PHYS 4D
Solution to HW 3

January 20, 2011

Problem Giancoli 31-34 (II) Estimate the radiation pressure due to a 75-W bulb at a distance of 8.0 cm from the center of the bulb. Estimate the force exerted on your fingertip if you place it at this point.

Solution: Using Eq. 31-21a,

$$P = \frac{\overline{S}}{c} = \frac{\frac{P}{4\pi d^2}}{c} = \frac{\frac{75W}{4\pi(8.0\times10^{-2}m)^2}}{3.00\times10^8 m/s} = 3.1\times10^{-6}N/m^2.$$

The force is pressure times area. We approximate the area of a fingertip to be $1.0cm^2$,

$$F = PA = \left(3.1\times10^{-6}N/m^2\right)\left(1.0\times10^{-4}m^2\right) = 3.1\times10^{-10}N.$$

Problem Giancoli 31-35 (II) Laser light can be focused (at best) to a spot with a radius r equal to its wavelength λ. Suppose that a 1.0- W beam of green laser light ($\lambda = 5\times10^{-7}m$) is used to form such a spot and that a cylindrical particle of about that size (let the radius and height equal r) is illuminated by the laser as shown in Fig. 31-23. Estimate the acceleration of the particle, if its density equals that of water and it absorbs the radiation. [This order-of-magnitude calculation convinced researchers of the feasibility of "optical tweezers," p. 829.]

Solution:

$$\overline{S}A = \frac{dU}{dt} = 1.0W.$$

$$a = \frac{F}{m} = \frac{PA}{m} = \frac{\frac{\overline{S}}{c}A}{m} = \frac{\frac{1}{c}\frac{dU}{dt}}{\rho_{H_2O}\pi r^2 r} = 8\times10^6 m/s^2.$$

Problem Giancoli 31-37 What size should the solar panel on a satellite orbiting Jupiter be if it is to collect the same amount of radiation from the Sun as a 1.0-m^2 solar panel on a satellite orbiting Earth?

Solution: The intensity from a point source is inversely proportional to the distance from the source,

$$\frac{I_{Earth}}{I_{Jupiter}} = \frac{r^2_{Sun-Jupiter}}{r^2_{Sun-Earth}} = \frac{\left(7.78\times10^{11}m\right)^2}{\left(1.496\times10^{11}m\right)^2} = 27.0$$

So it would take an area of $27m^2$ at Jupiter to collect the same radiation as a $1.0m^2$ solar panel at the Earth.

Problem Giancoli 31-38 (1) What is the range of wavelengths for (a) FM radio (88MHz to 108MHz) and (b) AM radio (535kHz to 1700kHz)?

Solution:

(a) For FM radio we have the following

$$\lambda_{\min} = \frac{c}{f} = \frac{\left(3.0\times10^8 m/s\right)}{\left(1.08\times10^8 Hz\right)} = 2.78m, \lambda_{\max} = \frac{c}{f} = \frac{\left(3.0\times10^8 m/s\right)}{\left(8.8\times10^7 Hz\right)} = 3.41m.$$

(b) For AM radio we have the following

$$\lambda_{\min} = \frac{c}{f} = \frac{\left(3.0\times10^8 m/s\right)}{\left(1.7\times10^6 Hz\right)} = 180m, \lambda_{\max} = \frac{c}{f} = \frac{\left(3.0\times10^8 m/s\right)}{\left(5.35\times10^5 Hz\right)} = 561m.$$

Problem Giancoli 31-39 (I) Estimate the wavelength for 1.9-GHz cell phone reception.
Solution:

$$\lambda = \frac{c}{f} = \frac{\left(3.0 \times 10^8 m/s\right)}{\left(1.9 \times 10^9 Hz\right)} = 0.16m.$$

