PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #4

(1) Consider a d-dimensional ultrarelativistic gas of classical indistinguishable particles with a dispersion $\varepsilon(\boldsymbol{p}) = c\,|\boldsymbol{p}|$.

(a) Find an expression for the grand potential $\Omega(T, V, \mu)$.

(b) Find the average number of particles $N(T, V, \mu)$.

(c) Find the entropy $S(T, V, \mu)$.

(d) Express the RMS fluctuations in the number of particle number, $(\Delta N)_{\rm RMS}$, in terms of the volume V, temperature T, and the pressure p.

(2) Consider again the d-dimensional classical ultrarelativistic gas with $\varepsilon(\boldsymbol{p}) = cp$.

(a) If $d = 3$, find the momentum distribution function $g(\boldsymbol{p})$.

(b) Again for $d = 3$, find a general formula for the moments $\langle\, |\boldsymbol{p}|^k \,\rangle$.

(c) Repeat parts (a) and (b) for the case $d = 2$.

(d) In $d = 3$, what is the distribution function $f(\boldsymbol{v})$ for velocities?

(3) A classical gas of indistinguishable particles in three dimensions is described by the Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \Big\{ A\,|\boldsymbol{p}_i|^3 - \mu_0 H S_i \Big\}\ ,$$

where A is a constant, and where $S_i \in \{-1\,,\,0\,,\,+1\}$ (*i.e.* there are three possible spin polarization states).

(a) Compute the free energy $F_{\sf gas}(T, H, V, N)$.

(b) Compute the magnetization density $m_{\sf gas} = M_{\sf gas}/V$ as a function of temperature, pressure, and magnetic field.

The gas is placed in thermal contact with a surface containing $N_{\sf s}$ adsorption sites, each with adsorption energy $-\Delta$. The surface is metallic and shields the adsorbed particles from the magnetic field, so the field at the surface may be approximated by $H = 0$.

(c) Find the Landau free energy for the surface, $\Omega_{\sf surf}(T, N_{\sf s}, \mu)$.

(d) Find the fraction $f_0(T, \mu)$ of empty adsorption sites.

(e) Find the gas pressure $p^*(T, H)$ at which $f_0 = \frac{1}{2}$.