PHYS 4D
Solution to HW 4

January 28, 2011

Problem Giancoli 32-1 (I) When you look at yourself in a 60-cm-tall plane mirror, you see the same amount of your body whether you are close to the mirror or far away. (Try it and see.) Use ray diagrams to show why this should be true.

Solution: See Fig.(1). Because the angle of incidence must equal the angle of reflection, we see from the ray diagrams that the ray that reflects to your eye must be as far below the horizontal line to the reflection point on the mirror as the top is above the line, regardless of your position.

Problem Giancoli 32-3 (II) Two plane mirrors meet at a 135° angle, Fig. 32-45. If light rays strike one mirror at 38° as shown, at what angle ϕ do they leave the second mirror?

Solution: See Fig.(2). The law of reflection can be applied twice. $\theta = 38°$, $\alpha = 135°$,

$$\theta + \alpha + \phi = 180° \Rightarrow \phi = 7°.$$

Problem Giancoli 32-5 (II) Show that if two plane mirrors meet at an angle ϕ, a single ray reflected successively from both mirrors is deflected through an angle of 2ϕ independent of the incident angle. Assume $\phi < 90°$ and that only two reflections, one from each mirror, take place.

Solution: See Fig.(3). The incoming ray is represented by line segment DA. For triangle ABC,

$$\phi + (90° - \theta_1) + (90° - \theta_2) = 180° \Rightarrow \phi = \theta_1 + \theta_2.$$

For triangle ABD,

$$\alpha + 2\theta_1 + 2\theta_2 = 180° \Rightarrow \alpha = 180° - 2\phi.$$

At point D, we see that the deflection is

$$\beta = 180° - \alpha = 2\phi.$$

Problem Giancoli 32-7 (II) Stand up two plane mirrors so they form a 90° angle as in Fig. 32-47. When you look into this double mirror, you see yourself as others see you, instead of reversed as in a single mirror. Make a ray diagram to show how this occurs.

Solution: See Fig.(4)

Problem Giancoli 32-11 (I) When walking toward a concave mirror you notice that the image flips at a distance of $0.50m$. What is the radius of curvature of the mirror?

Solution: The image flips at the focal point, which is half the radius of curvature. Thus the radius is $1.0m$.
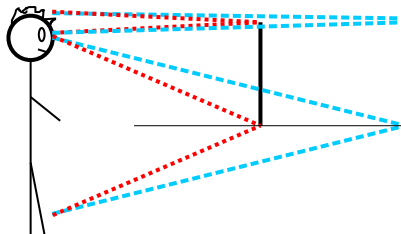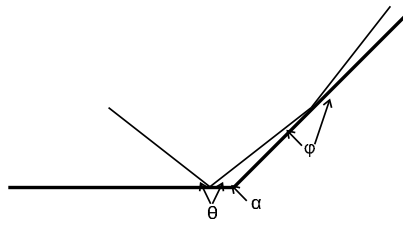

Figure 1: Problem 32-1

Figure 2: Problem 32-3
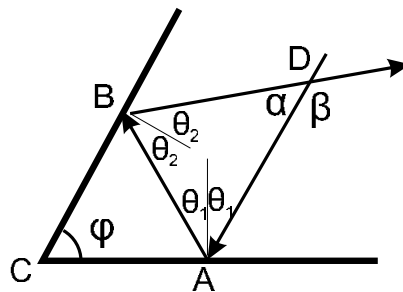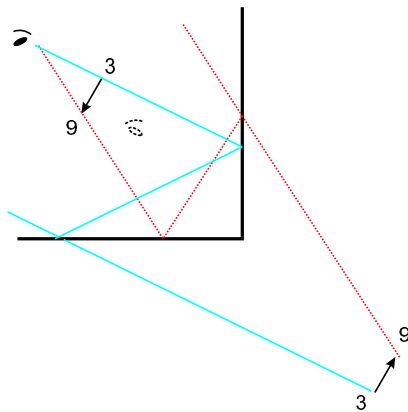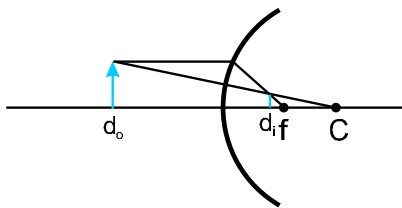

Figure 3: Problem 32-5

Figure 4: Problem 32-7

Figure 5: Problem 32-18

Problem Giancoli 32-12 (II) A small candle is $35cm$ from a concave mirror having a radius of curvature of $24cm$.