Problem Giancoli 31-42 (II) A satellite beams microwave radiation with a power of 12 kW toward the Earth's surface, 550 km away. When the beam strikes Earth, its circular diameter is about 1500 m. Find the rms electric field strength of the beam at the surface of the Earth.
Solution:

$$\begin{aligned} S &= \frac{P}{A} = c\varepsilon_0 E_{rms}^2 \rightarrow \\ E_{rms} &= \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{1.2 \times 10^4 W}{\pi \left(750m\right)^2 \left(3.0 \times 10^8 m/s\right) \left(8.85 \times 10^{-12} C^2/N \cdot m^2\right)}} = 1.6V/m. \end{aligned}$$

Problem Giancoli 31-55 A point source emits light energy uniformly in all directions at an average rate P_0 with a single frequency f. Show that the peak electric field in the wave is given by

$$E_0 = \sqrt{\frac{\mu_0 c P_0}{2\pi r^2}}.$$

Solution:

$$\begin{aligned} S &= \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \frac{1}{2} c\varepsilon_0 E_0^2 = \frac{1}{2} c \left(\frac{1}{c^2 \mu_0}\right) E_0^2 \\ &\rightarrow \frac{1}{2} c \left(\frac{1}{c^2 \mu_0}\right) E_0^2 = \frac{P_0}{4\pi r^2} \\ &\rightarrow E_0 = \sqrt{\frac{P_0 c \mu_0}{2\pi r^2}}. \end{aligned}$$

Problem Giancoli 31-56 Suppose a 35-kW radio station emits EM waves uniformly in all directions. (a) How much energy per second crosses a 1.0-m 2 area 1.0 km from the transmitting antenna?
Solution: (a) The power crossing a given area is the intensity times the area. The intensity is the total power through the area of a sphere centered at the source,

$$P = IA = \frac{P_0}{A_{total}} A = \frac{35000W}{4\pi \left(1.0 \times 10^3 m\right)^2} \left(1.0m\right)^2 = 2.8mW.$$

(b) What is the rms magnitude of the E field at this point, assuming the station is operating at full power? What is the rms voltage induced in a 1.0-m-long vertical car antenna (c) $r_1 = 1.0$ km away, (d) $r_2 = 50$ km away?
Solution: (b) We find the rms value of the electric field from the intensity, which is the magnitude of the Poynting vector.

$$\begin{aligned} S &= c\varepsilon_0 E_{rms}^2 = \frac{P}{4\pi r^2} \\ &\rightarrow E_{rms} = \sqrt{\frac{P}{4\pi r_1^2 c\varepsilon_0}} = 1.024V/m. \end{aligned}$$

(c) The voltage over the length of the antenna is the electric field times the length of the antenna.

$$V_{rms} = E_{rms} d = \left(1.024V/m\right)\left(1.0m\right) = 1.0V.$$

(d) We calculate the electric field at the new distance, and then calculate the voltage

$$E_{rms} = \sqrt{\frac{P}{4\pi r_2^2 c\varepsilon_0}} = 2.048 \times 10^{-2} V/m$$

$$V_{rms} = E_{rms}d = \left(2.048 \times 10^{-2} V/m\right)(1.0m) = 2.0 \times 10^{-2} V.$$

Problem Giancoli 31-58 In free space ("vacuum"), where the net charge and current flow is zero, the speed of an EM wave is given by $v = 1/\sqrt{\varepsilon_0 \mu_0}$. If, instead, an EM wave travels in a nonconducting ("dielectric") material with dielectric constant K, then $v = 1/\sqrt{K\varepsilon_0 \mu_0}$. For frequencies corresponding to the visible spectrum (near 5×10^{14} Hz), the dielectric constant of water is 1.77. Predict the speed of light in water and compare this value (as a percentage) with the speed of light in a vacuum.

Solution: The speed of light in water is

$$\begin{aligned} v_{water} &= \frac{1}{\sqrt{K\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}c = \frac{1}{\sqrt{1.77}}\left(3.0 \times 10^8 m/s\right) = 2.25 \times 10^8 m/s \\ \frac{v_{water}}{c} &= \frac{1}{\sqrt{K}} = \frac{1}{\sqrt{1.77}} = 75.2\%. \end{aligned}$$