PHYS 4D
Solution to HW 2

January 15, 2011

Problem Giancoli 31-22 (I) The **E** field in an EM wave has a peak of $26.5mV/m$. What is the average rate at which this wave carries energy across unit area per unit time?

Solution: The average rate at which this wave carries energy across unit area per unit time is the average magnitude of the Poynting vector.

$$\bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\left(8.85 \times 10^{-12} C^2/N \cdot m^2\right)\left(3 \times 10^8 m/s\right)(0.0265V/m) = 9.32 \times 10^{-7} W/m^2.$$

Problem Giancoli 31-24 (II) How much energy is transported across a $1.00cm^2$ area per hour by an EM wave whose E field has an rms strength of $32.8mV/m$?

Solution: The energy transported across unit area per unit time is the magnitude of the Poynting vector.

$$\begin{aligned} S &= c\varepsilon_0 E_{rms}^2 = \frac{\Delta U}{A\Delta t} \\ \frac{\Delta U}{\Delta t} &= c\varepsilon_0 E_{rms}^2 A \\ &= \left(3 \times 10^8 m/s\right)\left(8.85 \times 10^{-12} C^2/N \cdot m^2\right)(0.0328V/m)^2 \left(1 \times 10^{-4} m^2\right)(3600s/h) \\ &= 1.03 \times 10^{-6} J/h. \end{aligned}$$

Problem Giancoli 31-26 (II) If the amplitude of the B field of an EM wave is $2.5 \times 10^{-7} T$,
(a) what is the amplitude of the E field?
Solution:

$$E_0 = cB_0 = \left(3 \times 10^8 m/s\right) \times 2.5 \times 10^{-7} T = 75V/m$$

(b) What is the average power per unit area of the EM wave?
Solution: The average power per unit area is given by the Poynting vector

$$I = \frac{E_0 B_0}{2\mu_0} = \frac{75V/m \times 2.5 \times 10^{-7} T}{2 \times (4\pi \times 10^{-7} Ns^2/C^2)} = 7.5W/m^2.$$

Problem Giancoli 31-28 (II) A $15.8 - mW$ laser puts out a narrow beam $2.00mm$ in diameter. What are the rms values of E and B in the beam?

Solution: The power output per unit area is the intensity and also is the magnitude of the Poynting vector. Use Eq. 31-19a with rms values

$$\begin{aligned} S &= \frac{P}{A} = c\varepsilon_0 E_{rms}^2 \Rightarrow \\ E_{rms} &= \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{0.0158W}{\pi\left(10^{-3} m\right)^2 (3 \times 10^8 m/s)(8.85 \times 10^{-12} C^2/N \cdot m^2)}} \\ &= 1376.3V/m. \\ B_{rms} &= \frac{E_{rms}}{c} = \frac{1376.3V/m}{(3 \times 10^8 m/s)} = 4.59 \times 10^{-6} T. \end{aligned}$$

Problem Giancoli 31-29 Estimate the average power output of the Sun, given that about $1350W/m^2$ reaches the upper atmosphere of the Earth.

Solution: The radiation from the Sun has the same intensity in all directions, so the rate at which it reaches the Earth is the rate at which it passes through a sphere centered at the Sun with a radius equal to the Earth's orbit radius. The $1350W/m^2$ is the intensity, or the magnitude of the Poynting vector.

$$S = \frac{P}{A} \Rightarrow P = SA = 4\pi \left(1.496 \times 10^{11} m\right)^2 \left(1350W/m^2\right) = 3.80 \times 10^{26} W.$$

Problem Giancoli 31-31 (II) How practical is solar power for various devices? Assume that on a sunny day, sunlight has an intensity of $1000W/m^2$ at the surface of Earth and that, when illuminated by that sunlight, a solar-cell panel can convert 10% of the sunlight's energy into electric power. For each device given below, calculate the area A of solar panel needed to power it.

(a) A calculator consumes $50mW$. Find A in cm^2. Is A small enough so that the solar panel can be mounted directly on the calculator that it is powering?

Solution:

$$A = \frac{P}{l} = \frac{50 \times 10^{-3} W}{100W/m^2} = 5 \times 10^{-4} m^2$$

So yes.

(b) A hair dryer consumes $1500W$. Find A in m^2. Assuming no other electronic devices are operating within a house at the same time, is A small enough so that the hair dryer can be powered by a solar panel mounted on the house's roof?

Solution:

$$A = \frac{P}{l} = \frac{1500W}{100W/m^2} = 15m^2$$

So yes.

(c) A car requires $20hp$ for highway driving at constant velocity (this car would perform poorly in situations requiring acceleration). Find A in m^2. Is A small enough so that this solar panel can be mounted directly on the car and power it in "real time"?

Solution:

$$A = \frac{P}{l} = \frac{20hp\,(746W/hp)}{100W/m^2} = 149m^2$$

This would require a square panel of side length about $12m$. So no.

Problem Giancoli 31-32 (III)

(a) Show that the Poynting vector $\mathbf{S}$ points radially inward toward the center of a circular parallel-plate capacitor when it is being charged as in Example 31-1.

Solution: In a cylindrical coordinate, $(\hat{\mathbf{r}}, \hat{\theta}, \hat{\mathbf{z}})$ are orthogonal coordinates. If we denote $\hat{\mathbf{e}}_1 = \hat{\mathbf{r}}, \hat{\mathbf{e}}_2 = \hat{\theta}, \hat{\mathbf{e}}_3 = \hat{\mathbf{z}}$, they satisfy $\hat{\mathbf{e}}_k = \varepsilon_{ijk}\hat{\mathbf{e}}_i \times \hat{\mathbf{e}}_j$. Now, $\hat{\mathbf{E}} = \hat{\mathbf{z}}, \hat{\mathbf{B}} = \hat{\theta}$. So $\hat{\mathbf{S}} = -\hat{\mathbf{r}}$, that is $\mathbf{S}$ points radially inward toward the center of a circular parallel-plate capacitor.

