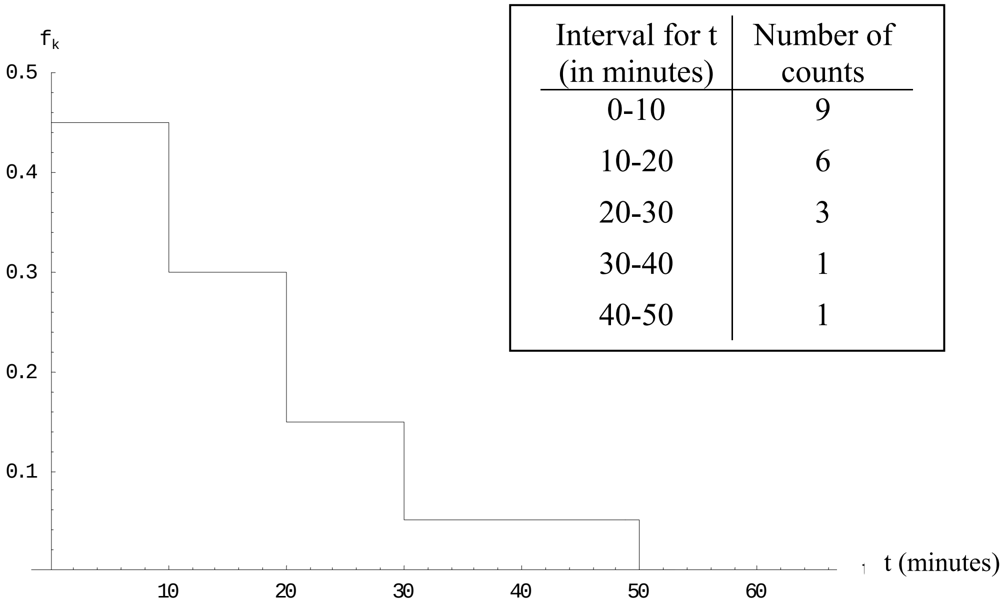
Physics 2BL Homework Set 03

Taylor Problems: 5.2, 5.6, 5.20, 5.36

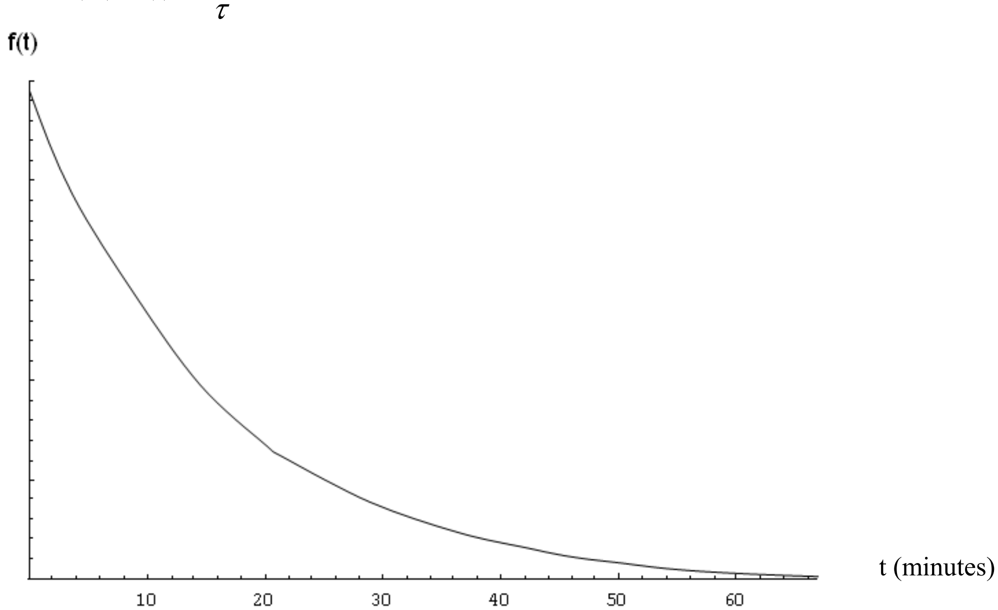
5.2:



