PHYS 4D
Solution to HW 7

February 21, 2011

Problem Giancoli 35-2 (I) Monochromatic light falls on a slit that is $2.60 \times 10^{-3} mm$ wide. If the angle between the first dark fringes on either side of the central maximum is 32.0° (dark fringe to dark fringe), what is the wavelength of the light used?

Solution: The angle from the central maximum to the first dark fringe is equal to half the width of the central maximum. Using the angle and Eq. 35-1, we calculate the wavelength used

$$\begin{aligned}\theta_1 &= \frac{1}{2}\Delta\theta = \frac{1}{2}32.0^\circ = 16.0^\circ,\\ \sin\theta_1 &= \frac{\lambda}{D} \Rightarrow \lambda = 717nm.\end{aligned}$$

Problem Giancoli 35-4 (II) Consider microwaves which are incident perpendicular to a metal plate which has a 1.6-cmslit in it. Discuss the angles at which there are diffraction minima for wave-lengths of (a) $0.50cm$, (b) $1.0cm$, and (c) $3.0cm$.

Solution: (a) Using Eq. 35-2, $m = 1, 2, 3, ...$ to calculate the possible diffraction minima, when the wavelength is $0.50cm$,

$$D\sin\theta_m = m\lambda \Rightarrow \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right). \Rightarrow \theta_1 = 18.2^\circ, \theta_2 = 38.7^\circ, \theta_3 = 69.6^\circ, \theta_4 = no\, solution.$$

θ_1, θ_2 and θ_3 give three diffraction minima.

(b) Using the new wavelength and repeat the above process, we find

$$\theta_1 = 38.7^\circ, \theta_2 = no\, solution.$$

θ_1 gives the only diffraction minima.

(c) Using the new wavelength and repeat the above process, we find

$$\theta_1 = no\, solution.$$

No diffraction minima.

Problem Giancoli 35-6 (II) Monochromatic light of wavelength $633nm$ falls on a slit. If the angle between the first bright fringes on either side of the central maximum is 35°, estimate the slit width. .

Solution: The angle from the central maximum to the first bright maximum is half the angle between the ffirst bright maxima on either side of the central maximum. The angle to the first maximum is about halfway between the angle to the first and second minima. We use Eq. 35-2, setting $m = \frac{3}{2}$, to calculate the slit width, D.

$$\begin{aligned}\theta_1 &= \frac{1}{2}\Delta\theta = \frac{1}{2}35.0^\circ = 17.5^\circ,\\ D &= \frac{m\lambda}{\sin\theta_1} = 3.2\mu m.\end{aligned}$$

Problem Giancoli 35-11 (II) Coherent light from a laser diode is emitted through a rectangular area $3.0\mu m \times 1.5\mu m$ (horizontal-by-vertical). If the laser light has a wavelength of $780nm$, determine the angle between the first diffraction minima (a) above and below the central maximum, (b) to the left and right of the central maximum.

Solution: (a) For vertical diffraction, we use the height of the slit ($1.5\mu m$) as the slit width in Eq. 35-1 to calculate the angle between the central maximum to the first minimum. The angular separation of the first minima is twice this angle.

$$\begin{aligned}\sin\theta_1 &= \frac{\lambda}{D} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = 31.3°. \\ \Delta\theta &= 2\theta_1 = 63°.\end{aligned}$$

(b) To find the horizontal diffraction, we use the width of the slit $3.0\mu m$.

$$\begin{aligned}\sin\theta_1 &= \frac{\lambda}{D} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = 15.07°. \\ \Delta\theta &= 2\theta_1 = 30°.\end{aligned}$$

Problem Giancoli 35-14 (III) (a) Explain why the secondary maxima in the single-slit diffraction pattern do not occur precisely at $\beta/2 = (m + \frac{1}{2})\pi$ where $m = 1, 2, 3, \cdots$. (b) By differentiating Eq. 35-7 with respect to β show that the secondary maxima occur when $\beta/2$ satisfies the relation $tan(\beta/2) = \beta/2$. (c) Carefully and precisely plot the curves $y = \beta/2$ and $y = tan\beta/2$. From their intersections, determine the values of β for the first and second secondary maxima. What is the percent difference from $\beta/2 = (m + \frac{1}{2})\pi$?

Solution: (a) The secondary maxima do not occur precisely where $\sin\beta/2$ is a maximum, that is at $\beta/2 = (m + \frac{1}{2})\pi$ where $m = 1, 2, 3, \cdots$, because the diffraction intensity (Eq.35-7) is the ratio of the sine function and $\beta/2$. Near the maximum of the sine function, the denominator of the intensity function causes the intensity to decrease more rapidly than the sine function causes it to increase. This results in the intensity reaching a maximum slightly before the sine function reaches its maximum.

(b) We set the derivative of Eq. 35-7 with respect to β equal to zero to determine the intensity extrema

$$0 = \frac{dI}{d\beta} = \frac{d}{d\beta} I_0 \left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0 \left(\frac{\sin(\beta/2)}{\beta/2}\right)\left(\frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2}\right)$$

When the first term in brackets is zero, the intensity is a minumum, so the intensity is a maximum when the second term in brackets is zero.

$$0 = \frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2} \Rightarrow \beta/2 = \tan(\beta/2).$$

(c) The first and secondary maxima are found where these two curves intersect, or $\beta_1 = 8.987$ and $\beta_2 = 15.451$. We calculate the percent difference between these and the maxima of the since curve, $\beta_1' = 3\pi$ and $\beta_2' = 5\pi$.

$$\begin{aligned}\frac{\Delta\beta}{\beta}|_1 &= \frac{\beta_1 - \beta_1'}{\beta_1'} = \frac{8.987 - 3\pi}{3\pi} = -4.64\%, \\ \frac{\Delta\beta}{\beta}|_2 &= -1.64\%.\end{aligned}$$

Problem Giancoli 35-31 (I) A source produces first-order lines when incident normally on a 12,000-line/cm diffraction grating at angles $28.8°, 36.7°, 38.6°$ and $47.9°$. What are the wavelengths?

