PHYS 4D
Solution to HW 5

February 5, 2011

Problem Giancoli 32-43 (II) A light beam strikes a 2.0-cm-thick piece of plastic with a refractive index of 1.62 at a 45° angle. The plastic is on top of a 3.0-cm- thick piece of glass for which $n = 1.47$. What is the distance D in Fig. 32-48?

Solution: The beam forms the hypotenuse of two right trangles as it passes through the plastic and then the glass. The upper angle of the triangle is the angle of refraction in that medium. Note that the sum of the opposite sides is equal to the displacement D. First, we calculate the angles of refraction in each medium using Snell's Law (Eq. 32-5)

$$\sin 45° = n_1 \sin \theta_1 = n_2 \sin \theta_2.$$
$$\theta_1 = \sin^{-1}(\frac{\sin 45°}{n_1}) = \sin^{-1}(\frac{\sin 45°}{1.62}) = 25.88°,$$
$$\theta_2 = \sin^{-1}(\frac{\sin 45°}{n_2}) = \sin^{-1}(\frac{\sin 45°}{1.47}) = 28.75°.$$

We then use the trigonometric identity for tangent to calculate the two opposite sides, and sum to get the displacement.

$$D = D_1 + D_2 = h_1 \tan \theta_1 + h_2 \tan \theta_2 = 2.0cm \tan 25.88° + 3.0cm \tan 28.75° = 2.6cm.$$

Problem Giancoli 32-45 (II) In searching the bottom of a pool at night, a watchman shines a narrow beam of light from his flashlight, 1.3 m above the water level, onto the surface of the water at a point 2.5 m from his foot at the edge of the pool (Fig. 32-50). Where does the spot of light hit the bottom of the pool, measured from the bottom of the wall beneath his foot, if the pool is $2.1m$ deep?

Solution: We find the angle of incidence from the distances

$$\tan \theta_1 = \frac{l_1}{h_1} = \frac{2.5m}{1.3m} = 1.9231, \theta_1 = 62.53°.$$

For the refraction from air into water, we have

$$n_{air} \sin \theta_1 = n_{water} \sin \theta_2,$$
$$\sin \theta_2 = \frac{1.00}{1.33} \sin 62.53° \Rightarrow \theta_2 = 41.84°$$

We find the horizontal distance from the edge of the pool from

$$l = l_1 + l_2 = l_1 + h_2 \tan \theta_2$$
$$= 2.5m + 2.1m \tan 41.84° = 4.4m$$

Problem Giancoli 32-54 (II) A parallel beam of light containing two wavelengths, $\lambda_1 = 465nm$ and $\lambda_2 = 652nm$, enters the silicate flint glass of an equilateral prism as shown in Fig. 32-54. At what angle does each beam leave the prism (give angle with normal to the face)? See Fig. 32-28.

Solution: The indices of refraction are estimated from Fig. 32-28 as 1.64 for $465nm$ and 1.62 for $652nm$. Consider the refraction at the first surface.

$$n_{air} \sin \theta_a = n \sin \theta_b$$
$$1.0 \sin 45° = 1.64 \sin \theta_{b1} \Rightarrow \theta_{b1} = 25.5°,$$
$$1.0 \sin 45° = 1.62 \sin \theta_{b2} \Rightarrow \theta_{b2} = 25.9°$$

We find the angle of incidence at the second surface from the upper triangle.

$$(90° - \theta_b) + (90° - \theta_c) + A = 180° \Rightarrow \theta_{c1} = 34.5°, \theta_{c2} = 34.1°.$$

Apply Snell's Law at the second surface,

$$
\begin{aligned}
n \sin \theta_c &= n_{air} \sin \theta_d \\
1.64 \sin 34.5° &= 1.00 \sin \theta_{d1} \Rightarrow \theta_{d1} = 68.4°, \\
1.62 \sin 34.1° &= 1.00 \sin \theta_{d2} \Rightarrow \theta_{d2} = 65.2°.
\end{aligned}
$$

Problem Giancoli 32-59 (II) A beam of light is emitted in a pool of water from a depth of $72.0cm$. Where must it strike the air-water inter- face, relative to the spot directly above it, in order that the light does not exit the water?

