PHYS 4D
Solution to HW 6

February 10, 2011

Problem Giancoli 34-4 (II) Monochromatic light falls on two very narrow slits $0.048mm$ apart. Successive fringes on a screen $6.00m$ away are $8.5cm$ apart near the center of the pattern. Determine the wavelength and frequency of the light.

Solution: For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = l\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$. Adjacent fringes will have $\Delta m = 1$.

$$
\begin{aligned}
d\frac{x}{l} &= d\sin\theta = m\lambda \Rightarrow x = \frac{\lambda m l}{d}. \\
x_1 &= \frac{\lambda m l}{d}, x_2 = \frac{\lambda(m+1)l}{d} \Rightarrow \Delta x = x_2 - x_1 = \frac{\lambda l}{d} \\
\lambda &= \frac{d\Delta x}{l} = \frac{(0.048mm) \times (8.5cm)}{6.00m} = 6.8 \times 10^{-7}m \\
f &= \frac{c}{\lambda} = \frac{3.0 \times 10^8 m/s}{6.8 \times 10^{-7}m} = 4.4 \times 10^{14} Hz
\end{aligned}
$$

Problem Giancoli 34-8 (II) Light of wavelength $680nm$ falls on two slits and produces an interference pattern in which the third-order bright fringe is $38mm$ from the central fringe on a screen $2.6m$ away. What is the separation of the two slits?

Solution:
$$
d\frac{x}{l} = m\lambda \Rightarrow d = \frac{\lambda m l}{x} = \frac{(680nm)3(2.6m)}{38mm} = 1.4 \times 10^{-4}m.
$$

Problem Giancoli 34-12 (II) In a double-slit experiment it is found that blue light of wavelength $480nm$ gives a second-order maximum at a certain location on the screen. What wavelength of visible light would have a minimum at the same location?

Solution: We equate the expression from Eq. 34-2a for the second order blue light to Eq. 34-2b, since the slit separation and angle must be the same for the two conditions to be met at the same location.

$$
\begin{aligned}
d\sin\theta &= m\lambda_b = 2(480nm) = 960nm; \\
d\sin\theta &= (m' + \frac{1}{2})\lambda, m' = 0, 1, 2, \cdots
\end{aligned}
$$

$$
\begin{aligned}
(m' + \frac{1}{2})\lambda &= 960nm \\
m' &= 0 \Rightarrow \lambda = 1920nm; \\
m' &= 1 \Rightarrow \lambda = 640nm; \\
m' &= 2 \Rightarrow \lambda = 384nm;
\end{aligned}
$$

The only one visible is $640nm$, $384nm$ is near the low wavelength limit for visible light.

Problem Giancoli 34-15 (II) Light of wavelength $470nm$ in air falls on two slits $6.00 \times 10^{-2}mm$ apart. The slits are immersed in water, as is a viewing screen $50.0cm$ away. How far apart are the fringes on the screen?

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Solution: In the water,

$$\lambda_n = \lambda/n.$$

$$d\frac{x}{l} = d\sin\theta = m\lambda_n \Rightarrow x = \frac{\lambda_n ml}{d};$$

$$x_1 = \frac{\lambda_n ml}{d}, x_2 = \frac{\lambda_n(m+1)l}{d} \Rightarrow \Delta x = x_2 - x_1 = \frac{\lambda_n l}{d} = \frac{\lambda l}{nd} = 2.94 \times 10^{-3}m.$$

Problem Giancoli 34-16 (II) A very thin sheet of plastic ($n = 1.60$) covers one slit of a double-slit apparatus illuminated by 680-nm light. The center point on the screen, instead of being a maximum, is dark. What is the (minimum) thickness of the plastic?

Solution: To change the center point from constructive interference to destructive interference, the phase shift produced by the introduction of the plastic must be equivalent to half a wavelength. The wavelength of the light is shorter in the plastic than in the air, so the number of wavelengths in the plastic must be 1/2 greater than the number in the same thickness of air. The number of wavelengths in the distance equal to the thickness of the plate is the thickness of the plate divided by the appropriate wavelength.

$$N_{plastic} - N_{air} = \frac{t}{\lambda_{plastic}} - \frac{t}{\lambda} = \frac{t}{\lambda}(n_{plastic} - 1) = \frac{1}{2}$$

$$\Rightarrow t = \frac{\lambda}{2(n_{plastic} - 1)} = 570nm.$$

Problem Giancoli 34-18 (II) Derive an expression similar to Eq. 34-2 which gives the angles for which the double-slit intensity is one-half its maximum value, $I_\theta = \frac{1}{2}I_0$.