Solution: We use Eq. 35-13 to calculate the wavelengths from the given angles. The slit separation, d, is the inverse of the number of lines per cm, N. We assume that $12,000$ is good to 3 significant figures

$$D\sin\theta = m\lambda \Rightarrow \lambda = \frac{D\sin\theta}{m} = \frac{\sin\theta}{Nm}.$$

$$\lambda_1 = 401nm, \lambda_2 = 498nm, \lambda_3 = 520nm, \lambda_4 = 618nm.$$

Problem Giancoli 35-33 (I) A grating has 6800 lines/ cm. How many spectral orders can be seen (400 to $700nm$) when it is illuminated by white light?

Solution: Because the angle increases with wavelength, to have a complete order we use the largest wavelength. We set the maximum angle is 90° to determine the largest integer m in Eq. 35-13.

$$d\sin\theta = m\lambda \Rightarrow m = \frac{\sin\theta}{\lambda N} = \frac{\sin 90°}{700nm \times 6800/cm} = 2.1.$$

Thus, two full spectral orders can be seen on each side of the central maximum, and a portion of the third order.

Problem Giancoli 35-37 (II) A diffraction grating has 6.0×10^5 lines/m. Find the angular spread in the second-order spectrum between red light of wavelength $7.0 \times 10^{-7}m$ and blue light of wavelength $4.5 \times 10^{-7}m$.

Solution: Using Eq.35-13, we find the second order angles for the maximum and minimum wavelength, where the slit separation distance is the inverse of the number of lines per cm. Subtracting these two angles gives the angular width.

$$\begin{aligned} d\sin\theta &= m\lambda \Rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N). \\ m &= 2, \\ \theta_1 &= \sin^{-1}\left(2 \times 7.0 \times 10^{-7}m \times 6.0 \times 10^5/m\right) = 57.1^\circ, \\ \theta_2 &= \sin^{-1}\left(2 \times 4.5 \times 10^{-7}m \times 6.0 \times 10^5/m\right) = 32.7^\circ. \\ \Delta\theta &= 24^\circ. \end{aligned}$$

Problem Giancoli 35-40 (II) Two first-order spectrum lines are measured by a 9650 line/cm spectroscope at angles, on each side of center, of $+26^\circ 38\prime, +41^\circ 02\prime$ and $-26^\circ 18\prime, -40^\circ 27\prime$. Calculate the wavelengths based on these data.

Solution: The diffraction angles is $\frac{1}{2}$ the difference between the angles on opposite sides of the center. Then we solve Eq.35-13 for the wavelength, with d equal to the inverse of the number of lines per cm.

$$\begin{aligned} \theta_1 &= \frac{\theta_r - \theta_l}{2} = \frac{26^\circ 38\prime - (-26^\circ 18\prime)}{2} = 26.47^\circ. \\ \lambda_1 &= d\sin\theta_1 = \frac{\sin\theta_1}{N} = \frac{\sin 26.47^\circ}{9650/cm} = 462nm. \end{aligned}$$

$$\begin{aligned} \theta_2 &= \frac{\theta_{2r} - \theta_{2l}}{2} = \frac{41^\circ 02\prime - (-40^\circ 27\prime)}{2} = 40.742^\circ. \\ \lambda_2 &= d\sin\theta_2 = \frac{\sin\theta_2}{N} = \frac{\sin 40.74^\circ}{9650/cm} = 676nm. \end{aligned}$$

Problem Giancoli 35-44 (II) **Missing orders** occur for a diffraction grating when a diffraction minimum coincides with an interference maximum. Let D be the width of each slit and d the separation of slits. (a) Show that if $d = 2D$, all even orders $(m = 2, 4, 6, ...)$ are missing. (b) Show there will be missing orders whenever

$$\frac{d}{D} = \frac{m_1}{m_2}$$

where m_1 and m_2 are integers. (c) Discuss the case $d = D$, the limit in which the space between slits becomes negligible.

Solution: (a) Missing orders occur when the angle to the interference maxima (Eq. 34-2a) is equal to the angle of a diffraction minimum (Eq. 35-2). We set $d = 2D$ and show that the even interference orders are missing.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \Rightarrow \frac{m_1}{m_2} = \frac{d}{D} = 2. \Rightarrow m_1 = 2m_2.$$

Since $m_1 = 1, 2, 3, 4, ...$ all even orders of m_1 correspond to the diffraction minima and will be missing from the interference pattern.

(b) Setting the angle of interference maxima equal to the angle of diffraction minimum, with the orders equal to integers we determine the relationship between the slit size and separation that will produce missing orders.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \Rightarrow \frac{d}{D} = \frac{m_1}{m_2}.$$

(c) When $d = D$, all interference maxima will overlap with diffraction minima so that no fringes will exist. This is expected because if the slit width and separation distance are the same, the slits will merge into one single opening.