Physics 2BL Homework Set 05

Taylor Problems: 8.6, 8.10, 8.24

8.6: $A = \dfrac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{N\Sigma x^2 - (\Sigma x)^2}$, $B = \dfrac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$, $\Delta = N\Sigma x^2 - (\Sigma x)^2$

$$\Delta = 2(x_1^2 + x_2^2) - (x_1 + x_2)^2 = 2x_1^2 + 2x_2^2 - x_1^2 - x_2^2 - 2x_1x_2 = x_1^2 + x_2^2 - 2x_1x_2 = (x_1 - x_2)^2$$

$$A = \frac{(x_1^2 + x_2^2)(y_1 + y_2) - (x_1 + x_2)(x_1y_1 + x_2y_2)}{(x_1 - x_2)^2} = \frac{x_1^2y_2 + x_2^2y_1 - x_1x_2y_2 - x_1x_2y_1}{(x_1 - x_2)^2}$$

$$B = \frac{2(x_1y_1 + x_2y_2) - (x_1 + x_2)(y_1 + y_2)}{(x_1 - x_2)^2} = \frac{x_1y_1 + x_2y_2 - x_1y_2 - x_2y_1}{(x_1 - x_2)^2}$$

Now check to see if $y_1 = A + Bx_1$ & $y_2 = A + Bx_2$ by plugging x_1 & x_2 into $y = A + Bx$:

(x_1,y_1): $$y = A + Bx_1 = \frac{x_1^2y_2 + x_2^2y_1 - x_1x_2y_2 - x_1x_2y_1 + x_1^2y_1 + x_1x_2y_2 - x_1^2y_2 - x_1x_2y_1}{(x_1 - x_2)^2}$$

$$= \frac{(x_1^2 - 2x_1x_2 + x_2^2)y_1}{(x_1 - x_2)^2} = \frac{(x_1 - x_2)^2}{(x_1 - x_2)^2}y_1 = y_1 \quad \text{So } y_1 = A + Bx_1$$

(x_2,y_2): $$y = A + Bx_2 = \frac{x_1^2y_2 + x_2^2y_1 - x_1x_2y_2 - x_1x_2y_1 + x_1x_2y_1 + x_2^2y_2 - x_1x_2y_2 - x_2^2y_1}{(x_1 - x_2)^2}$$

$$= \frac{(x_1^2 - 2x_1x_2 + x_2^2)y_2}{(x_1 - x_2)^2} = \frac{(x_1 - x_2)^2}{(x_1 - x_2)^2}y_2 = y_2 \quad \text{So } y_2 = A + Bx_2$$

Thus, the least-squares line passes through both (x_1,y_1) & (x_2,y_2).

8.10: $(x_1,y_1) = (1,2)$, $(x_2,y_2) = (2,3)$, $(x_3,y_3) = (3,2)$
$w_1 = 1/0.5^2 = 4$, $w_2 = 1/0.5^2 = 4$, $w_3 = 1/1^2 = 1$

(A) Weighted: $\Sigma wx^2 = 29$, $\Sigma wy = 22$, $\Sigma wx = 15$, $\Sigma wxy = 38$

$$\Delta = \Sigma w \Sigma wx^2 - (\Sigma wx)^2 = (9)(29) - 15^2 = 36$$

$$A = \frac{\Sigma wx^2 \Sigma wy - \Sigma wx \Sigma wxy}{\Delta} = \frac{(29)(22) - (15)(38)}{36} = 1.889$$

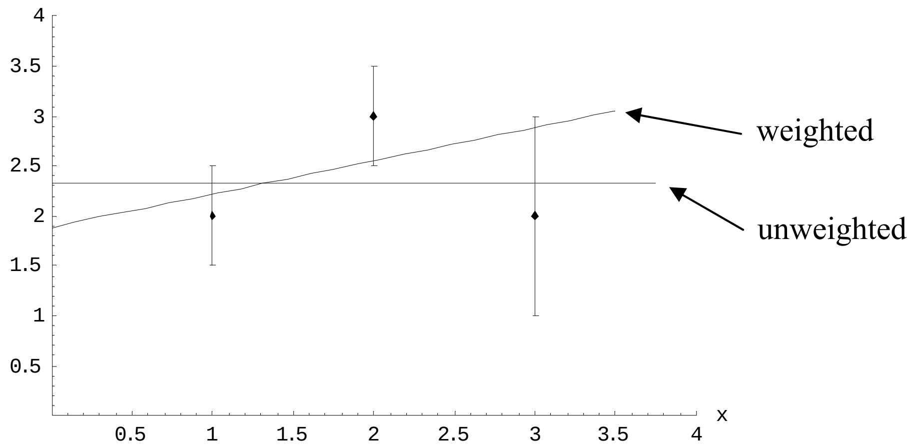
$$B = \frac{\Sigma w \Sigma wxy - \Sigma wx \Sigma wy}{\Delta} = \frac{(9)(38) - (15)(22)}{36} = 0.333$$

