PHYSICS 140A : STATISTICAL PHYSICS
MIDTERM EXAMINATION

(1) A particle has a g_0-fold degenerate ground state with energy $\varepsilon_0 = 0$ and a g_1-fold degenerate excited state with energy $\varepsilon_1 = \Delta$. A collection of N such particles is arranged on a lattice. Since each particle occupies a distinct position in space, the particles are regarded as distinguishable.

(a) Find the free energy $F(T, N)$.

(b) Find the entropy $S(T, N)$. Sketch $S(T, N)$ *versus* T for fixed N, taking care to evaluate the limiting values $S(T = 0, N)$ and $S(T = \infty, N)$.

Suppose now that the ground state is magnetic, such that in an external field H, the g_0 ground state energy levels are split into $g_0/2$ levels with energy $\varepsilon_{0,+} = +\mu_0 H$ and $g_0/2$ levels with energy $\varepsilon_{0,-} = -\mu_0 H$. (We take g_0 to be even in this case.) The states with energy $\varepsilon_1 = \Delta$ remain g_1-fold degenerate.

(c) Find $F(T, N, H)$.

(d) Find the sero field magnetic susceptibility,

$$\chi(T) = \frac{1}{N} \left(\frac{\partial M}{\partial H} \right)_{H=0} ,$$

where M is the magnetization.