball; it falls just like when standing on the ground. You can play catch or pour wine on a plane, even though for someone watching from outside the plane the ball or wine would travel in a parabola. The principle of relativity says one cannot tell whether or not one is moving in a frame with constant velocity (except by looking outside at someone else). So consider a mass floating in an orbiting rocket ship; not touching anything, just floating. Where does it get it marching orders from? Newton says both the mass and ship get their orders from the distant Earth. Einstein says the mass gets its orders locally. A free falling frame is a “local inertial frame” so since there is nothing inside the spaceship pushing on the mass, it stays still with respect to the spaceship. In fact, according to Einstein both the space ship and mass are sampling the local curvature of spacetime which is what is causing them to orbit. Things move in “straight lines” in inertial frames; the mass can veer, but only responding to structure of spacetime right there. Newton says the mass would go “straight” in his ideal all pervading reference frame but the Earth deflects it.

How do you tell if a frame is inertial? Easy, just check every particle, light ray, etc. to see if they move in straight lines at constant speed. So inside the space ship it is an inertial frame and everything moves simply. Simple? Too simple! Where is gravity at all? How do we see the curvature?

Fig. 2: Balls in a space ship

Consider two balls in a space ship. We put them side by side 25 m apart. If the space ship is in orbit, the balls just float there. They don’t move apart or together, and if there were no windows, there would be no way to tell they were in orbit above the Earth or in the middle of space far from any star or planet. Now, instead of in orbit, drop the entire space ship from a height of 250 meters above the earth. The ship and balls both fall straight down, and will hit the ground 7 seconds later ($t = \sqrt{2d/g}$). While falling, the balls still seem to be floating in deep space away from all forces. However, if you check carefully there is a small effect. Going straight down towards the Earth’s center, the balls are about 1×10^{-3} m closer together when they hit. $l = \theta r$, $dl/l = dr/r$, $dl = ldr/r = (25)(250)/6.4 \times 10^6 = 1 \times 10^{-3}$m. Watching this from the ground it is clear what is happening, but inside it seems as if the balls are attracting each other. After 7 seconds they have moved about 1 mm closer. This is not actually the gravity attraction between the two balls, but is the "tidal" force and in fact proves that the space is curved. Note that if your measuring instruments had an accuracy of worse than 1 mm, then this attraction could not be detected. We say that to an accuracy of 1 mm and a time under 7 seconds this 25 m wide space is a local inertial frame, but for longer times, or better accuracy, it is not. Smaller size ships and shorter times give more approximately inertial frames. However, if you add enough small frames together you can detect the curvature. Consider a ring of balls above the Earth's surface each separated by 25 m and drop them all together. After 7 seconds they are all 1 mm closer. In each frame you can't see it, but by adding up all the frames you see that entire circle around the Earth has shrunk. The factor is 1 mm/25000 mm = 1/25000, and the distance from the center of the Earth shrinks by same factor $(1/25000)(6.4 \times 10^6)m = 250m$. Note this is just like the distance around a line of latitude shrinks for surveyors rolling balls toward north pole. The smallness of this effect in a single spaceship actually shows the smallness of the curvature of spacetime, which is part of the reason GR is not easy to experimentally distinguish from Newton.