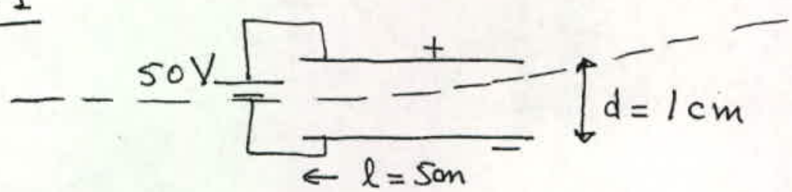
Physics 2D **Quiz 3 Solutions** Winter 2010

Problem 1



$B = 7.5 \times 10^{-4}\,T$

$F = qE + qvB$; no deflection $\Rightarrow F = 0 \Rightarrow E = vB$

$\Rightarrow v = \dfrac{E}{B}$; $E = \dfrac{V}{d} = \dfrac{50\,V}{1\,cm} = 5000\,\dfrac{V}{m}$

$$v = \frac{5000}{7.5 \times 10^{-4}}\,\frac{m}{s} \Rightarrow \boxed{v = 6.667 \times 10^{6}\,\frac{m}{s} = v_x} \text{ (a)}$$

Deflection angle:

$\tan\theta = \dfrac{v_y}{v_x}$, $v_y = a \cdot t$, $a = \dfrac{eE}{m_e}$, $t = \dfrac{l}{v_x}$

$$\Rightarrow v_y = \frac{eE}{m_e} \cdot \frac{l}{v_x} \Rightarrow v_y = \frac{e}{m_e}\,\frac{V}{d}\,\frac{l}{v_x}$$

$$\tan\theta = \frac{e}{m_e}\,\frac{V}{d}\,\frac{l}{v_x^2} = \frac{e}{m_e} \cdot \frac{5000 \cdot 5 \times 10^{-2}}{(6.667 \times 10^{6})^2}$$

with $\dfrac{e}{m_e} = 1.76 \times 10^{11}\,C/kg$

$$\tan\theta = 0.99 \Rightarrow \boxed{\theta = 44.7^\circ}$$

Problem 2

According to Rutherford's formula,

$$\frac{\Delta n(\phi_1)}{\Delta n(\phi_2)} = \frac{\sin^4(\phi_2/2)}{\sin^4(\phi_1/2)} \Rightarrow$$

$$\Rightarrow \Delta n(\phi_1) = \frac{\sin^4(\phi_2/2)}{\sin^4(\phi_1/2)} \Delta n(\phi_2); \quad \text{for } \phi_2 = 180^\circ, \sin^4 \phi_2/2 = 1;$$

for $\phi_1 = 45^\circ$, $\sin^4 \phi_1/2 = 0.0214 \Rightarrow$ for $\Delta n(\phi_2) = 100$, we have

$$\boxed{\Delta n(\phi_1) = \frac{1}{0.0214} \times 100 = 4663 \text{ particles scattered at } 45^\circ}$$

$\boxed{\text{same for Al and Au}}$

(b) Results will deviate when kinetic energy is large enough that distance of closest approach = radius of nucleus.

$$K_\alpha = \frac{2Z ke^2}{R} \; ; \quad \text{for Al, } K_\alpha = \frac{2.13.14.4}{4.9 \times 10^{-5}} eV = 7.64 MeV$$

$$\text{For Au, } K_\alpha = \frac{2.79.14.4}{7.3 \times 10^{-5}} eV = 31.2 MeV$$

So: for Al, there are deviations from Rutherford formula for $K_\alpha > 7.64$ MeV. Fewer particles are scattered at large angles $\Rightarrow$ more particles are scattered at 45° than predicted by Rutherford's formula.

For Au, same with $K_\alpha > 31.2$ MeV

Problem 3

Energy is $E_n = -E_0 \frac{Z^2}{n^2}$

For H, $Z=1$, $E^{H}_{n_H} = -\frac{E_0}{n_H^2}$; for He, $Z=2 \Rightarrow E^{He}_{n_{He}} = -\frac{4E_0}{n_{He}^2}$

If $E_{n_H} = E_{n_{He}} \Rightarrow \frac{E_0}{n_H^2} = \frac{4E_0}{n_{He}^2} \Rightarrow \boxed{n_{He} = 2n_H}$

Since $L = n\hbar$, angular momentum of He^+ electron is twice angular momentum of H electron.

(b) For He, energies are:

$E_n = -\frac{4E_0}{n^2} \Rightarrow E_1 = -54.4\,eV,\ E_2 = -13.6\,eV$, etc.

Lowest energy that can be absorbed is $E_2 - E_1 = 40.8\,eV$, corresponds to wavelength $\lambda = \frac{hc}{E_2 - E_1} = \frac{12{,}400}{40.8}\,\mathring{A} = 304\,\mathring{A}$.

Since the light has $\lambda \geq 1000\,\mathring{A}$, He^+ can't absorb any photons.

For H: $E_n = -E_0/n^2$, lowest energies are:

$E_1 = -13.6\,eV,\quad E_2 = -3.4\,eV,\ E_3 = -1.511\,eV,\ E_4 = -0.85\,eV$

$E_2 - E_1 = 10.2\,eV,\ \lambda = \frac{hc}{10.2\,eV} = \boxed{1216\,\mathring{A} \text{ is absorbed}}$

$E_3 - E_1 = 12.09\,eV,\ \lambda = \frac{hc}{12.09\,eV} = \boxed{1025.7\,\mathring{A} \text{ is absorbed}}$

$E_4 - E_1 = 12.75\,eV,\ \lambda = 972.5\,\mathring{A} < 1000\,\mathring{A} \Rightarrow$ not absorbed.

Emission: $2 \to 1 \Rightarrow \lambda = 1216\,\mathring{A},\ 3 \to 1 \Rightarrow \boxed{\lambda = 1025.7\,\mathring{A}}$

and $3 \to 2 \Rightarrow E_3 - E_2 = 1.889\,eV \Rightarrow \lambda = \frac{hc}{1.889\,eV} = \boxed{6565\,\mathring{A}}$