(B) Unweighted: $\Sigma x^2 = 14$, $\Sigma y = 7$, $\Sigma x = 6$, $\Sigma xy = 14$, $N=3$

$$\Delta = N\Sigma x^2 - (\Sigma x)^2 = (3)(14) - 6^2 = 6$$

$$A = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{\Delta} = \frac{(14)(7)-(6)(14)}{6} = 2.333$$

$$B = \frac{N\Sigma xy - \Sigma x \Sigma y}{\Delta} = \frac{(3)(14)-(6)(7)}{6} = 0$$



* The weighted line yields the best fit through the error bars.
* The unweighted line yields the best fit through the data points.

8.24: $y = A \sin(\omega t) + B \cos(\omega t)$, $\omega = 10$ rad/s

"x": Time t (0.1sec)	-4	-2	0	2	4
"y": Position y (cm)	3	-16	6	9	-8

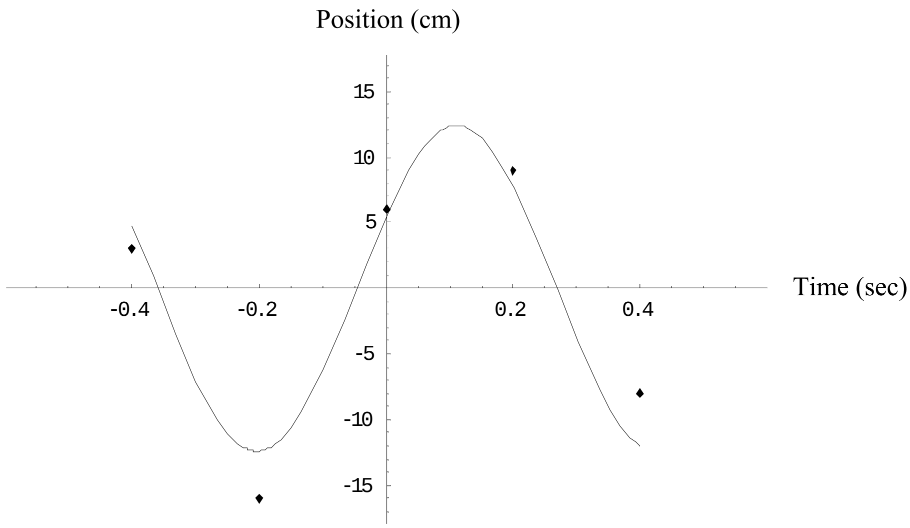
$$A \Sigma [f(x_i)]^2 + B \Sigma f(x_i)g(x_i) = \Sigma y_i f(x_i) \qquad (8.41)$$
$$A \Sigma f(x_i)g(x_i) + B \Sigma [g(x_i)]^2 = \Sigma y_i g(x_i)$$

Let $f(t_i) = \cos(\omega t_i)$ & $g(t_i) = \sin(\omega t_i)$

Time (sec)	Position (cm)	ω (rad·sec)	f(t)=cos(ωt)	g(t)=sin(ωt)	$[f(x_i)]$^2	$[g(x_i)]$^2	$f(x_i)g(x_i)$	$y_i f(x_i)$	$y_i g(x_i)$
-0.4	3	10	-0.6536436	0.75680249	0.42725	0.57275	-0.49468	-1.96093	2.27040
-0.2	-16	10	-0.4161468	-0.9092974	0.17317	0.82682	0.37840	6.65834	14.5487
0	6	10	1	0	1	0	0	6	0
0.2	9	10	-0.4161468	0.90929742	0.17317	0.82682	-0.3784	-3.74532	8.18367
0.4	-8	10	-0.6536436	-0.756802	0.42725	0.57275	0.49467	5.22914	6.05442
		SUM	-1.1395809	0	2.20085	2.79914	0	12.1812	31.0572

$A \cdot 2.20085 + B \cdot 0 = 12.1812 \rightarrow A = 5.53477$

$A \cdot 0 + B \cdot 2.79914 = 31.0572 \rightarrow B = 11.0953$



For 3 of 5 data points, the fit is pretty good with "a couple of centimeters" as the tolerance. However, the fit is not as good near the peaks.