PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #4

(1) A strange material obeys the equation of state $E(S,V,N) = a\,S^7/V^4N^2$, where a is a dimensionful constant.

(a) What are the SI dimensions of a?

(b) Find the equation of state relating p, T, and $n = N/V$.

(c) Find the coefficient of thermal expansion $\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ and the isothermal compressibility $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$. Express your answers in terms of p and T.

(d) ν moles of this material execute a Carnot cycle between reservoirs at temperatures T_1 and T_2. Find the heat Q and work W for each leg of the cycle, and find the cycle efficiency η.

(2) The entropy of a thermodynamic system $S(E,V,N)$ is given by

$$S(E,V,N) = a\,E^\alpha\,V^\beta\,N^\gamma\ ,$$

where a is a dimensionful constant.

(a) Extensivity of S imposes a condition on (α,β,γ). Find this constraint.

(b) Even with the extensivity condition satisfied, the system may violate one or more stability criteria. Find the general conditions on (α,β,γ) which are thermodynamically permissible.

(3) For an ideal gas, find the difference $C_\varphi - C_V$ for the following functions φ. You are to assume N is fixed in each case.

(a) $\varphi(p,V) = p^3\,V^2$

(b) $\varphi(p,T) = p\,e^{T/T_0}$

(c) $\varphi(T,V) = VT^{-1}$

(4) Find an expression for the energy density $\varepsilon = E/V$ for a system obeying the Dieterici equation of state,

$$p(V - Nb) = Nk_{\rm B}T\,e^{-Na/Vk_{\rm B}T}\ ,$$

where a and b are constants. Your expression for $\varepsilon(v,T)$ should involve an integral which can be expressed in terms of the exponential integral,

$$\mathsf{Ei}(x) = \int\limits_{-\infty}^{x} dt\ \frac{e^t}{t}\ .$$