Physics 2D — Quiz 1 Solutions — Winter 2011

Problem 1

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) \quad ; \quad \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.866^2}} = 2$$

(a) $t' = 1$ year , $x' = 0$ $\Rightarrow$ $\boxed{t = \gamma t' = 2 \text{ years}}$

(b) Use instead:

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Need x, the position of the ship at time t. Clearly, $x = vt$

$$\Rightarrow \quad t' = \gamma\left(t - \frac{v}{c^2}vt\right) = \gamma t\left(1 - \frac{v^2}{c^2}\right) = \frac{t\,(1-v^2/c^2)}{\sqrt{1-v^2/c^2}} =$$

$$= t\sqrt{1-v^2/c^2} = \frac{t}{\gamma} \Rightarrow \boxed{t = \gamma t'}$$

Answer is same as (a), as should be since question is the same, only procedure is different.

(c) The time it takes light to reach Penelope from position $x = vt$ is:

$$t_2 = \frac{x}{c} = \frac{vt}{c} \Rightarrow \text{total time is}$$

$$t_{total} = t + t_2 = t\left(1 + \frac{v}{c}\right) = 2\text{years}\,(1 + 0.866) =$$

$$\boxed{= 3.73 \text{ years}}$$

Problem 2

(a) Put frame S' on ship A: $v = 0.6c$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}, \quad \text{with } u_x = -0.8c \Rightarrow$$

$$u'_x = \frac{-0.8c - 0.6c}{1 + 0.8 * 0.6} = -\frac{1.4}{1.48}c = \boxed{-0.946c} \quad (a)$$

= speed of B measured from A.

By symmetry, speed of A measured from B is the same.

(b) $L = \frac{L_p}{\gamma}$. For B, $\gamma = \frac{1}{\sqrt{1-0.8^2}} = \frac{5}{3} = 1.667$

$$\Rightarrow L = L_p \cdot \frac{3}{5} = 300 \cdot \frac{3}{5}\ \text{m} = \boxed{180\,\text{m}} \quad (b)$$

(c) As measured from ship A:

$$\gamma = \frac{1}{\sqrt{1-0.946^2}} = 3.085$$

$$\Rightarrow L = \frac{L_p}{\gamma} = \frac{300\,\text{m}}{3.085} = \boxed{97.25\,\text{m}} \quad (c)$$

Problem 3

Momentum conservation:

$$\boxed{\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2}$$

Energy conservation: (a)

$$\boxed{M = \gamma_1 m_1 + \gamma_2 m_2}$$

With: $\boxed{u_1 = 0.8c = \frac{4}{5}c \;;\; \gamma_1 = \frac{5}{3} \;;\; u_2 = 0.6c = \frac{3}{5}c \;;\; \gamma_2 = \frac{5}{4}}$

So:

$$\frac{\cancel{5}}{3} \cdot \frac{4}{\cancel{5}} m_1 = \frac{\cancel{5}}{4} \cdot \frac{3}{\cancel{5}} \cdot m_2 \Rightarrow \frac{4}{3} m_1 = \frac{3}{4} m_2 \Rightarrow m_2 = \frac{16}{9} m_1$$

$$M = \frac{5}{3} m_1 + \frac{5}{4} m_2 = \frac{5}{3} m_1 + \frac{5}{\cancel{4}} \cdot \frac{\overset{4}{\cancel{16}}}{9} m_1 = \frac{15}{9} m_1 + \frac{20}{9} m_1 = \frac{35}{9} m_1$$

(b)

$$\Rightarrow \boxed{m_1 = \frac{9}{35} M} \qquad m_2 = \frac{16}{9} \cdot \frac{9}{35} M \Rightarrow \boxed{m_2 = \frac{16}{35} M}$$

Mass deficit: $\Delta M = M - m_1 - m_2 = M - \frac{25}{35} M \Rightarrow \boxed{\Delta M = \frac{10}{35} M}$

Kinetic energies:

$$\frac{K_1}{c^2} = (\gamma_1 - 1) m_1 = \left(\frac{5}{3} - 1\right) \frac{9}{35} M = \frac{2}{\cancel{3}} \cdot \frac{\overset{3}{\cancel{9}}}{35} M = \frac{6}{35} M$$

$$\frac{K_2}{c^2} = (\gamma_2 - 1) m_2 = \left(\frac{5}{4} - 1\right) \frac{16}{35} M = \frac{1}{\cancel{4}} \cdot \frac{\overset{4}{\cancel{16}}}{35} M = \frac{4}{35} M$$

Hence: $\boxed{\frac{K_1 + K_2}{c^2} = \frac{6}{35} M + \frac{4}{35} M = \frac{10}{35} M = \Delta M}$ (c)