Solution: The intensity as a function of angle from the central maximum is given by Eq. 34-6.

$$I_\theta = I_0 \cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) = \frac{1}{2}I_0$$

$$\Rightarrow \cos\left(\frac{\pi d\sin\theta}{\lambda}\right) = \pm\frac{1}{\sqrt{2}}.$$

$$\Rightarrow \frac{\pi d\sin\theta}{\lambda} = \cos^{-1}(\pm\frac{1}{\sqrt{2}}) = 45° \pm n(90°) = \frac{\pi}{4} \pm n\frac{\pi}{2}.$$

$$\Rightarrow 2d\sin\theta = (\frac{1}{2} \pm n)\lambda$$

For $\theta \geq 0$, we just take the plus sign,

$$2d\sin\theta = (\frac{1}{2} + n)\lambda, n = 0, 1, 2, \cdots.$$

Problem Giancoli 34-19 (II) Show that the angular full width at half maximum of the central peak in a double-slit interference pattern is given by $\Delta\theta = \lambda/2d$ if $\lambda \ll d$.

Solution: Similar to the above problem, we can get

$$\frac{\pi d\sin\theta_{1/2}}{\lambda} = \frac{\pi}{4}$$

And $\lambda \ll d$, then $\theta_{1/2} \approx \sin\theta_{1/2} = \frac{\lambda}{4d} \Rightarrow \Delta\theta = \lambda/2d$.

Problem Giancoli 34-27 (II) A thin film of oil ($n_o = 1.50$) with varying thickness floats on water ($n_w = 1.33$). When it is illuminated from above by white light, the reflected colors are as shown in Fig. 34-26. In air, the wavelength of yellow light is 580nm. (a) Why are there no reflected colors at point A? (b) What is the oil's thickness t at point B?

Solution:

(a) When illuminated from above at A, a light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected from the oil-air interface undergoes no phase shift. If the iol thickness at A is negligible compared to the wavelength of the light, then there is no significant shift in phase due to a path distance traveled by

a ray in the oil. Thus the light reflected from the two surfaces will distructively interfere for all visible wavelengths, and the oil will appear black when viewed from above.

(b) From the discussion in part (a), the ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected from the oil-air interface undergoes no phase shift due to the reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{oil}}\right) 2\pi$. For constructive interference, the net phase change must be a multiple of 2π.

$$\begin{aligned} \phi_{net} &= \phi_2 - \phi_1 = \left(\frac{2t}{\lambda_{oil}}\right) 2\pi - \pi = m(2\pi). \\ &\Rightarrow t = \frac{1}{2}(m + \frac{1}{2})\frac{\lambda}{n_o}. \end{aligned}$$

From the diagram, we see that point B is the second thickness that yields constructive interference for $580nm$, and so we use $m = 1$. (The first location that yields constructive interference would be for $m = 0$)

$$t = \frac{1}{2}(m + \frac{1}{2})\frac{\lambda}{n_o} = \frac{1}{2}(1 + \frac{1}{2})\frac{580nm}{1.50} = 290nm.$$

Problem Giancoli 34-40 (II) What is the wavelength of the light entering an interferometer if 384 bright fringes are counted when the movable mirror moves $0.125mm$?

Solution: From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so correspondings to a mirror motion of $\frac{1}{2}\lambda$. Let N be the number of fringe shifts produced by a mirror movement of Δx.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \Rightarrow \lambda = \frac{2\Delta x}{N} = \frac{2(0.125mm)}{384} = 651nm.$$

Problem Giancoli 34-41 (II) A micrometer is connected to the movable mirror of an interferometer. When the micrometer is tightened down on a thin metal foil, the net number of bright fringes that move, compared to the empty micrometer, is 272. What is the thickness of the foil? The wavelength of light used is $589nm$.

Solution: The thickness of the foil is the distance that the mirror moves during the 272 fringe shifts.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \Rightarrow \Delta x = \frac{1}{2}N\lambda = \frac{1}{2}272 \times 589nm = 8.01 \times 10^{-5}m.$$