Solution: We find the critical angle for light leaving the water:

$$n_{water} \sin \theta_1 = n_{air} \sin \theta_2 \Rightarrow \theta_1 = \sin^{-1}\left(\frac{n_{air}}{n_{water}}\right) = \sin^{-1}\frac{1.00}{1.33} = 48.75°$$

If the light is indicent at a greater angle than this, it will totally reflect. Find R from the diagram.

$$R > H \tan \theta_1 = (72.0cm) \tan 48.75° = 82.1cm.$$

Problem Giancoli 32-60 (II) A ray of light, after entering a light fiber, reflects at an angle of $14.5°$ with the long axis of the fiber, as in Fig. 32-56. Calculate the distance along the axis of the fiber that the light ray travels between successive reflections off the sides of the fiber. Assume that the fiber has an index of refraction of 1.55 and is $1.40 \times 10^{-4}m$ in diameter.

Solution: The ray reflects at the same angle, so each segment makes a $14.5°$ angle with the side. We find the distance l between reflections from the definition of the tangent function.

$$\tan \theta = \frac{d}{l} \Rightarrow l = \frac{d}{\tan \theta} = \frac{1.40 \times 10^{-4}m}{\tan 14.5°} = 5.41 \times 10^{-4}m.$$

Problem Giancoli 33-5 (II) A 105-mm-focal-length lens is used to focus an image on the sensor of a camera. The maximum distance allowed between the lens and the sensor plane is $132mm$. (a) How far ahead of the sensor should the lens be if the object to be photographed is $10.0m$ away?

Solution: We find the image distance from Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{(10.0m)(0.105m)}{10.0m - 0.105m} = 0.106m.$$

(b) $3.0m$ away?

Solution:

$$d_i = \frac{d_o f}{d_o - f} = \frac{(3.0m)(0.105m)}{3.0m - 0.105m} = 0.109m.$$

(c) $1.0m$ away?

Solution:

$$d_i = \frac{d_o f}{d_o - f} = \frac{(1.0m)(0.105m)}{1.0m - 0.105m} = 0.117m.$$

(d) What is the closest object this lens could photograph sharply?

Solution:

$$\frac{1}{d_{o_{min}}} + \frac{1}{d_{i_{max}}} = \frac{1}{f} \Rightarrow d_{o_{max}} = \frac{d_{i_{max}} f}{d_{i_{max}} - f} = \frac{(0.132m)(0.105m)}{0.132m - 0.105m} = 0.513m$$

Problem Giancoli 33-9 (II) An object is located $1.50m$ from an 8.0-D lens. By how much does the image move if the object is moved (a) $0.90m$ closer to the lens, and (b) $0.90m$ farther from the lens?

Solution: First, find the original image distance from Eqs. 33-1 and 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P \Rightarrow d_i = \frac{d_o}{P d_o - 1} = \frac{1.50m}{(8.0D) \times 1.50m - 1} = 0.1364m$$

a) With $d_o = 0.6m$, the new image distance

$$d_i = \frac{d_o}{Pd_o - 1} = \frac{0.60m}{(8.0D) \times 0.60m - 1} = 0.1579m$$

The image moved $0.02m$ away from the lens.

b) With $d_o = 2.4m$, the new image distance

$$d_i = \frac{d_o}{Pd_o - 1} = \frac{2.40m}{(8.0D) \times 2.40m - 1} = 0.1319m$$

The image moved $0.004m$ toward the lens.

Problem Giancoli 33-12 (II) (a) A 2.80-cm-high insect is $1.30m$ from a 135-mm-focal-length lens. Where is the image, how high is it, and what type is it? (b) What if $f = -135mm$?