Interval for t (in minutes)	Number of counts
0-10	9
10-20	6
20-30	3
30-40	1
40-50	1

5.6:

(A) $f(t) = \frac{1}{\tau} e^{-t/\tau}$



(B) Eqn. 5.13 states: $\int_{-\infty}^{\infty} f(t)dt = 1$

We are told f(t) = 0 for t < 0 in our case, so

$\int_{-\infty}^{\infty} f(t)dt = \int_{0}^{\infty} f(t)dt = \int_{0}^{\infty} (1/\tau)Exp(-t/\tau)dt$ Let u = -t/ τ, du = -(1/ τ)dt → dt = - τ du

$\int_{-\infty}^{\infty} f(t)dt = \int_{0}^{-\infty} (-\tau/\tau)e^{u}du = -\int_{0}^{-\infty} e^{u}du = e^{0} - e^{-\infty} = 1 - 0 = 1$, as expected

(C) Eqn. 5.15 states: $\bar{t} = \int_{-\infty}^{\infty} tf(t)dt$

Since f(t) = 0 for t < 0, we have $\int_{-\infty}^{\infty} tf(t)dt = \int_{0}^{\infty} tf(t)dt$

$\int_{-\infty}^{\infty} tf(t)dt = \int_{0}^{\infty} (t/\tau)Exp(-t/\tau)dt$ Let u = -t/ τ, du = -(1/ τ)dt → dt = - τ du

$$= \int_{0}^{-\infty} ue^{u}(-\tau)du = -\tau\int_{0}^{-\infty} ue^{u}du = \tau\int_{-\infty}^{0} ue^{u}du$$

$$= \tau\left[ue^{u}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} e^{u}du \right] = \tau\left[0 - 0 - e^{u}\Big|_{-\infty}^{0} \right] = \tau[1-0] = \tau$$

<u>5.20</u>: Mean height $\bar{h}$ = 69”, σ = 2”

(A) How many men between 67” and 71” = $\bar{h} \pm 1\sigma$

68% should be within the range of $\bar{h} \pm 1\sigma$.
→ N = 1000 · 68% = 680 men

(B) How many men taller than 71” = $\bar{h} + 1\sigma$

If 68% should be within the range of $\bar{h} \pm \sigma$, 32% should be outside that range.
Assuming it is symmetric, 16% should be lower than $\bar{h} - \sigma$ and 16% should be above $\bar{h} + \sigma$.
→ N = 1000 · 16% = 160 men

(C) How many men taller than 75” = $\bar{h} + 3\sigma$

For same reasons as above, half of the men outside the range of $\bar{h} \pm 3\sigma$
→ N = (1/2) (1-0.997) · 1000 = 1.5 ≈ 2 men

(D) How many men between 65” and 67” = $(\bar{h} - 2\sigma)$ and $(\bar{h} - \sigma)$

How many inside $\bar{h} + 2\sigma$ = (0.9545) 1000 = 954.5
How many inside $\bar{h} + \sigma$ = (0.68) 1000 = 680
(1/2) (955 – 680) = 137.5 ≈ 138 men

5.36: $x_A = 13 \pm 1$, $x_B = 15 \pm 1$

(A) $|x_A - x_B| = 2 \pm \sqrt{1^2 + 1^2} = 2 \pm 1.4$

(B) $t = \dfrac{|x_A - x_B|}{\sigma} = \dfrac{2}{1.4} = 1.4$

Prob(outside 1.4 σ) = 1 – Prob(within 1.4 σ)

= 1 – 0.8385 ≈ 0.16

Prob(outside 1.4 σ) = 16%

Assuming 5% tolerance, this discrepancy is not significant.