PHYSICS 2D | QUIZ 4 | WINTER QUARTER 2011
PROF. HIRSCH | | FEBRUARY 18th, 2011

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \to -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \to -v$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1+v/c}\ /\ \sqrt{1-v/c}$ (approaching)

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\ MeV/c^2$ Proton: $m_p = 938.26\ MeV/c^2$ Neutron: $m_n = 939.55\ MeV/c^2$

Atomic mass unit: $1\ u = 931.5\ MeV/c^2$; electron volt: $1\text{eV} = 1.6\times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67\times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda, T) = N_\lambda(\lambda)\times \overline{E}(\lambda, T) = \dfrac{8\pi}{\lambda^4}\times \dfrac{hc/\lambda}{e^{hc/\lambda k_BT} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV\,A$; $k_B = (1/11{,}600)eV\ /K$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi$, $\phi \equiv$ work function; Bragg equation: $n\lambda = 2d\sin\vartheta$

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243A$; Coulomb constant: $ke^2 = 14.4\ eV\,A$

Coulomb force: $F = \dfrac{kq_1q_2}{r^2}$; Coulomb potential: $V = \dfrac{kq}{r}$; Coulomb energy: $U = \dfrac{kq_1q_2}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Rutherford scattering: $\Delta n = C\dfrac{Z^2}{K_\alpha^2}\dfrac{1}{\sin^4(\phi/2)}$; $\hbar c = 1{,}973\ eV\,A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097\times 10^7\ m^{-1} = \dfrac{1}{911.3A}$

Bohr atom: $E_n = -\dfrac{ke^2Z}{2r_n} = -E_0\dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e(ke^2)}{2\hbar^2} = 13.6eV$; $K = \dfrac{m_e v^2}{2}$; $U = -\dfrac{ke^2Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_e ke^2} = 0.529A$; $L = m_e vr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$; Heisenberg: $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Schrodinger equation: $-\dfrac{\hbar^2}{2\text{m}}\dfrac{\partial^2\Psi}{\partial x^2} + \text{U(x)}\Psi(\text{x,t}) = \text{i}\hbar\dfrac{\partial\Psi}{\partial t}$; $\Psi(\text{x,t}) = \psi(\text{x})\text{e}^{-\text{i}\frac{\text{E}}{\hbar}t}$

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Time – independent Schrodinger equation: $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2}+\mathrm{U(x)}\psi(\mathrm{x})=\mathrm{E}\psi(\mathrm{x})$; $\int_{-\infty}^{\infty} dx\ |\psi(x)|^2=1$

∞ square well: $\psi_{\mathrm{n}}(x)=\sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n=\frac{\pi^2\hbar^2n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e}=3.81eVA^2$ (electron)

Justify all your answers to all problems. Write clearly.

<u>Problem 1</u> (10 points)

An electron at time t=0 is described by the wave packet $\psi(x)=C(e^{i(k_1+k_2)x}+e^{i(k_1-k_2)x})$ with k_1=1Å, k_2=0.1Å, and C a constant.

(a) Make a plot of $\mathrm{Re}(\psi(x))$ (the real part of $\psi(x)$). Indicate in the plot the two x-values closest to the origin where $\psi(x)$=0.
(b) Calling Δx the distance between the two x-values found in (a), give its value in Å.
(c) Find the value of Δp for this electron in units eV/c.
(d) Show that the values of Δx and Δp found in (b) and (c) are consistent with the uncertainty principle $\Delta x\Delta p\sim\hbar$.

<u>Problem 2</u> (10 points)

A particle is described by the wavefunction
$\psi(x)=C\sqrt{4-x^2}$
for x in the interval (-2,2) and $\psi(x)=0$ for x outside that interval.

(a) How much more likely is it to find this particle around x=0 than it is to find it around x=1? Find the ratio of probabilities.
(b) Find the value of the constant C.
(c) What is the probability of finding this particle at position x in the range (0,2)?
(d) What is the probability of finding this particle at position x in the range (-1,1)?

<u>Problem 3</u> (10 points)

An electron is in the ground state of a one-dimensional box. Light with wavelengths in the range 1000Å to 5000Å is incident.

(a) Find one possible value for the length of this box (in Å) that will give rise to no absorption line for wavelengths in that range. (There is an infinite number of right answers for this question of course, to get credit you need to justify your answer).
(b) Assuming <u>a single</u> absorption line is seen for wavelengths in that range, find the minimum length of this box, in Å.
(c) Assuming a single absorption line is seen for wavelengths in that range, find the maximum length of this box, in Å.

Hint: use $\hbar^2\pi^2/2m_e=37.6\ eV\,A^2$