(b) Integrate $\mathbf{S}$ over the cylindrical boundary of the capacitor gap to show that the rate at which energy enters the capacitor is equal to the rate at which electrostatic energy is being stored in the electric field of the capacitor (Section 24-4). Ignore fringing of $\mathbf{E}$.

Solution: We evaluate the Poynting vector, and then integrate it over the curved cylindrical surface between the capacitor plates. The magnetic field (from Example 31-1) is

$$B = \frac{1}{2}\mu_0\varepsilon_0 r_0 \frac{dE}{dt}|_{r=r_0},$$

$\mathbf{E} \perp \mathbf{B}$, so

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B} = (-\hat{\mathbf{r}})\,\frac{1}{2}\varepsilon_0 r_0 E \frac{dE}{dt}|_{r=r_0}$$

In calculating $\int\int \mathbf{S}\cdot d\mathbf{A}$ for energy flow into the capacitor volume, note that both $\mathbf{S}$ and $d\mathbf{A}$ pointing inward, and that S is constant over the surface,

$$\begin{aligned}\int\int \mathbf{S}\cdot d\mathbf{A} &= \int\int S dA = S\int\int dA = S2\pi r_0 d \\ &= \frac{1}{2}\varepsilon_0 r_0 E\frac{dE}{dt}|_{r=r_0} 2\pi r_0 d = \varepsilon_0 \pi d r_0^2 E\frac{dE}{dt}|_{r=r_0}\end{aligned}$$

The energy stored in the capacitor $U =$ the energy density $u\times$ the volume of the capacitor V. By Eq. 24-6

$$\begin{aligned}U &= u\cdot V = \left(\frac{1}{2}\varepsilon_0 E^2\right)\left(d\pi r_0^2\right). \\ \frac{dU}{dt} &= \varepsilon_0 E d\pi r_0^2\frac{dE}{dt}.\end{aligned}$$

Thus, we have

$$\int\int \mathbf{S}\cdot d\mathbf{A} = \frac{dU}{dt}.$$

Problem Giancoli 31-43 A 1.60-m-long FM antenna is oriented parallel to the electric field of an EM wave. How large must the electric field be to produce a 1.00-mV (rms) voltage between the ends of the antenna? What is the rate of energy transport per square meter?

Solution: The electric field is found from the desired voltage and the length of the antenna. Then use that electric field to calculate the magnitude of the Poynting vector.

$$E_{rms} = \frac{V_{rms}}{d} = \frac{10^{-3}V}{1.60m} = 6.25\times 10^{-4}V/m.$$

$$\begin{aligned}S &= c\varepsilon_0 E_{rms}^2 = c\varepsilon_0\frac{V_{rms}^2}{d^2} \\ &= \left(3\times 10^8 m/s\right)\left(8.85\times 10^{-12}C^2/N\cdot m^2\right)\left(6.25\times 10^{-4}V/m\right)^2 \\ &= 1.04\times 10^{-9}W/m^2.\end{aligned}$$

Problem Giancoli 31-48 Cosmic microwave background radiation fills all space with an average energy density of $4\times 10^{-14}J/m^3$.

(a) Find the rms value of the electric field associated with this radiation.

Solution: The rms value of the electric field associated with this radiation is found from Eq. 24-6

$$\begin{aligned}u &= \frac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E_{rms}^2 \\ E_{rms} &= \sqrt{\frac{u}{\varepsilon_0}} = \sqrt{\frac{4\times 10^{-14}J/m^3}{8.85\times 10^{-12}C^2/N\cdot m^2}} = 0.07V/m.\end{aligned}$$

(b) How far from a 7.5-kW radio transmitter emitting uniformly in all directions would you find a comparable value?

Solution: A comparable value can be found using the magnitude of the Poynting vector

$$\begin{aligned}\bar{S} &= c\varepsilon_0 E_{rms}^2 = \frac{P}{4\pi r^2}\Rightarrow \\ r &= \frac{1}{E_{rms}}\sqrt{\frac{P}{4\pi c\varepsilon_0}} = \frac{1}{0.07V/m}\sqrt{\frac{7500W}{4\pi\left(3\times 10^8 m/s\right)\left(8.85\times 10^{-12}C^2/N\cdot m^2\right)}} = 7km.\end{aligned}$$

Problem Giancoli 31-52 How large an emf (rms) will be generated in an antenna that consists of a circular coil 2.2cm in diameter having 320 turns of wire, when an EM wave of frequency 810kHz transporting energy at an average rate of $1.0 \times 10^{-4} W/m^2$ passes through it? [Hint: you can use Eq. 29-4 for a generator, since it could be applied to an observer moving with the coil so that the magnetic field is oscillating with the frequency $f = \omega/2\pi$.]

Solution: Using Eq. 29-4, which says $\mathcal{E} = \mathcal{E}_0 \sin \omega t = NBA\omega \sin \omega t \Rightarrow$

$$\begin{aligned}
\mathcal{E}_{rms} &= NA\omega B_{rms} = NA\omega \sqrt{\frac{\mu_0 \bar{S}}{c}} \\
&= 320 \times \pi \times \left(1.1 \times 10^{-2} m\right)^2 \times 2\pi \times 8.1 \times 10^5 Hz \times \sqrt{\frac{(4\pi \times 10^{-7} Ns^2/C^2) \times 1.0 \times 10^{-4} W/m^2}{(3 \times 10^8 m/s)}} \\
&= 4.0 \times 10^{-4} V.
\end{aligned}$$