PHYSICS 225A
HOMEWORK 7

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DUE: Tuesday May 28, 2012

Do the following problem from Shutz Chapter 11: 7

1. Calculate the Schwarzchild radius and "density" for the following black hole (BH) masses. (density: $\rho = M/(\frac{4}{3}\pi R_{SC}^3)$)
(a) Primordial: $m = 2 \times 10^{15}$ grams
(b) Smallest real BH: $m = 8M_\odot$
(c) BH at center of Milky Way: $m = 4 \times 10^7 M_\odot$
(d) BH at center of M87 galaxy: $m = 6.6 \times 10^9 M_\odot$
Convert all densities to gm/cm^3, think of something that is the similar in size to each black hole and compare each density to the density of water.

2. Calculate the gravitational redshift of the Ly alpha line of Hydrogen (rest frame wavelength $\lambda_0 = 1215.67$ Angstroms) and also the wavelength as measured far away for photons emitted from:
(a) the surface of a neutron star
(b) the surface of the Sun
(c) the surface of the Earth
(d) 1.5 Schwarzchild radius from center of a black hole.
Guess as to whether each redshift is measurable or not by current spectrographs on current telescopes.

3. Standing on the roof of a building here in San Diego, you shine your green laser pointer onto a person standing 200 ft below you. Supposing its wavelength is exactly 5340 Angstroms when it leaves the laser, calculate the wavelength when it hits the person below.

4. A robot worker is constructing a shell at radius $r = 6M$, around a $6M_\odot$ black hole. It drops a wrench, which falls to the already completed floor 10 meters below. (You may assume 10 coordinate meters below) How fast is that wrench going when it hits the floor? Do the calcuation both for the shell worker frame and the Schwarzschild coordinate frame by performing the following steps.

a) Use the radial and time Schwarzschild geodesic equations to find the equation for radial infall for an object that starts at rest at $r = r_0$.

b) Define the "shell" frame as the frame that is static at a given r (i.e. the one we are in standing on the Earth surface). Switch from dt to dt_{shell} and from r to dr_{shell} to find: $v_{\text{shell}} = \frac{dr_{\text{shell}}}{dt_{\text{shell}}}$.

c) Evalute the above and express results results for v and v_{shell} in meters/second.

d) If the wrench weighs 5 kg, how many joules of energy are released when it hits the floor? Would another worker robot hit by that wrench survive?

e) Estimate the gravitational acceleration g in the shell frame using $g_{\text{shell}} \approx (v_{\text{shell}}(r_0) - v_{\text{shell}}(r))/\Delta t_{\text{shell}}$, and estimating the time for the wrench to fall by approximating the average speed of the wrench as one-half the speed when it hits the floor.

f) Give the answer above in "gee"s, that is, give your approximate acceleration in units of 9.8m/s^2. Could a person work in that environment? (Fighter pilots risk blacking out when they experience around 7 gee's.)