Solution:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{(1.30m)(0.135m)}{1.30m - 0.135m} = 0.151m,$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \Rightarrow h_i = -\frac{d_i}{d_o} h_o = -\frac{0.151m}{1.30m} 2.80cm = -0.324m.$$

The image is behind the lens a distance $0.151m$, is real and inverted.

Problem Giancoli 33-15 (II) Show analytically that the image formed by a converging lens (a) is real and inverted if the object is beyond the focal point $(d_o > f)$, and

Solution:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = \frac{d_o f}{d_o - f},$$

$$m = -\frac{d_i}{d_o} = -\frac{f}{d_o - f}$$

So when $d_o > f$, the image distance is positive, producing a real image, and the magnification is negative, which gives an inverted image.

(b) is virtual and upright if the object is within the focal point $(d_o < f)$.

Solution: From the above equations, when $d_o < f$, the image distance is negative, producing a virtual image, and the magnification is positive, which gives an upright image.

(c) Next, describe the image if the object is itself an image (formed by another lens), and its position is on the opposite side of the lens from the incoming light, for $-d_o > f$, and

Solution: We set $-d_0 = f$ and calculate the limiting image distance and magnification.

$$d_i = \frac{(-f) f}{(-f) - f} = \frac{f}{2}, m = -\frac{d_i}{d_o} = -\frac{f}{(-f) - f} = \frac{1}{2}.$$

We also take the limit of large negative object distance

$$d_i = \frac{(-\infty) f}{(-\infty) - f} = f, m = -\frac{d_i}{d_o} = -\frac{f}{(-\infty) - f} = 0.$$

For these limiting cases, we see that when $-d_o > f$, the image is real and upright with $\frac{1}{2}f < d_i < f$ and $0 < m < \frac{1}{2}$.

(d) for $0 < -d_o < f$.

Solution: We take the limiting condition $d_o \to 0$, and determine the resulting image distance and magnification

$$d_i = \frac{(0) f}{(0) - f} = 0, m = -\frac{d_i}{d_o} = -\frac{f}{(0) - f} = 1.$$

From this limit and that found in part (c), we see that when $0 < -d_o < f$, the image is real and upright, with $0 < d_i < \frac{1}{2}f$ and $\frac{1}{2} < m < 1$.

Problem Giancoli 33-42 (II) Reading glasses of what power are needed for a person whose near point is $105cm$, so that he can read a computer screen at $55cm$? Assume a lens-eye distance of $1.8cm$.

Solution: The screen placed $55cm$ from the eye, or $53.2cm$ from the lens, is to produce a virtual image $105cm$ from the eye, or $103.2cm$ from the lens. Find the power of the lens from Eqs 33-1 and 33-2.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.532m} + \frac{1}{-1.032m} = 0.91D.$$

Problem Giancoli 33-49 (II) A nearsighted person has near and far points of 10.6 and $20.0cm$ respectively. If she puts on contact lenses with power $P = -4.00D$, what are her new near and far points?

Solution: Find the object distance for the contanct lens to form an image at the eye's near point, using Eqs. 33-2 and 33-1.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P \Rightarrow d_o = \frac{d_i}{Pd_i - 1} = \frac{-0.106m}{(-4.00D) \times (-0.106m) - 1} = 0.184m$$

Likewise find the object distance for the contact lens to form an image at the eye's far point.

$$d_o = \frac{d_i}{Pd_i - 1} = \frac{-0.200m}{(-4.00D) \times (-0.200m) - 1} = 1.00m$$

Problem Giancoli 33-61 (1) The overall magnification of an astronomical telescope is desired to be $35\times$. If an objective of $88cm$ focal length is used, what must be the focal length of the eyepiece? What is the overall length of the telescope when adjusted for use by the relaxed eye?

Solution: We find the focal length of the eyepiece from the magnification by Eq. 33-7

$$M = -\frac{f_o}{f_e} \Rightarrow f_e = -\frac{f_o}{M} = -\frac{88cm}{35} = 2.5cm.$$

For both object and image far away, the separation of the lenses is the sum of the focal lengths.

$$f_o + f_e = 88cm + 2.5cm = 90.5cm.$$