(a) What is the focal length of the mirror?

Solution: The focal length is half the radius of curvature, so

$$f = \frac{1}{2}r = \frac{1}{2}24cm = 12cm.$$

(b) Where will the image of the candle be located?

Solution: Use Eq. 32-2,

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{35cm \times 12cm}{35cm - 12cm} = 18.3cm$$

(c) Will the image be upright or inverted?

Solution: The image will be inverted since the magnification $m = -\frac{d_i}{d_o}$ is negative.

Problem Giancoli 32-13 (II) You look at yourself in a shiny 9.2-cm-diameter Christmas tree ball. If your face is $25.0cm$ away from the ball's front surface, where is your image? Is it real or virtual? Is it upright or inverted?

Solution: The ball is a convex mirror with a focal length

$$f = \frac{1}{2}r = \frac{1}{2}(-4.6cm) = -2.3cm,$$

$$d_i = \frac{d_o f}{d_o - f} = \frac{25.0cm \times (-2.3cm)}{25.0cm - (-2.3cm)} = -2.1cm$$

$$m = -\frac{d_i}{d_o} = \frac{2.1cm}{25.0cm} = 0.084.$$

My image is $2.1cm$ behind the surface of the ball, virtual and upright.

Problem Giancoli 32-15 (II) A dentist wants a small mirror that, when $2.00cm$ from a tooth, will produce a $4.0\times$ upright image. What kind of mirror must be used and what must its radius of curvature be?

Solution: $d_o = 2.00cm$, $m = 4.0$

$$\begin{aligned}
d_i &= -md_o, \\
f &= \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{-md_o}{1-m} = \frac{-4 \times 2.00}{1-4}cm = 2.67cm \\
r &= 2f = 5.3cm.
\end{aligned}$$

Because the focal length is positive, the mirror is concave.

Problem Giancoli 32-18 (II) An object $3.0mm$ high is placed $18cm$ from a convex mirror of radius of curvature $18cm$.

(a) Show by ray tracing that the image is virtual, and estimate the image distance.

Solution: See Fig.(5).The image is virtual. $d_i = -6cm$.

(b) Show that the (negative) image distance can be computed from Eq. 32-2 using a focal length of $-9.0cm$.

Solution:
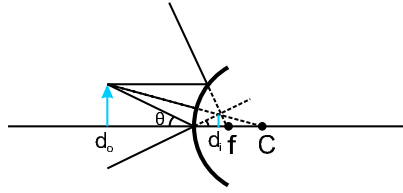$$d_i = \frac{d_o f}{d_o - f} = \frac{(18cm)(-9cm)}{(18cm)-(-9cm)} = -6.0cm$$

Figure 6: Problem 32-21

(c) Compute the image size, using Eq. 32-3.
Solution:

$$m = -\frac{d_i}{d_o},$$

$$h_i = mh_o = -\frac{d_i}{d_o}h_o = -\frac{-6.0cm}{18cm} \times 3.0mm = 1.0mm.$$

Problem Giancoli 32-21 (II) Show, using a ray diagram, that the magnification m of a convex mirror is $m = -d_i/d_o$, just as for a concave mirror. [Hint: Consider a ray from the top of the object that reflects at the center of the mirror.]

Solution: See Fig.(6). Consider the ray reflects from the center of the mirror, and note that $d_i < 0$.

$$\tan\theta = \frac{h_o}{d_o} = \frac{h_i}{-d_i} \Rightarrow m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$