PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #2

(1) Compute the density of states $D(E, V, N)$ for a three-dimensional gas of particles with Hamiltonian $\hat{H} = \sum_{i=1}^{N} A\,|\boldsymbol{p}_i|^4$, where A is a constant. Find the entropy $S(E, V, N)$, the Helmholtz free energy $F(T, V, N)$, and the chemical potential $\mu(T, p)$.

(2) Consider a gas of classical spin-$\frac{3}{2}$ particles, with Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m} - \mu_0 H \sum_i S_i^z \ ,$$

where $S_i^z \in \left\{ -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2} \right\}$ and H is the external magnetic field. Find the Helmholtz free energy $F(T, V, H, N)$, the entropy $S(T, V, H, N)$, and the magnetic susceptibility $\chi(T, H, n)$, where $n = N/V$ is the number density.

(3) Compute the RMS volume fluctuations in the $T - p - N$ ensemble.

(4) For the system described in problem (1), compute the distribution of speeds $\bar{f}(v)$. Find the most probable speed, the mean speed, and the RMS speed.